INSTITUTE OF INFORMATION AND COMMUNICATION TECHNOLOGIES
BULGARIAN ACADEMY OF SCIENCES

One more a Specific Application of Hough/Radon Transforms in Image Processing

a seminar lecture by Dimo Dimov, IICT-BAS

(Aula Seminari ex Elettronica)
University of Pavia
Oct. 25, 2018

10/26/2018 (BG SOFIA 30) Erasmus+ (I PAVIA 01) Dimo Dimov, dtdim@iinf.bas.bg http://mmsip.bas.bg/staff/dimov.html

Acknowledgements

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The lecture plan:

Part 1: Simulation and/or verification of visual stimuli to help experimental modeling of voluntary saccadic eye movements during human decision making
(to a case of Hough | Radon Transform's usage, HT|RT)

Part 2: What to choose if a discrete HT|RT is necessary, the fastest performance or the exact performance - speed against precision.

Part 1: an Image Processing Problem

Let's consider a current project of IICT-BAS, entitled

"Modeling of voluntary saccadic eye movements during decision making" (https://www.researchgate.net/project/Modelling-of-voluntary-saccadic-eye-movements-during-decision-making)

The project, in a collaboration with INB (the Inst. of Neuro-Biology) at BAS, is coordinated by Prof. Petia Koprinkova (IICT-BAS).

The project objectives are:
- To create a unified biologically plausible model of voluntary saccadic eye movements during visual tasks, combining the model for processing of visual motion information (a previous development of the team), with a model describing the human brain structures involved in the decision making processes and learning, based on punish/reward principle (called reinforcement learning)
- To investigate age-related changes in the model parameters.
- To study the age-related changes in humans based on the statistical characteristics of decision making processes by means of the theory of plausible and paradoxial reasoning.

The project advance can be seen by the current publications of the team on the site.

An interest for this seminar will be the project work package 4 "Experimental Data Processing", where it was necessary to develop a software for verification of test stimuli.

Part 1: an Image processing problem: Verification of Test Stimuli.

Visual stimuli simulation and/or verification's tools have to be designed to help experimental modeling of voluntary saccadic eye movements during human brain decision making.

Visual stimuli (video clips) are prepared in advance (see some AVI-s demo).

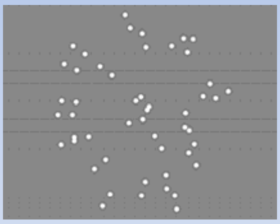
Brief description of the visual stimuli experiments:

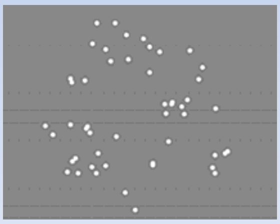
- the visual stimuli are displayed on the computer screen;
- an eye tracking technology is used to obtain currently the actor gaze coordinates on the screen;
- the actor should gazing the screen to indicate a guess, where the stimuli center is situated, on the left or on the right from the screen center;
- the series of actor's gaze coordinates are currently saved for next processing and evaluation of psychological characteristics of the actor.

Part 1: an Image processing problem: Verification of Test Stimuli.

A plenty of conventional approaches are known to generate visual stimuli (video clips) and to verify them from the both viewpoints, of the actor, who participates in the experiments, as well as of the researcher, who conducts the experiments.

Three types of stimuli were used: "static", "flicker" and "combined". In each of them a video frame consisted of 25 pairs of dots separated by fixed distance (so-called "Glass-patterns", [1]). The dots were positioned in such a way that 72% of them (18 pairs) formed radial patterns with center ("Focus-of-Expansion" (FoE), [2]), shifted to the left or to the right of the screen center. The three conditions differed by their dynamic properties. A typical frame =>





<= The 4th stimuli type is "motion", where FoE is virtually imagined by pairs of corresponding test points in each two consecutive frames. Instead of FoE, the term "Center of Motion (COM)" is often used in the case [3].

Part 1: an Image processing problem: Verification of Test Stimuli.

Additional information about the above matter can be found also from:

[1] Ross, J., "The perceived direction and speed of global motion in Glass pattern sequences", Elsevier, Vision Research 44 (2004) 441–448.

[2] Beardsley, S. A., L. M. Vaina, "How Can a Patient Blind to Radial Motion Discriminate Shifts in the Center-of-Motion?", Springer, Journal of Computational Neuroscience 18 (2005) 55–66.

