The Exact (ρ,θ)-Hough Transform -- Definition and Performance

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The Exact (ρ,&thetas;)-Hough Transform: Definition and Performance

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The (ρ,&thetas;)-interpretation of Hough Transform (HT) is a well-known projective technique that is often used in image processing considering its facilities to localize long stretched objects in a given image. In our earlier work, a definition of "exact HT" has been introduced for both given grids (XsizexYsize) and (PsizexΘsize) of the input image and of the HT result, respectively, considering the (ρ,&thetas;)-HT like a Radon transform (RT). A few iterative approaches have been also proposed there to approximate the exact HT for reducing the performance complexity to not much bigger one than cubic. However, these approaches become less acceptable with increasing of the input image grid as in the case of astronomical images, e.g. when HT is applied for identification of flare stars in archive images of stellar chains. In this work an analytic solution is proposed for distribution of the exact (ρ,&thetas;)-HT model into the chosen HT-grid of the output.

Contents

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3. The exact (ρ,θ) HT definition

4. The exact (ρ,θ) HT performance

5. Discussion & Conclusion

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Hough transform to stress on elongated objects in images

[2, 7, 8, 9, 12]

- The original HT, by Hough (1961):

  a line $L(x,y|k,b)$: $(y=kx+b)$ in the object (input image) space ⇔ the point $L_{HT}(k,b)$ of the HT parameter space;

  A line (as a continuum of points) <=> a point (as a continuum of lines crossing it)

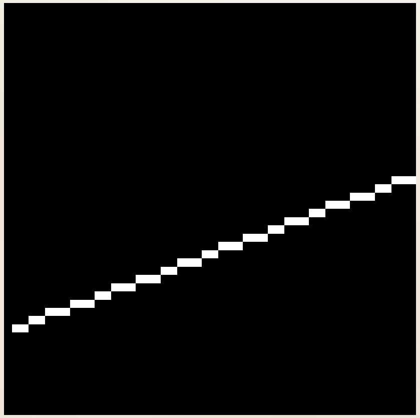
  Troubles with HT representation of vertical lines ($k \to \pm\infty$)

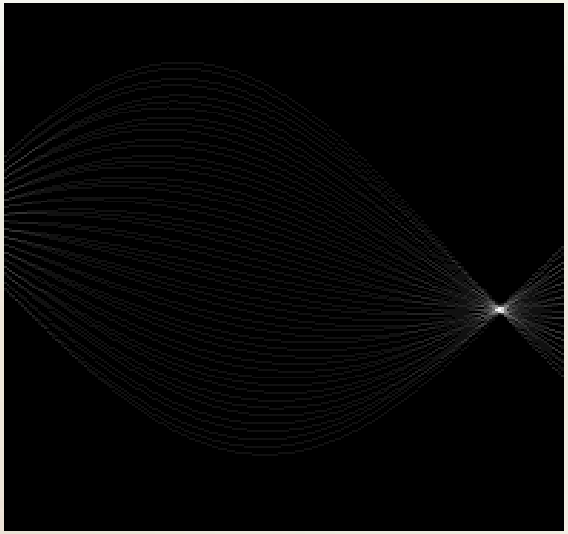
- The (ρ,θ)HT, a modification of HT by Duda & Hart (1982):

  based on the normal equation of a line: $L(x,y|\rho,\theta)$: $x.\cos(\theta)+y.\sin(\theta)=\rho$

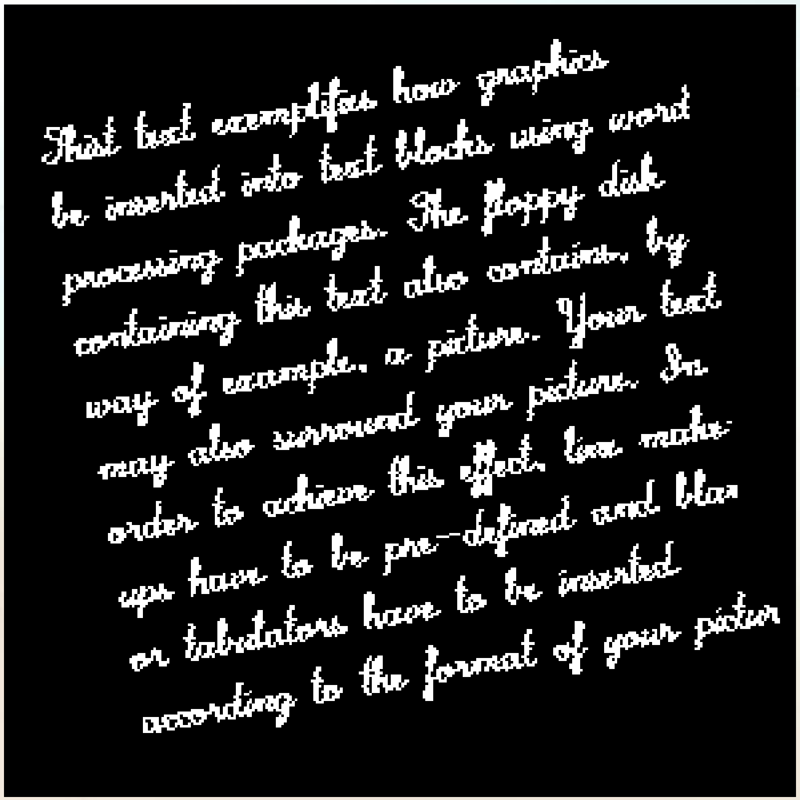
  A line (as a continuum of points) <=> a point (as a continuum of sinusoids crossing it)