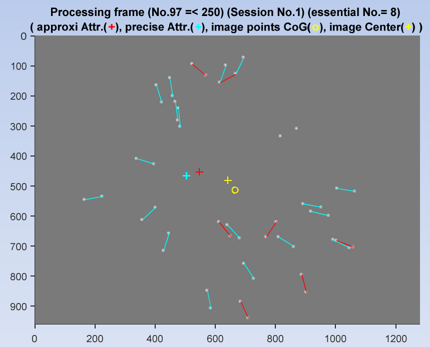
[3] Niehorster, D. D., J. C. K. Cheng, Li Li, "Optimal combination of form and motion cues in human heading perception", Journal of Vision (2010) 10(11):20, 1–15 (http://www.journalofvision.org/content/10/11/20)

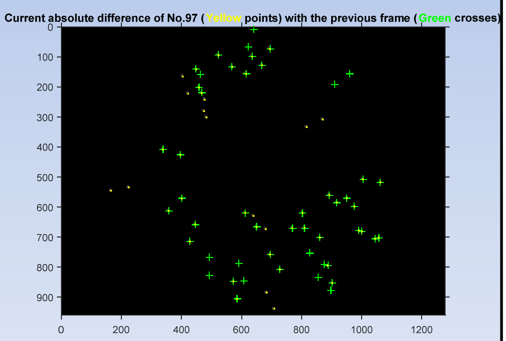
Part 1: an Image processing problem : Testing of a “flicker” type stimuli

A Matlab demo: (on 'fli-7.avi‘) (i.e., the FoE is 7 units on the left)

(by “m1_cD1F_common_Avi_stimuli_Directly_SingleFrame.m”)

The flicker stimuli processing is performed over each single frame of the video. An essential info about the current frame processing is shown above the both figures (visible if debug regime is chosen).



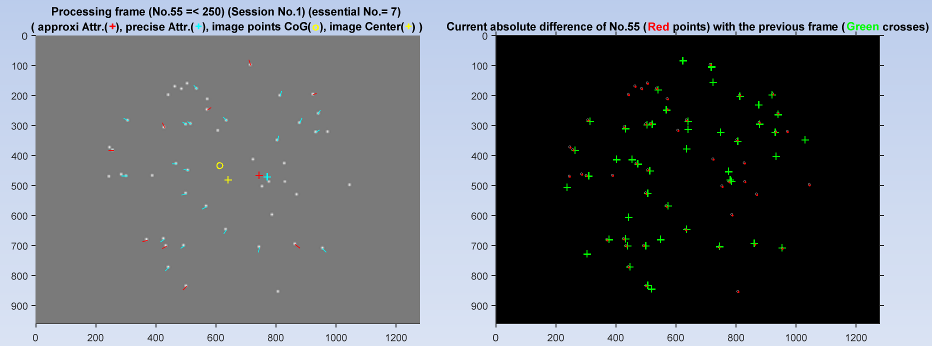


Part 1: an Image processing problem: Testing of a "motion" type stimuli

A Matlab demo: (on 'mov+7.avi') (i.e., the FoE is 7 units on the right)

(by "m2_mD2F_analyse_MOT_Avi_stimuli_Directly_CoupleFrames.m")

The "motion" stimuli processing is performed on each couple of consecutive frames of the video. An essential info about the current frame processing is shown above the both figures (visible if debug regime is chosen).



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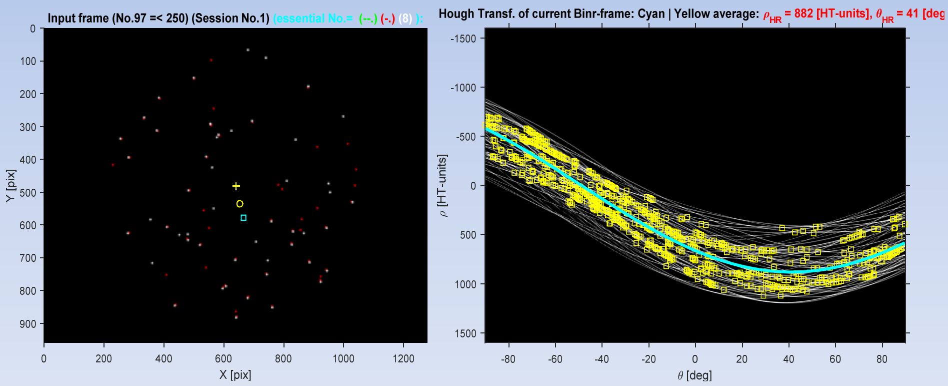
Part 1: an Image processing problem: A test using a Hough transform for stimuli frames

A Matlab demo: (on 'mov+1.avi')

(by "m3_mH3F_analyse_MOT_Avi_stimuli_Hough_TripleFrames.m")

The "motion" stimuli processing, via Hough transform, but over each triple of consecutive frames, for a more precise evaluation of the essential points "life" (visual sparking).



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http://mmsip.bas.bg/staff/dimov.html

Part 1: an Image processing problem: Let's analyze the above demo

Pure geometric approaches have been used for the first two demos:

- The number "nP50" of points is fixed, nP50 = 50. The base distance "dL60" (between the substantial pairs of points in the frame) is also fixed, dL60 = 60 pix.
- The current value of FoE is evaluated (by a quadratic criterion, LSM) as the virtual point, nearest to the 25 straight lines (nL25 <=> nP50/2) determined by the 25 pairs of points (i.e., segments, line-cuts) of length closest to dL60. The so-calculated FoE, has been called "FoE of the 1st order" and marked as FoE-1.
- Finally, the wanted FoE is evaluated as "FoE from the 2nd order" (FoE-2) after isolation of nL07, (nL07 = 7) outliers, i.e. the outermost segments from the nL25 already selected ones (towards FoE-1).

The parameter values, dL60, nP50, nL25 and nLo7, are fixed (by default) according to currently available videos (stimuli), but can be manipulated.

Unlike the first 3 cases (static, flicker and combined), where virtual line-cuts in the current frame are used, in the 4th case, the motion vectors on the corresponding test points are evaluated on each two consecutive frames.

Even if the moving speed is known in advance it is quite complex to determine the points' correspondence in successive frames (i.e. the visual sparks leaving 3, or more frames).

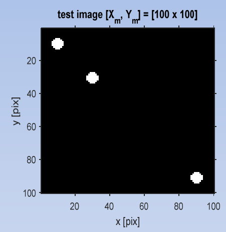
Part 1: an Image processing problem: Let's analyze the above demo

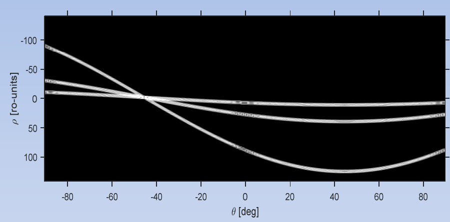
(!) Because of virtual lines wanted, it's good idea to look for them as "point spots" in the space of a Hough | Radon transform:

- If not fixed, the basic parameter dL60 could be "detected" by histogram approaches, but the experiments showed drastic deviations (errors) from time to time, what seems to be quite complex to overcome analyzing the histograms.
- The H|R approach does not give anything more in cases of fixed parameters, e.g. dL60. Some (theoretical) interest could be the Sinogram attractor, obtained by an LSM approach, but it is a pro-image of the attractor evaluated as a virtual (LSM) point in the input space. Practically, we only will add "computational" noise (and run time) due to the not enough precise hough() | radon() functions of Matlab (R20-10a| 14a| 15a| 18a);
- However, H|R approaches could be "irreplaceable", in cases of unknown "moving speed", or dL60, or for example, to model pairs of test points through the corresponding "diamond like" domains of Sinogram crossings in the H|R space. Unfortunately, all these ideas will require H|R procedures of very high precision, and enough speedy, of course.
- But, fortunately, such a procedure is already known, and it does not use the popular increase of memory to reach enough precise filling of the H|R accumulation space (see Dimov, D., A. Dimov, CompSysTech, Palermo, 2016)

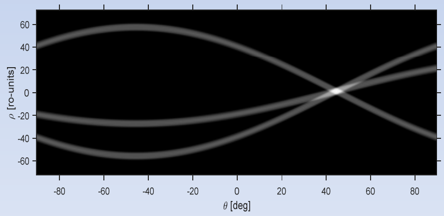
Part 1: an Image processing problem: Another idea of using a Hough transform in stimuli frames

One more idea: to evaluate (in Hough|Radon space) the lengths of virtual line-cuts among the input test points (circle spots).





hough() case



radon() case

The 3 input points (spots) are respectively represented by 3 Sinogram-shapes in the H|R accumulation space. Consider their cross area, i.e. the pro-image of the virtual line over the 3 input points (which are chosen collinear).

Part 1: an Image processing problem: Another idea of using a Hough transform in stimuli frames

Continues…

The Sinogram crossings' area can be split in 3 sub-areas:

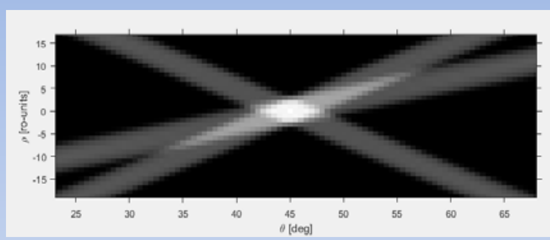
i) for the three shapes crossing area (the brightest conjunction, see the figures);

ii) the conjunction area of each 2 shapes; and

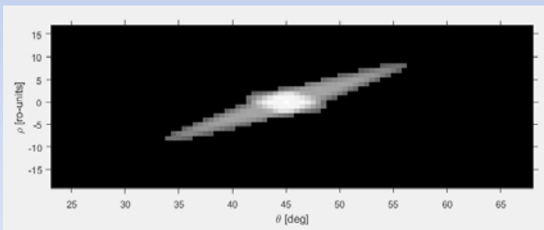
iii) the 3 Sinogram shapes' disjunction.