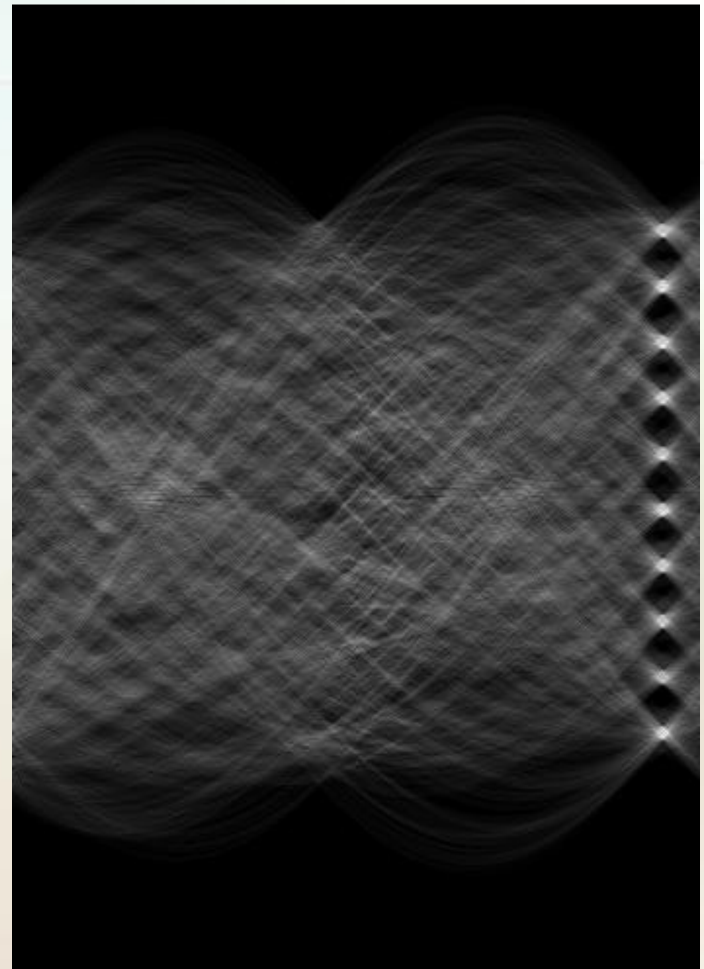
  ($|\rho| \le$ Diag(image), $-\pi/2 < \theta \le \pi/2$)



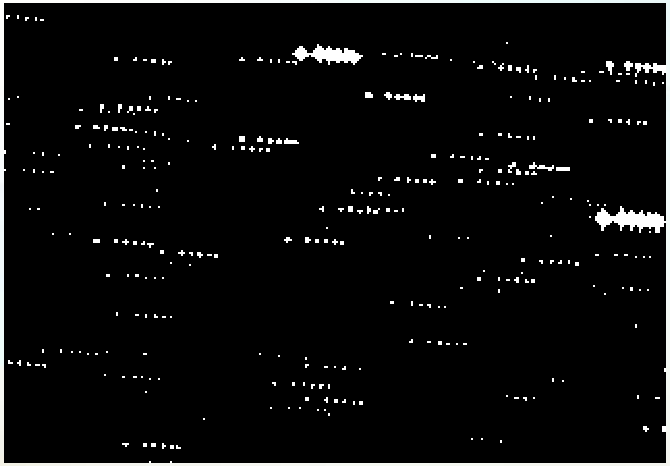


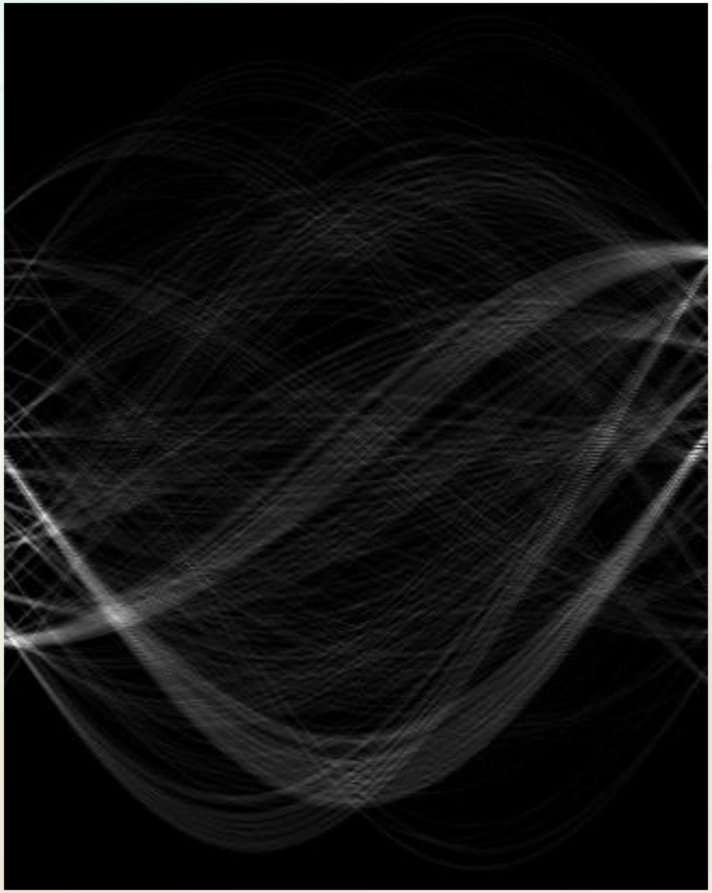
(ρ,θ)HT to recognize the slope of text rows in an image
(the most popular application of HT) [3, 5, 6, 7, 9]





Our case: (ρ,θ)HT to recognize stellar chains in an image **[1, 4]**





Segmentation of stellar chains, obtained by a specific astronomical method: multiple photo expositions (combined over one and the same photo plate) of the observed sky quadrant.

This is a problems to be solved in our project on Astroinformatics

…Hough transform to stress on elongated objects in images

- The traditional algorithm for (ρ,θ) HT performance: it transforms each image pixel and accumulate its HT representation (a cosinusoid) into HT space. To increase the representation precision you have to increase the HT space (i.e. array) size. And the processing complexity is ~ $X_{max}.Y_{max}.\theta_{max} \sim N^3$ *(!) A new approach is obviously necessary to speed up but to make more precise too [3, 8, 9, 11, 12, 13]*

*An **exact HT** can be defined using the fact that:*

- The (ρ,θ)HT is equivalent to the transform of Radon (1917): [7, 9, 11,13]

$$h(\rho,\theta) = \iint_{RoI} f(x,y)\,\delta\big(x\cos(\theta) + y\sin(\theta) - \rho\big)\,\mathrm{d}x\mathrm{d}y$$