Because of accumulative property of H|R space, these areas can be isolated by levels (of intensity).

(see the both magnifications herein)



magnified radon() case of interest



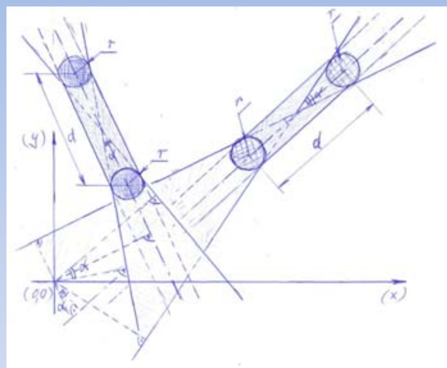
magnified radon() case with 1st level isolated

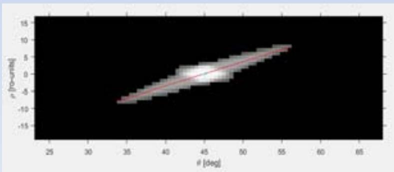
Part 1: an Image processing problem: Another idea of using a Hough transform in stimuli frames.

Continues …

See on the drawing : =>

- All the points (circle spots) have equal radii "*r*". The "real-line area" between 2 spots consist of all possible lines covering both the spots.
- Two line-cuts of equal length *d* are drawn. Obviously, nevertheless of their mutual position and orientation, the both "line cuts" have equal definitive angles α, $\alpha = 2.\arcsin(2r/d)$
- … which can be been measured in the H|R space (see the picture below).





<= Magnified radon() case with evaluated (in red) the distance "*d*" of line-cuts defined by the two spot-point pairs given on the above drawing, (i.e. the longest "*d*" among the 3 test circle spots).

End of Part 1

End of Part1

Thank you

(for your attention & endurance)

The author (dtdim@iinf.bas.bg) will be pleased to answer your questions on the pure informaticals part of this lecture.

For possible questions about the essence of the project itself, please contact directly the coordinator (pkoprinkova@bas.bg) or the other leading specialist (nbbocheva@hotmail.com)

End of Part 1

End of Part 1

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One more a Specific Application of Hough/Radon Transforms in Image Processing

Part 2.1: The Exact (ρ,θ)-Hough Transform -- Definition and Performance

a seminar lecture by Dimo Dimov, IICT-BAS

(Aula Seminari ex Elettronica)
University of Pavia
Oct. 25, 2018

The Exact (ρ,θ)-Hough Transform -- Definition and Performance

This lecture has been based (but not limited) on the results published in:

Dimov, D., and A. Dimov, The Exact (ρ,θ)-Hough Transform -- Definition and Performance, in Rachev B., and A. Smrikarov (Eds.), Proceedings of CompSysTech'16, Palermo, Italy, 23-24 June 2016 (ACM Int. Conf. Proc. Series, ACM PRESS, NY, USA, 2016, pp.198-205).

http://dl.acm.org/citation.cfm?id=2983523&CFID=865067031&CFTOKEN=54232689

You may find a similar title presentation (dated 15.11.2016) on the CVML site:

https://vision.unipv.it/events/DTDim_Erasmus_presentation_1_(UNIPV).pdf

Please, enjoy the above matter. The author will be pleased to answer your possible questions on it. Thank you.

End of Part 2.1

End of Part 2.1

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One more a Specific Application of Hough/Radon Transforms in Image Processing

Part 2.2: Fast Hough/Radon Transform using FFT and Fractional FFT

a seminar lecture by Dimo Dimov, IICT-BAS

(Aula Seminari ex Elettronica)
University of Pavia
Oct. 25, 2018

Part 2.2 lecture plan:

- The idea of simple (and robust) comparison between both, the exact H|RT and the fast(er) H|RT.
- Recent approaches to faster H|RT based on the Projection Slice Theorem (and via FFT)
- One more innovation for the most effective of them.
- Comparison results and a Discussion.

The comparison approach:

As already said in the previous lecture: The (ρ,θ)HT is equivalent to a Radon transform, and v.v.:

$$h(\rho,\theta) = \iint_{RoI} f(x,y)\,\delta\big(x\cos(\theta) + y\sin(\theta) - \rho\big)\,dx\,dy$$

$$(\rho,\theta) \in RoHT$$

And the similar is valid for respective discrete versions. The exact (ρ,θ)HT has been proposed like a direct discrete version of RT, having in mind equal input grids and equal output grids for both transforms (see Part 2.1 of the lecture).

Besides of both the transforms are linear, the comparison approach we need should consider the transforms, pixel by pixel, consecutively, what is the “weightiest” case of comparison. This means that eventual errors among different areas of comparison do not influence each other, if these areas are chosen as single pixels (i.e. the smallest areas of comparison).

In this line of thinking, the best image of comparison will be each “totally white” and squared one.