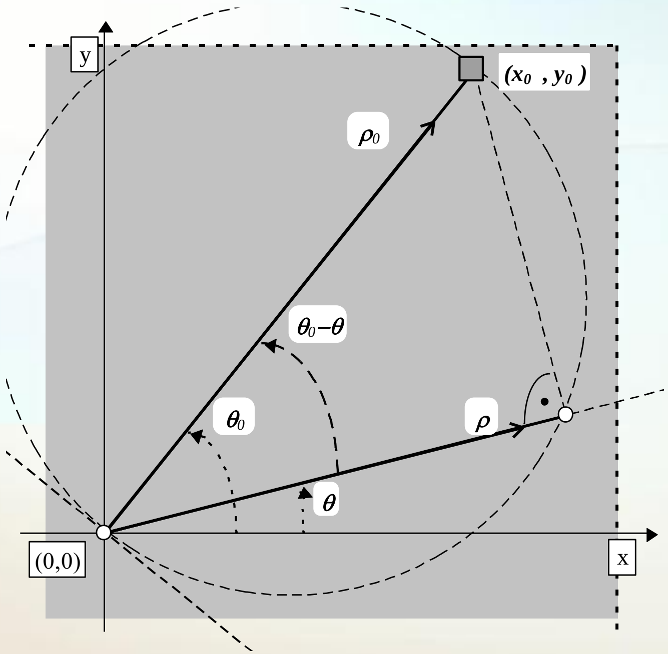
$$(\rho,\theta) \in \boldsymbol{RoHT}$$

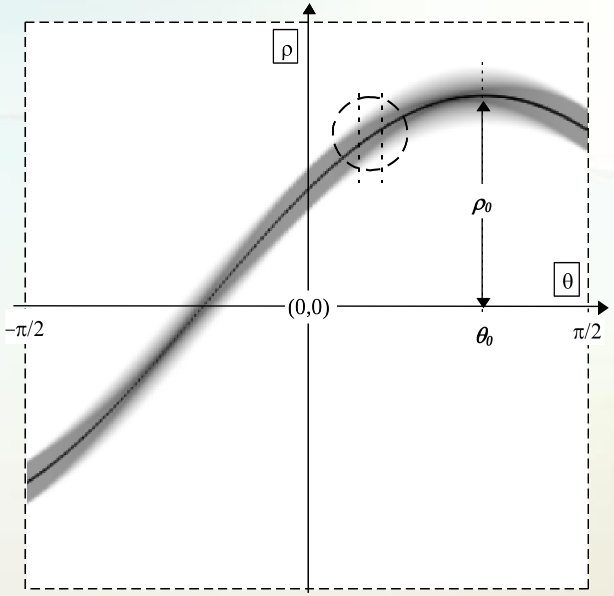
where ***RoI*** is the definition domain of the image $f = f(x,y)$, $(x,y) \in$ ***RoI***; ***RoHT*** is the definition domain for HT of the image, i.e. for $h = h(\rho,\theta)$; and $\delta(.)$ is the Dirac's function.

- Image processing uses direct (ρ,θ)HT (i.e. RT), while computer tomography – the inverse RT (i.e. $(\rho,\theta)HT^{-1}$)

Exact HT performance for the both given grids:

(the input grid and the chosen grid for HT output)



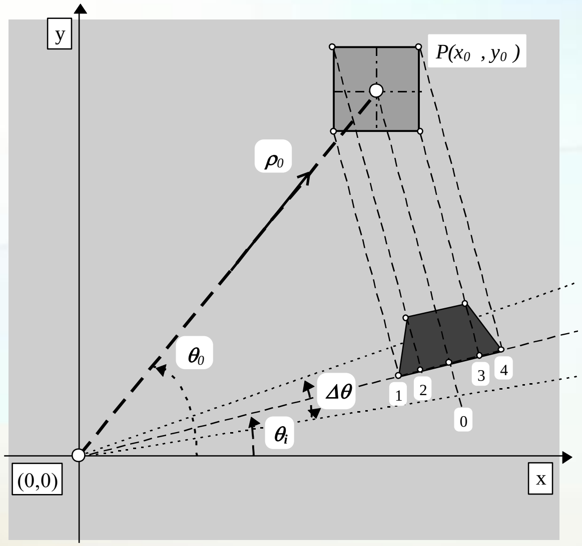


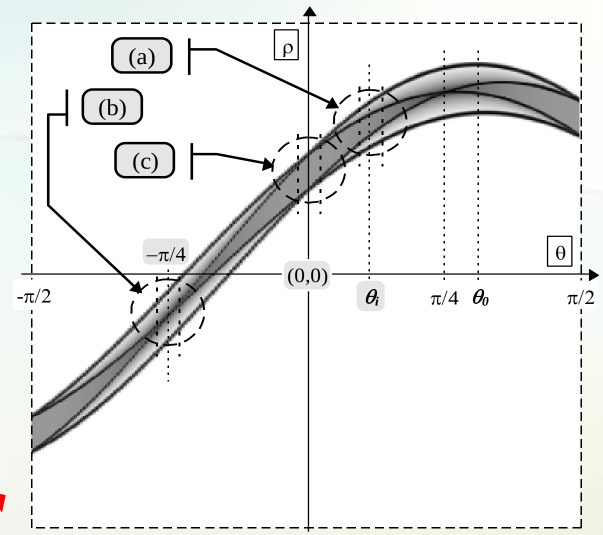
Geometric interpretation of the correspondence:

$$(x_0, y_0) \in \boldsymbol{RoI} \Leftrightarrow$$
$$\Leftrightarrow \{(\rho,\theta)\in \boldsymbol{RoHT} | \rho=\rho_0\cos(\theta_0-\theta)\}$$

The HT-image of a "real" pixel $P(x_0, y_0)$ of dimensions $[nx,\ n\Delta y]$, (n positive integer) relatively to the HT-image of its centre (x_0, y_0), i.e. to a "real" point of dimensions $[\Delta x,\ \Delta y]$.

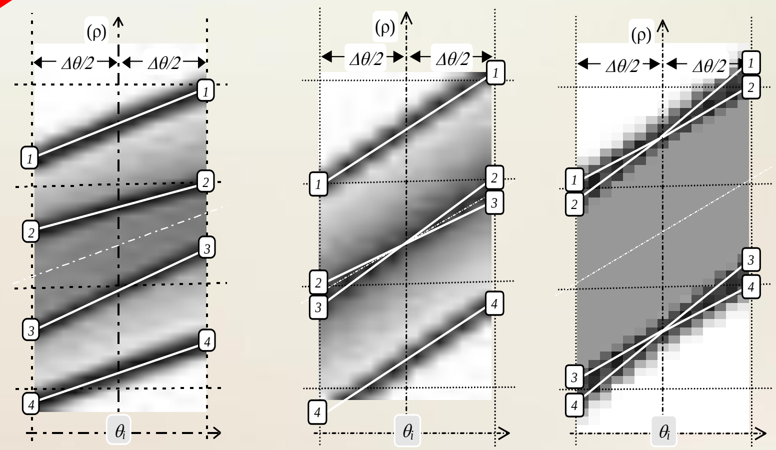
...Exact HT performance: representation of an image pixel





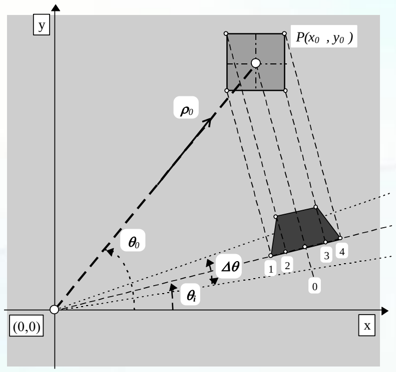
A real pixel, a projection of it, and

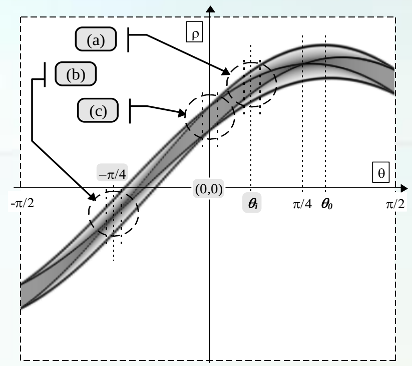
its **Generalised Cosinusoide (GC):**
i.e. the Cosine-shape representing a given real pixel and the 3 basic types for the shape edges (cut vertically in the HT space)
The shape edges are Cosinusoides.



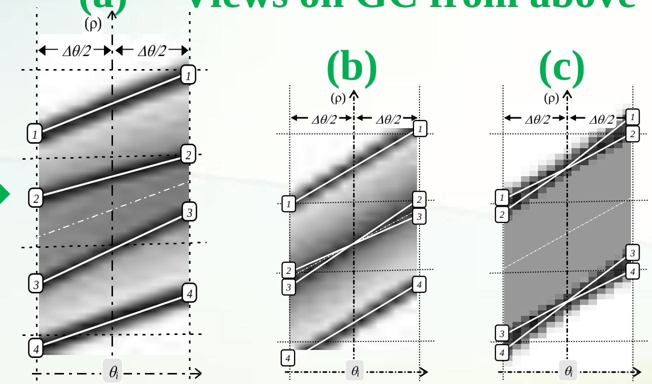
…Exact HT performance: the Cosine-shape by regular parts

(been cut by vertical strips of HT accumulation space)





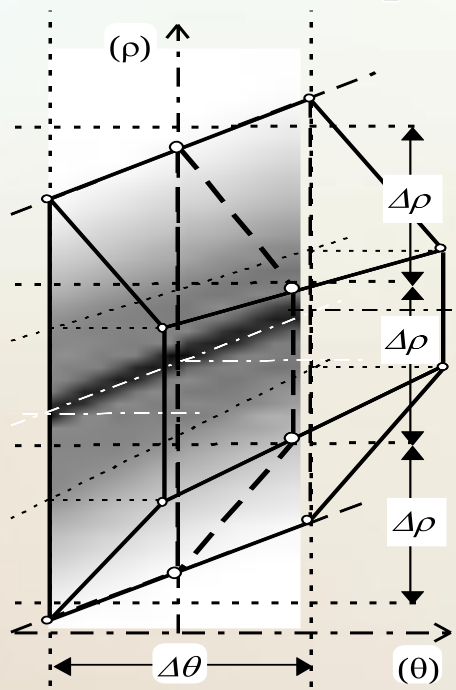
(a) Views on GC from above

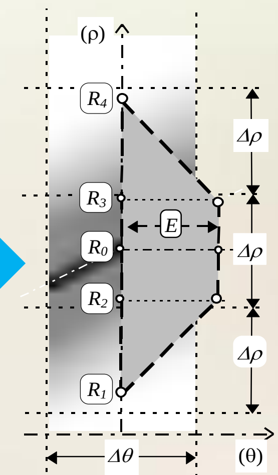


The **“Trapezium-like Cosine-shaped Hexahedron” (TC6) that is cut by a vertical strip of a HT-pixel width.**

Each TC6 vertical section is a symmetrical trapezium (ST) of constant area (=$f(x,y).\Delta x.\Delta y/\Delta\rho$)**.**

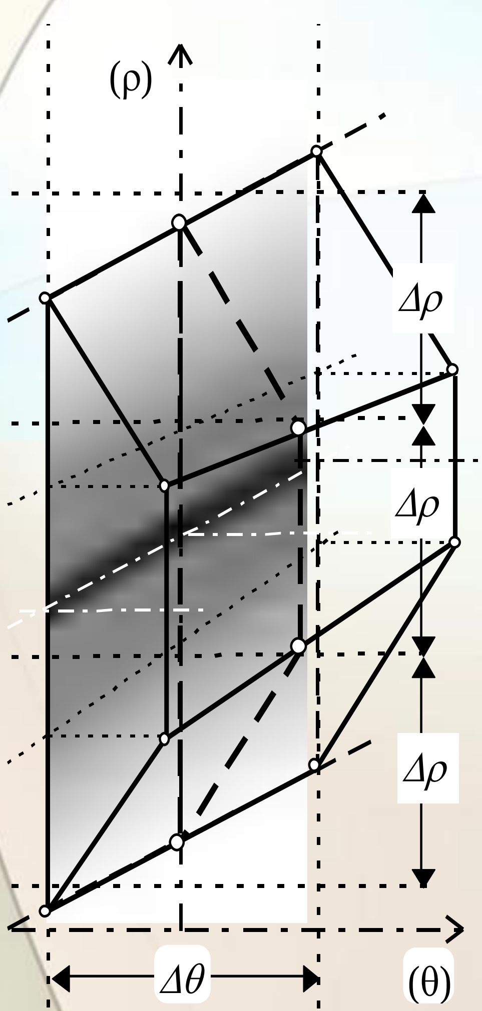
The general case of TC6 edges’ co-location is illustrated.





...Exact HT performance: TC6 contribution to HT pixels

(approximated and analytical solution)



Each TC6 of given Cosine-shape contributes for the HT value of the HT-pixels covered by this TC6 (they are 5 in the illustration).

Respective sub-volumes of TC6 have to be calculated for to obtain the "*exact HT*" (pixel by pixel in the HT space).

Possible ways of calculation:

-(approximating solution): dividing the TC6 vertically by *K* strips of equal width for each HT-pixel covered by this TC6.