The comparison approach:

Besides, the exact (ρ,θ)HT (cf. Part 2.1) is the most precise performance of (ρ,θ)HT, i.e. of the RT as well, by definition for a given size of the input and output grids. This means that any other performances (e.g., the faster one of interest herein) can/(is to) be compared by a simple difference of its output values towards those of the exact (ρ,θ)HT, pixel by pixel.

If both the transforms give enough low differences of results after statistical evaluations, then the next stage of comparison, namely by processing speed, could have a sense. Because of well known assessments for processing complexity of both transforms, a cubic against a quadratic ones, we can expect to found a “threshold” of the grid size N, which to help the decision for “preciseness or speed”.

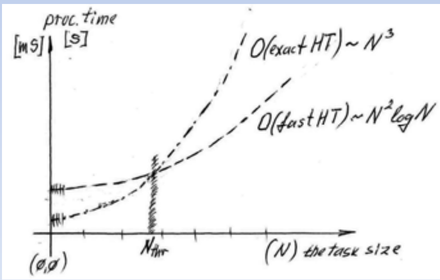
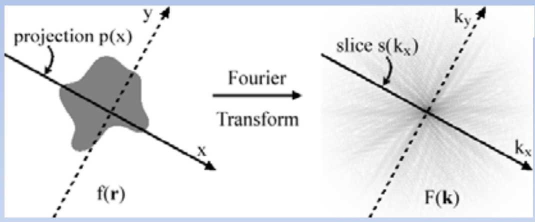


Fig.: The “threshold” N_{thr} to help the decision for preciseness or speed: =>

Faster H|RT-s based on Projection Slice Theorem:

- The Projection Slice Theorem (PST):
  (borrowed from https://en.wikipedia.org/wiki/Projection-slice_theorem)



If $f(x, y)$ is a two-dimensional function, then the projection of $f(x, y)$ onto the x axis is $p(x)$, where $p(x) = \int f(x,y)\,dy$. The Fourier transform of $f(x, y)$ is $F(\omega_x, \omega_y) = \iint f(x,y)\exp\left(-j(x\omega_x + y\omega_y)\right)dx\,dy$. Then, the slice $s(\omega_x)$

$$s(\omega_x) = F(\omega_x, 0) = \iint f(x,y)\exp(-jx\omega_x)\,dx\,dy = \int p(x)\exp(-jx\omega_x)\,dx$$

which is just the Fourier transform of $p(x)$.

Faster H|RT-s based on Projection Slice Theorem: …(1)

- Another (more comfortable) expression of PST:
- (ρ,θ)HT | RT : $h(\rho,\theta) = \iint f(x,y)\,\delta(x\sin(\theta) + y\cos(\theta) - \rho)\,dx\,dy$
- (ρ,θ)HT | RT -spectrum: $H_\rho(\omega,\theta) = \int h(\rho,\theta)\exp(-j\,\omega\rho)\,d\rho =$

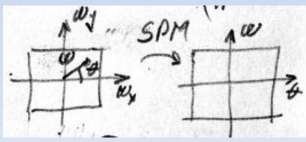
$$= \iint f(x,y)\exp\left(-j\omega\left(x\sin(\theta) + y\cos(\theta)\right)\right)dx\,dy$$

$$= \iint f(x,y)\exp\left(-j(x\omega_x + y\omega_y)\right)dx\,dy = F(\omega_x, \omega_y)$$

- hence:

$$h(\rho,\theta) = \frac{1}{2\pi}\iint H_\rho(\omega,\theta)\exp(+j\,\omega\rho)\,d\omega = \frac{1}{2\pi}\iint F(\omega\cos(\theta), \omega\sin(\theta))\exp(+j\,\omega\rho)\,d\omega$$

(!) Use an appropriate Simple Polar Mapping =>
before the final inverse FFT (by columns) to obtain
wanted $h(\rho,\theta)$ (i.e. an approximation of it):



(See a demo: https://mathinsight.org/polar_coordinates_mapping)