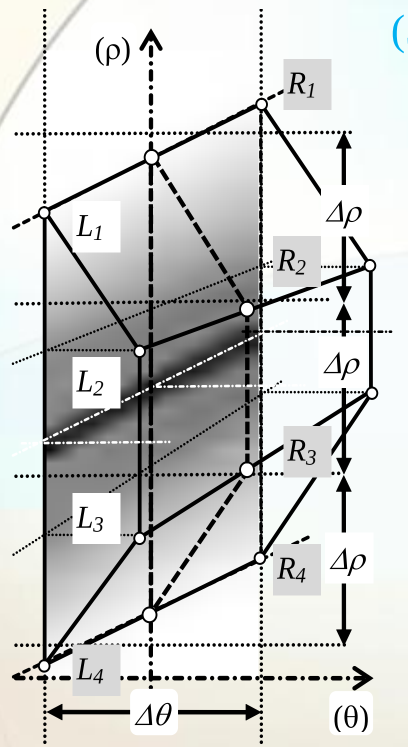
processing complexity ~ $(X_{max}.Y_{max}.\theta_{max})\ K \sim K.N^3$

- (analytical solution): ? cases of calculus

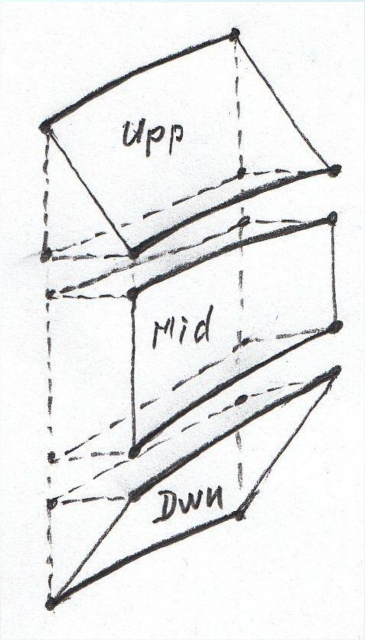
processing complexity ~ $X_{max}.Y_{max}.\theta_{max} \sim N^3$

programming complexity is higher (comparatively to approximated solution)

… The exact HT performance: an analytical solution



(5 HT-pixels are covered by the TC6 in this illustration)



3 main vertical components (counted top-down):

- upper pentahedron **Upp**=$\{R_1, R_2, L_1, L_2\}$,
- middle hexahedron **Mid**=$\{R_2, R_3, L_2, L_3\}$, and
- bottom pentahedron **Dwn**=$\{R_3, R_4, L_3, L_4\}$.

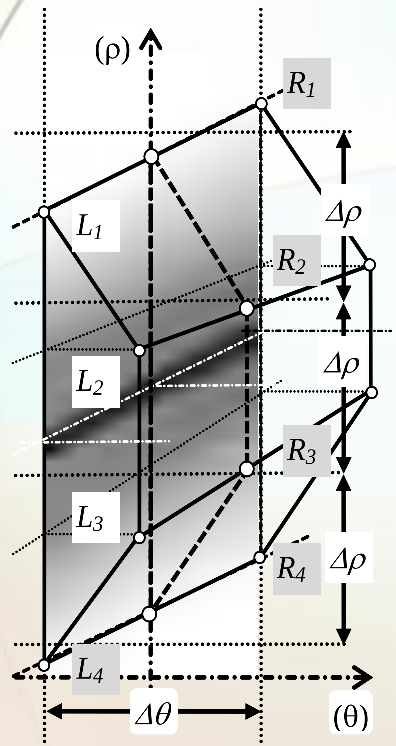
Each component will appear in 3 possible *types* according to the vertical position of the respective pairs of vertices, (L_i, L_{i+1}) and (R_i, R_{i+1}), i=1,2,3, and each type will have 4 possible appearances (2 horizontally symmetric and 2 vertically symmetric):

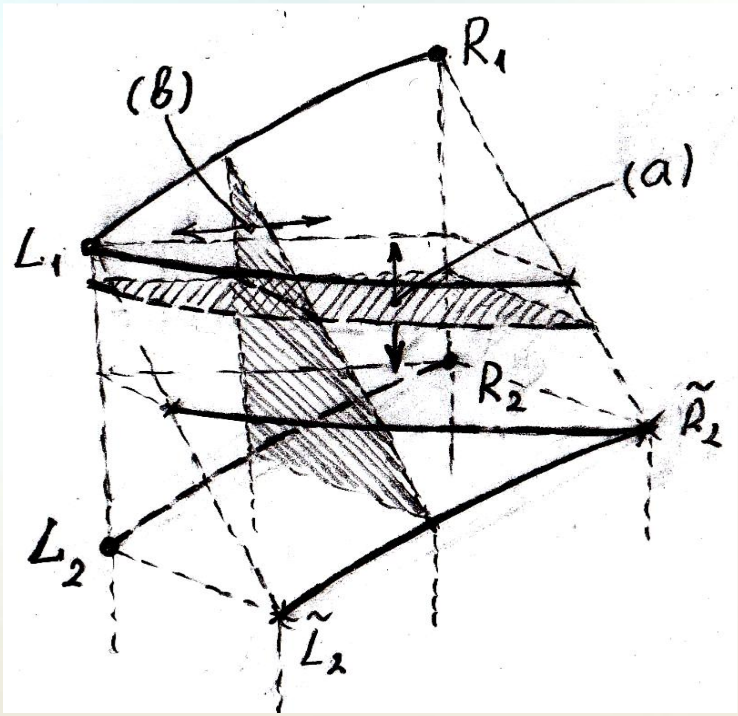
- a (**HalfAltern**) type: $_{(i)}(RLRL)_{(i+1)}$ and $_{(i)}(LRLR)_{(i+1)}$, also $_{(i+1)}(RLRL)_{(i)}$ and $_{(i+1)}(LRLR)_{(i)}$;
- a (**Alternative**) type: $_{(i)}(RRLL)_{(i+1)}$ and $_{(i)}(LLRR)_{(i+1)}$, also $_{(i+1)}(RRLL)_{(i)}$ and $_{(i+1)}(LLRR)_{(i)}$;
- a (**Central**) type: $_{(i)}(RLLR)_{(i+1)}$ and $_{(i)}(LRRL)_{(i+1)}$, also $_{(i+1)}(RLLR)_{(i)}$ and $_{(i+1)}(LRRL)_{(i)}$.

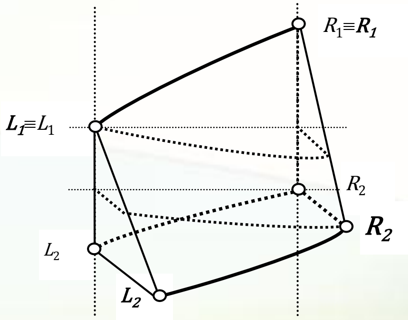
Combining the 3 components (Upp, Mid, Dwn) and their 12 appearances (see them above) we have a total of 36 possible cases; 9 of them (*basic varieties*) are illustrated in Table 1, and the remaining 27 can be got by horizontal and/or vertical symmetries among these 9 basic varieties. I.e. we need only 9 *basic computing modules* for the integration of all TC6-s.

… The exact HT performance: an analytical solution

(5 HT-pixels are covered by the TC6 in this illustration)





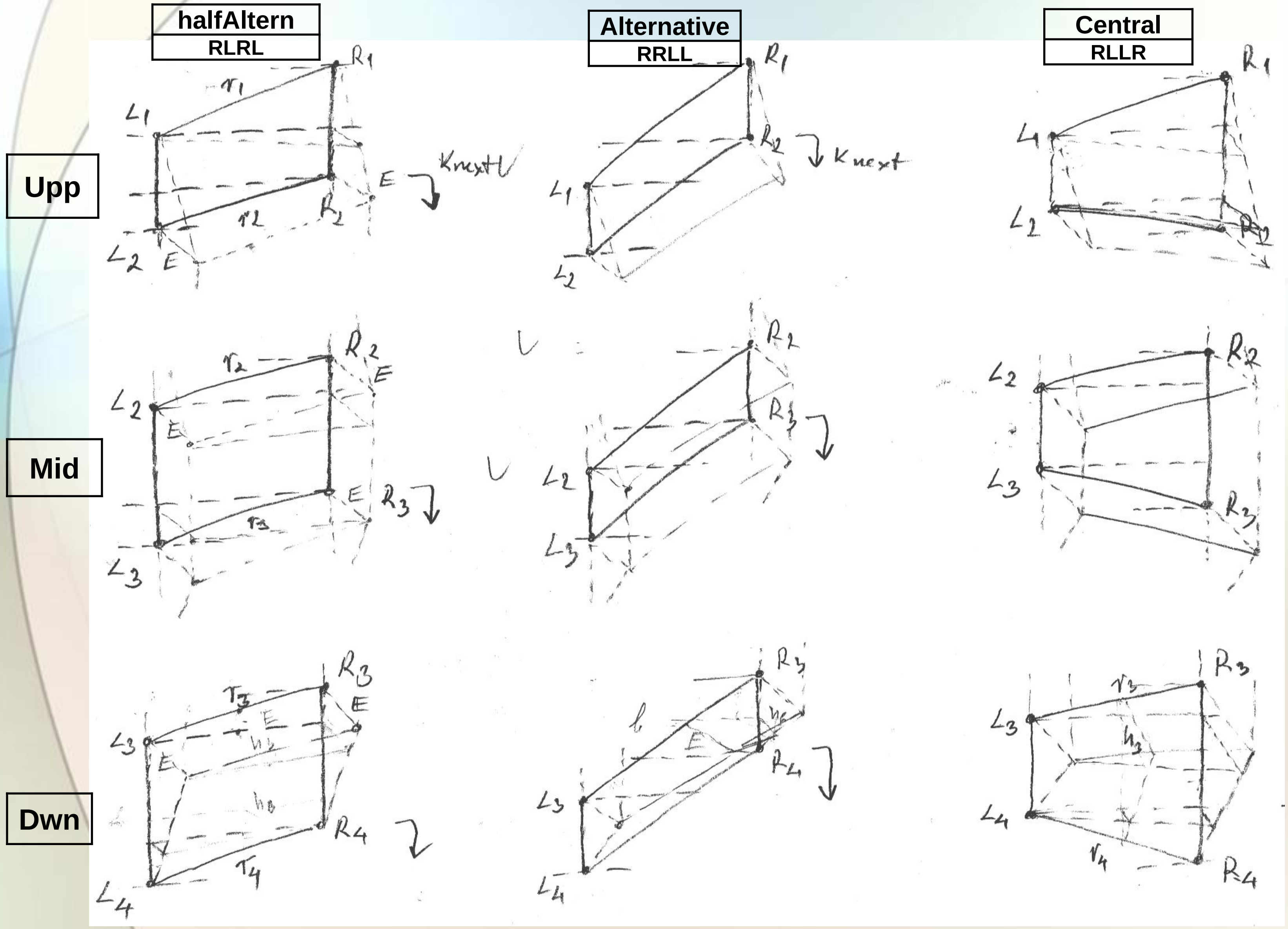


How to integrate:

-(a) "horizontal plane-cut ↔ vertical integration“ , bad choice (many non-linearities)

- (b) "vertical plane-cut ↔ horizontal integration" that we chose (!)

... The exact HT performance: 9 basic varieties to compute



... The exact HT performance:

Table 2. Volume calculus for the 3 basic types of TC6 (Upper) components

(halfAltern) RLRL		
RL	$V(b)=\left(b^2 I_{U1}(q)-2bI_{U2}(q)+I_{U3}(q)\right)\Big\|_{q=\alpha}^{q=\theta_R}$	$R_1 \geq b > L_1$, $\theta_L \leq \alpha(b) < \theta_R$, $\theta_R - \theta_L = \Delta_\theta$
LR	$V(b)=(b-L_1)\left((b+L_1)I_{U1}(q)-2I_{U2}(q)\right)\Big\|_{q=\theta_L}^{q=\theta_R}$	$L_1 \geq b > R_2$, $\theta_R - \theta_L = \Delta_\theta$
RL	$V(b)=(b-R_2)\left((b+R_2)I_{U1}(q)-2I_{U2}(q)\right)\Big\|_{q=\theta_L}^{q=\alpha}+$ $+\left(R_2 I_{U4}(q)-I_{U5}(q)+I_{U6}(q)-R_2 I_{U7}(q)-R_2^2 I_{U1}(q)+R_2 I_{U2}(q)\right)\Big\|_{q=\alpha}^{q=\theta_R}$	$R_2 \geq b > L_2$, $\theta_L \leq \alpha(b) < \theta_R$, $\theta_R - \theta_L = \Delta_\theta$
(Alternative) RRLL		
RR	$V(b)=\left(b^2 I_{U1}(q)-2bI_{U2}(q)+I_{U3}(q)\right)\Big\|_{q=\alpha}^{q=\theta_R}$	$R_1 \geq b > R_2$, $\theta_L \leq \alpha(b) < \theta_R$, $\theta_R - \theta_L = \Delta_\theta$
RL	$V(b)=\left(b^2 I_{U1}(q)-2bI_{U2}(q)+I_{U3}(q)\right)\Big\|_{q=\alpha}^{q=\beta}+\left(I_{U6}(q)-I_{U8}(q)\right)\Big\|_{q=\beta}^{q=\theta_R}$ $-\left(R_2^2 I_{U1}(q)-2R_2 I_{U2}(q)+I_{U3}(q)\right)\Big\|_{q=\theta_{R21}}^{q=\theta_R}$	$R_2 \geq b > L_1$, $\theta_L \leq \alpha(b) \leq \beta(b) < \theta_R$, $\theta_L \leq \theta_{R12} < \theta_R$, $\theta_R - \theta_L = \Delta_\theta$
LL	$V(b)=(b-L_1)\left((b+L_1)I_{U1}(q)-2I_{U2}(q)\right)\Big\|_{q=\theta_L}^{q=\alpha}+$ $+\left(L_1 I_{U4}(q)-I_{U5}(q)+I_{U6}(q)-L_1 I_{U7}(q)-L_1^2 I_{U1}(q)+L_1 I_{U2}(q)\right)\Big\|_{q=\alpha}^{q=\theta_{L12}}$	$L_1 \geq b > L_2$, $\theta_L \leq \alpha(b) < \theta_R$, $\theta_L \leq \theta_{L12} < \theta_R$, $\theta_R - \theta_L = \Delta_\theta$
(Central) RLLR		
RL	$V(b)=\left(b^2 I_{U1}(q)-2bI_{U2}(q)+I_{U3}(q)\right)\Big\|_{q=\alpha}^{q=\theta_R}$	$R_1 \geq b > L_1$, $\theta_L \leq \alpha(b) < \theta_R$, $\theta_R - \theta_L = \Delta_\theta$
LL	$V(b)=(b-L_1)\left((b+L_1)I_{U1}(q)-2I_{U2}(q)\right)\Big\|_{q=\theta_L}^{q=\theta_R}$	$L_1 \geq b > L_2$, $\theta_L < \theta_R$, $\theta_R - \theta_L = \Delta_\theta$
LR	$V(b)=(b-L_2)\left((b+L_2)I_{U1}(q)-2I_{U2}(q)\right)\Big\|_{q=\alpha}^{q=\theta_R}+$ $+\left(L_2 I_{U4}(q)-I_{U5}(q)+I_{U6}(q)-L_2 I_{U7}(q)-L_2^2 I_{U1}(q)+L_2 I_{U2}(q)\right)\Big\|_{q=\theta_L}^{q=\alpha}$	$L_2 \geq b > R_2$, $\theta_L \leq \alpha(b) < \theta_R$, $\theta_R - \theta_L = \Delta_\theta$