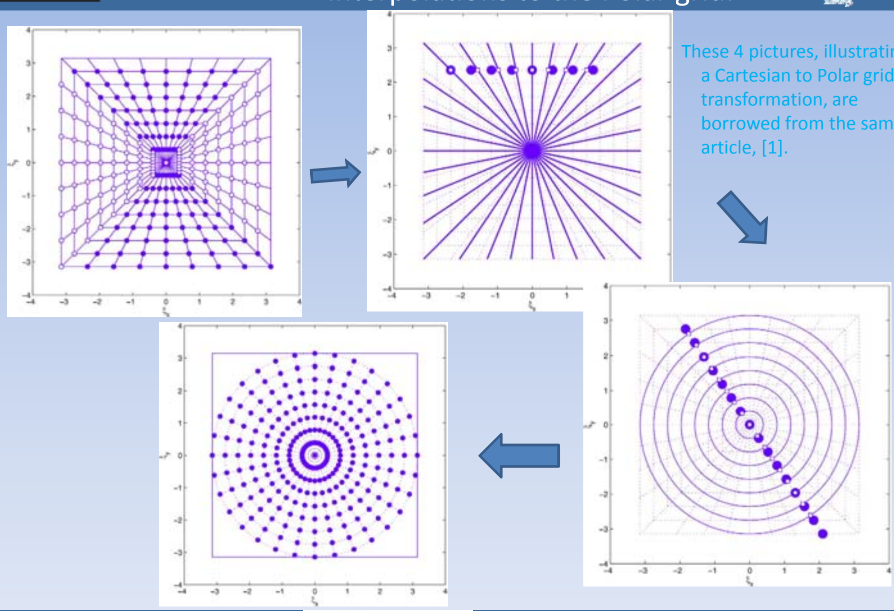
Faster H|RT-s based on Projection Slice Theorem: …(2)

- One more approach to fast H|RT (perhaps, the best one, cf. [1, 2]):
- The idea of this approach is to escape the SPM of the 2D Fourier spectrum $F(\omega_x, \omega_y)$. Really, SPM has a squared complexity, but it is too difficult to be perform enough precisely.
- Instead of SPM, a 3-steps' approximation is used, namely:

  (i) a Fractional FFT to convert the 2D Cartesian grid of $F(\omega_x, \omega_y)$ to the so called "Pseudo Polar" (PP) grid of $H_\rho(\omega, \theta)$, i.e. the 1D Fourier spectrum by ρ .

  (ii & iii) two interpolations of the PP-grid, first horizontally (by θ) and then vertically (by ρ) are implemented to reach the conventional Polar grid.

The Pseudo Polar grid and its both interpolations to the Polar grid:



These 4 pictures, illustrating a Cartesian to Polar grid transformation, are borrowed from the same article, [1].

…the Fractional Fourier transform:

- FrFT: $F_\alpha(\omega) = \int f(x)\exp(-j\,\omega x\alpha)\mathrm{d}x$, α real
- It is known a fast algorithm for the discrete FrFT:

$$F_\alpha(k) = \sum_{n=0}^{N-1} f(n)\exp\left(-j\frac{2\pi}{N}nk\alpha\right);\ \{0 \le k < N\}$$

  based on the so called "*Chirp-Z*" transform, e.g. see [3].
- The performance complexity of fast discrete FrFT can be evaluated to ~ $44N + 20N\log N$ operations.

Fast Hough/Radon Transform using FFT and Fractional FFT:

This lecture is based on the following documents:

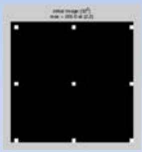
[1] Averbuch, A., R.R. Coifman, D.L. Donoho, M. Elad, M. Israeli. "Fast and accurate Polar Fourier transform", Appl. Comput. Harmon. Anal. 21 (2006) 145–167
[2] A. Averbuch, R.R. Coifman, D.L. Donoho, M. Israeli, J. Waldren, "Fast Slant Stack: A notion of Radon Transform for Data in a Cartesian Grid which is Rapidly Computible, Algebraically Exact, Geometrically Faithful and Invertible", SIAM J. Sci. Comput. (2001).
[3] Bailey, D. H., P. N. Shwarztrauber, "The Fractional Fourier Transform and Application", SIAM Review, vol. 33, no. 3., 1995.
[4] The Projection Slice Theorem, https://en.wikipedia.org/wiki/Projection-slice_theorem
[5] Polar coordinates mapping, https://mathinsight.org/polar_coordinates_mapping

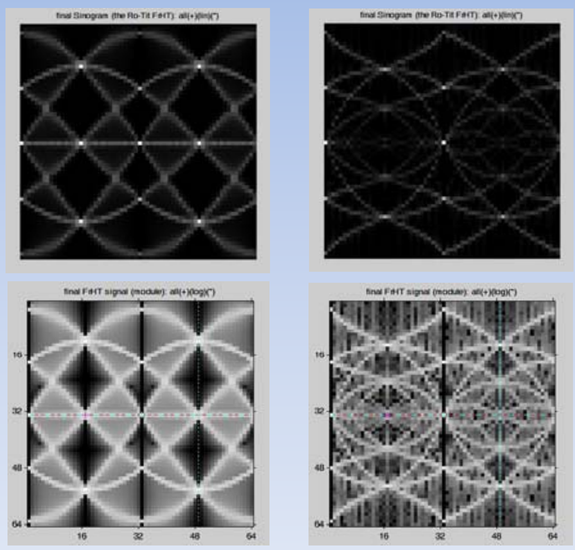
Nevertheless of been already presented at the Seminar time, a part of next slide-show will be suppressed because of possible Copy-Rights collisions with the author's future publication on this matter.