... The exact HT performance:

Table 2a. Solutions of basic integrals used in Table 2.

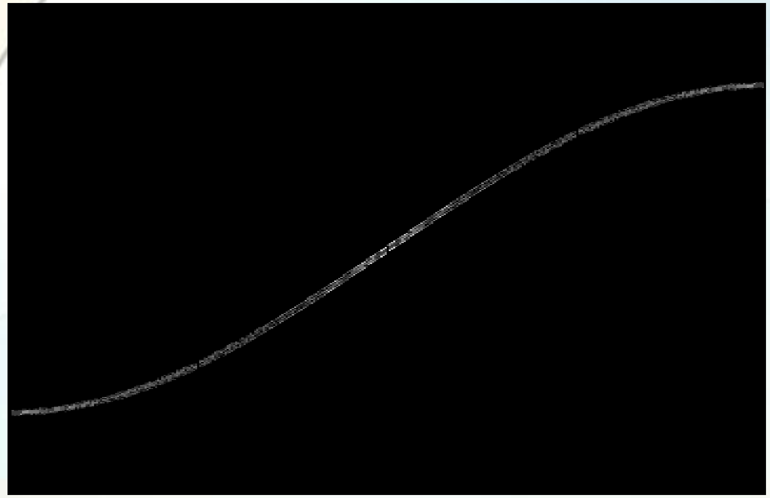
Integral	Pattern	Result formulae	A coefficient to multiply the final result if $q \in \ldots$			
			$(-\pi/2, -\pi/4]$	$(-\pi/4, 0]$	$(0, \pi/4]$	$(\pi/4, \pi/2]$
$I_{U1}(q)$	$\int \frac{dq}{(r_2 - r_1)2M}$	$\ln \lvert \mathrm{tg}(q) \rvert$	$-\lambda\Delta_\rho$	$-\lambda\Delta_\rho$	$\lambda\Delta_\rho$	$\lambda\Delta_\rho$
$I_{U2}(q)$	$\int \frac{r_1 dq}{(r_2 - r_1)2M}$	$x_1 \ln \left\lvert \mathrm{tg}\,\frac{q}{2} \right\rvert + y_1 \ln \left\lvert \left(1 + \mathrm{tg}\,\frac{q}{2}\right) \Big/ \left(1 - \mathrm{tg}\,\frac{q}{2}\right) \right\rvert$	$-\lambda$	$-\lambda$	λ	λ
$I_{U3}(q)$	$\int \frac{r_1^2 dq}{(r_2 - r_1)2M}$	$x_1^2 \ln \lvert \sin\ q \rvert - y_1^2 \ln \lvert \cos\ q \rvert + x_1 y_1 q$	$-\lambda/\Delta_\rho$	$-\lambda/\Delta_\rho$	λ/Δ_ρ	λ/Δ_ρ
$I_{U4}(q)$	$\int \frac{r_2 dq}{(r_2 - r_1)2M}$	$x_2 \ln \left\lvert \mathrm{tg}\,\frac{q}{2} \right\rvert + y_2 \ln \left\lvert \left(1 + \mathrm{tg}\,\frac{q}{2}\right) \Big/ \left(1 - \mathrm{tg}\,\frac{q}{2}\right) \right\rvert$	$-\lambda$	$-\lambda$	λ	λ
$I_{U5}(q)$	$\int \frac{r_2 r_1 dq}{(r_2 - r_1)2M}$	$x_2 x_1 \ln \lvert \sin\ q \rvert + (x_2 y_1 + x_1 y_2) q - y_2 y_1 \ln \lvert \cos\ q \rvert$	$-\lambda/\Delta_\rho$	$-\lambda/\Delta_\rho$	λ/Δ_ρ	λ/Δ_ρ
$I_{U6}(q)$	$\int \frac{r_2 dq}{2M}$	$x_2 \ln \lvert \sin q \rvert + y_2 q$	$\lambda \frac{\Delta_x}{\Delta_\rho}$	$-\lambda \frac{\Delta_x}{\Delta_\rho}$	$-\lambda \frac{\Delta_y}{\Delta_\rho}$	$-\lambda \frac{\Delta_y}{\Delta_\rho}$
		$x_2 q - y_2 \ln \lvert \cos q \rvert$	$-\lambda \frac{\Delta_y}{\Delta_\rho}$	$-\lambda \frac{\Delta_y}{\Delta_\rho}$	$\lambda \frac{\Delta_y}{\Delta_\rho}$	$\lambda \frac{\Delta_y}{\Delta_\rho}$
$I_{U7}(q)$	$\int \frac{dq}{2M}$	$\ln \left\lvert \mathrm{tg}\,\frac{q}{2} \right\rvert$	$\lambda\Delta_x$	$-\lambda\Delta_x$	$\lambda\Delta_x$	$\lambda\Delta_x$
		$\ln \left\lvert \left(1 + \mathrm{tg}\,\frac{q}{2}\right) \Big/ \left(1 - \mathrm{tg}\,\frac{q}{2}\right) \right\rvert$	$-\lambda\Delta_y$	$-\lambda\Delta_y$	$\lambda\Delta_y$	$\lambda\Delta_y$
$I_{U8}(q)$	$\int \frac{r_1 dq}{2M}$	$x_1 \ln \lvert \sin q \rvert + y_1 q$	$\lambda \frac{\Delta_x}{\Delta_\rho}$	$-\lambda \frac{\Delta_x}{\Delta_\rho}$	$\lambda \frac{\Delta_x}{\Delta_\rho}$	$\lambda \frac{\Delta_x}{\Delta_\rho}$
		$x_1 q - y_1 \ln \lvert \cos q \rvert$	$-\lambda \frac{\Delta_y}{\Delta_\rho}$	$-\lambda \frac{\Delta_y}{\Delta_\rho}$	$\lambda \frac{\Delta_y}{\Delta_\rho}$	$\lambda \frac{\Delta_y}{\Delta_\rho}$