Fast Hough/Radon Transform using FFT and Fractional FFT: **some test results …**

A comparison of the output precision using nine (8+1) representative points (pixels) of a square input image:

Linearly scaled intensity output: =>





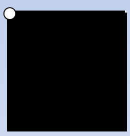
by my code | by the open code of [1]

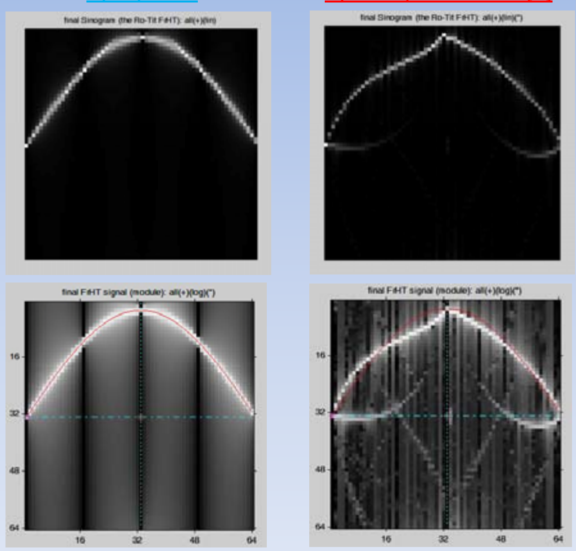
Logarithmic scaled intensity output: => (to stress on error areas)

Fast Hough/Radon Transform using FFT and Fractional FFT: **some test results (1) …**

The output precision compared through the left-top pixel (1, 1):

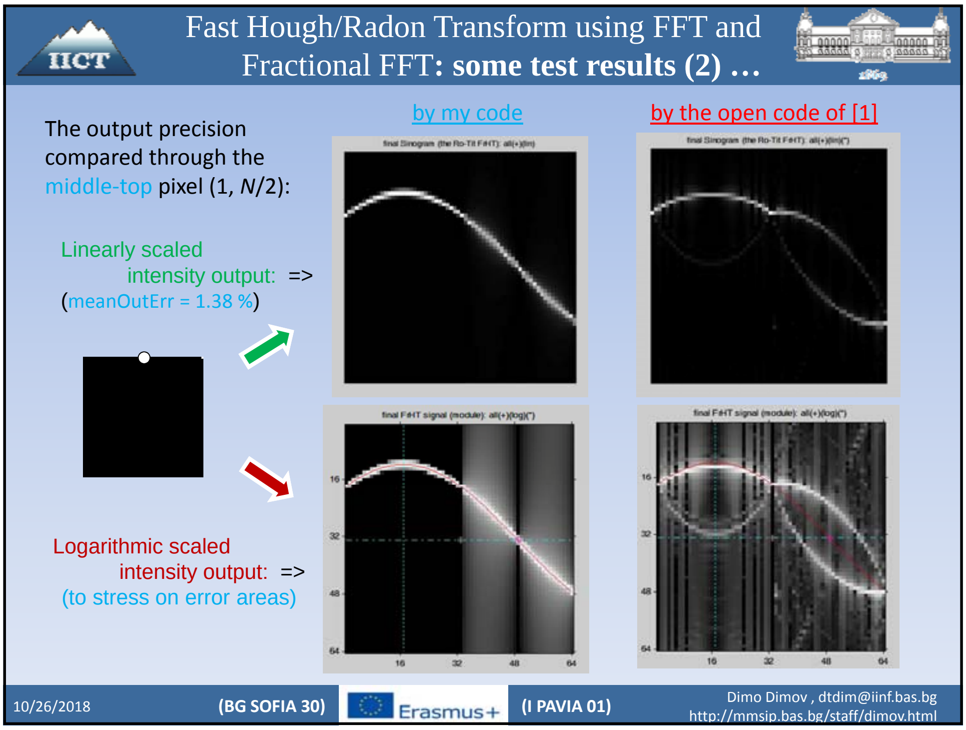
Linearly scaled intensity output: => (meanOutErr = 2.60 %)