... The exact HT performance:

where $r_i(q) = \rho_i \cos(q - \theta_i)$, $\rho_i = \sqrt{x_i^2 + y_i^2}$, $\theta_i = \mathrm{tg}(y_i / x_i)$, $(x_i, y_i) = (x_0 \pm \Delta_x / 2, y_0 \pm \Delta_y / 2), i = 1,2,3,4$;
$2M = r_4(q) + r_3(q) - r_2(q) - r_1(q)$; $q \in [\theta_L, \theta_R)$, $\Delta_x = \Delta_y = 1$, $\Delta_\theta = \pi / \Theta_{\max}$, $\Delta_\rho = \sqrt{X_{\max}^2 + Y_{\max}^2} / P_{\max}$;
$X_{\max}, Y_{\max}$ are the sizes of input image *f*, and $\Theta_{\max}, P_{\max}$ the output HT array size; $\lambda = \frac{f(x_0, y_0)}{2\Delta_x \Delta_y \Delta_\theta \Delta_\rho}$
$f(x_0, y_0)$ is given pixel value, $(x_0, y_0) \in X_{\max} \times Y_{\max}$.

Each volume $V=V(b)$ is calculated from the beginning of the respective variety. Thus, the corresponding HT pixel accumulates a volume $\Delta V(b)$, b is the HT-pixel distance (lower side) to the beginning of the TC6.

Experiments



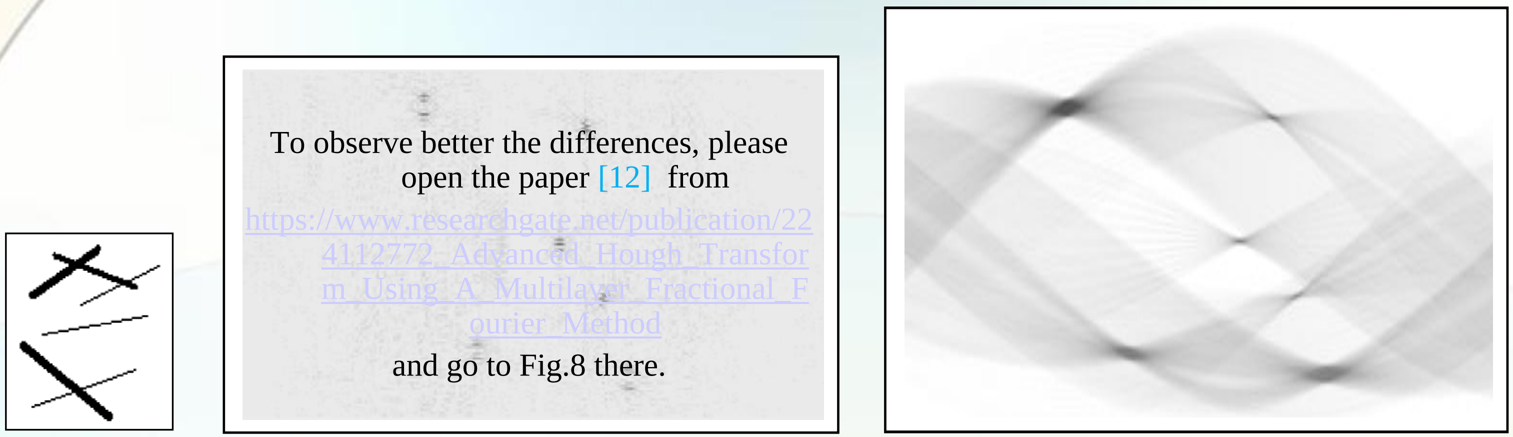
Comparison between both, iterative and precise, performances of (ρ,θ)HT:

- A test (image 65*65 of a "white" pixel (x,y)=(0,-32) on a "black" background);

- Small vertical differences (mean ~3.53%) between both Cosinusoide shapes (as expected);

- For better visibility the result is amplified to the maximal intensity;

Experiments …



(a) (b) (c)

Comparison with a FRFT (Fractional FT) performance of (ρ,θ)-HT, on an example borrowed from [12]: a) and b) an image and their HT [12], and c) our exact HT.

It can be seen that at least 3 of the 6 corresponding peaks are damaged in FRFT approach, appearing fuzzy in twin peaks each.

This, of course, does not discredit the particular application [12] (of hieroglyph recognition), but it confirms the thesis – in quick implementations of HT there is still a lot to be desired, at least in terms of accuracy.

Where to apply

- Generally in images of small resolution, for example:
  - in relatively small images (for example, with reduced resolution)
  - in determining the slope of relatively small objects (i.e. on small portions of the image);
  - …
- Wherever accuracy of HT or its individual projections dominate the processing time:
  - in test and setup of new algorithms and/or software for HT performance;
  - in a comparative analysis of experimental determination of error in other implementations of HT;
  - …
- Like most other algorithms for image processing, the proposed method involves efficient parallel implementations.

Conclusion marks

- An analytic performance of the exact (ρ,θ)HT has been proposed.
- The performance complexity is cubic $\sim X_{\text{size}} Y_{\text{size}} \Theta_{\text{size}}$, i.e. similar to the standard realizations of HT.
- Preciseness – maximal by definition (!)
- Consequently, at equal other conditions – the input image grid ($X_{\text{size}}, Y_{\text{size}}$, Δ_x , Δ_y) and the chosen grid for HT output (P_{size}, Θ_{size} , Δ_ρ , Δ_θ), there exists a limit of preciseness (i.e. a mean square error) which if be kept then our analytical approach will gain in processing speed.

Thank You

(for your questions ☺)