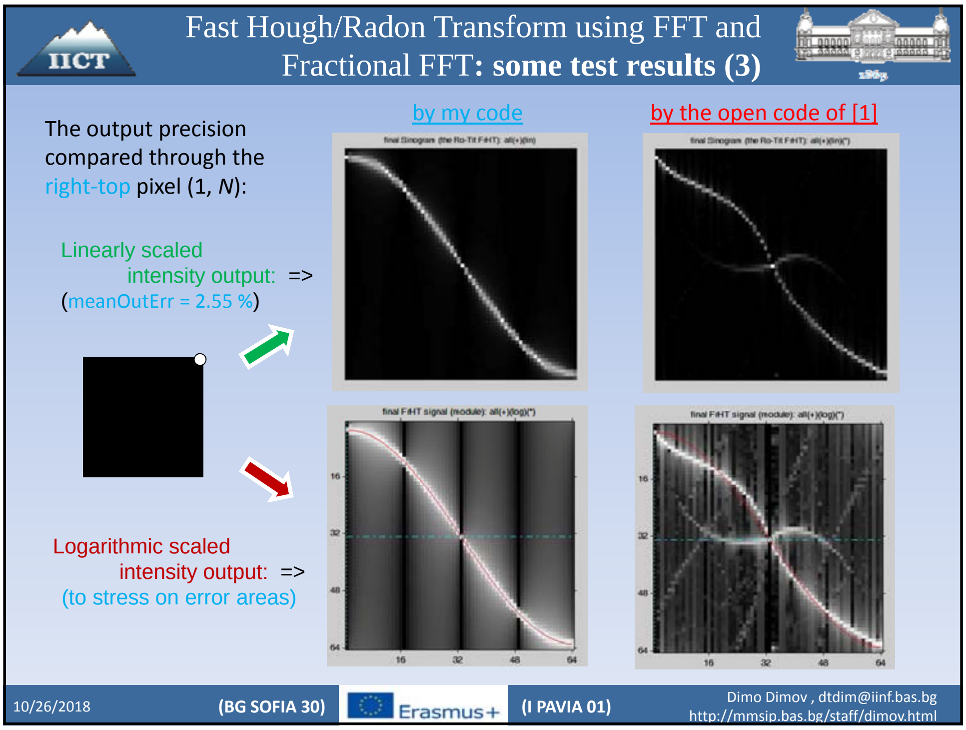




by my code | by the open code of [1]

Logarithmic scaled intensity output: => (to stress on error areas)


IICT
Fast Hough/Radon Transform using FFT and Fractional FFT: some test results (2) …
The output precision compared through the middle-top pixel (1, N/2):
Linearly scaled intensity output: =>
(meanOutErr = 1.38 %)
Logarithmic scaled intensity output: =>
(to stress on error areas)
by my code
by the open code of [1]
10/26/2018
(BG SOFIA 30)
Erasmus+
(I PAVIA 01)
Dimo Dimov , dtdim@iinf.bas.bg
http://mmsip.bas.bg/staff/dimov.html


IICT
Fast Hough/Radon Transform using FFT and Fractional FFT: some test results (3)
The output precision compared through the right-top pixel (1, N):
Linearly scaled intensity output: =>
(meanOutErr = 2.55 %)
Logarithmic scaled intensity output: =>
(to stress on error areas)
by my code
by the open code of [1]
10/26/2018
(BG SOFIA 30)
Erasmus+
(I PAVIA 01)
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Fast Hough/Radon Transform using FFT and Fractional FFT: **instead of Conclusion: …**

- The two articles cited above provide a solid foundation towards the "fast H|RT" problem solving, but the open source experiments are very disturbing.
- It is not clear why the authors conduct their method / approach assessments, not by individual pixels of the input image, but by pixel groups where they are looking for "the worst case" (for some reason).
- For some reason, the authors' open code (Matlab written, been available till 2016 at http://www.cs.technion.ac.il/~elad/PolarFFT.zip) does not give the expected sinogram as a pro-image of a pixel but some "crooked" resemblance of it, despite of my best attempts to rehabilitate this code (see the right side figures in the previous 3 slides).
- My structural innovation and correction of this code works quite better, giving an acceptable output similarity towards the exact (ρ,θ)HT (discussed in Part 2.1 of the lecture), see "red" sinusoide-curves in the figures of previous three slides.

Fast Hough/Radon Transform using FFT and Fractional FFT: **instead of Conclusion:**

- Each "red" sinusoide-curve represents the precise axis of respective TC6 (cf. Part 2.1), i.e. it is the pro-image of the centre of corresponding "white" pixel of interest (see test results in the last 3 slides).
- The output precision of the H|RT can be evaluated after isolation of the TC6-area (cf. Part.2.1), which is approximated in our case via an enough wide strip surrounding the "red" sinusoide-curve. The mean of the outside error ("outErr") after the TC6 area isolation can be used as a value of comparison.
- But, obviously, this could not be done for the original fast H|RT algorithm.
- At the same time, if we have a look on the logarithmically scaled intensity outputs, the mean outside error of the "better looking" fast H|RT code (on the left) seems greater than the one of the "quite crooked" fast H|RT (on the right). This ascertainment shows that there is still some to do for the perfection of both algorithms for fast H|RT.
- As for the N_{thr}, i.e. the threshold to prefer for "preciseness or speed" proposed above for the input grid size N, it can be expertly evaluated to $N_{thr} \approx$??? (at least for now).

End of Part 2.2

End of lecture

Thank you

(for your attention of interest)

The author (dtdim@iinf.bas.bg) will be pleased to answer your questions on this lecture, as well as to collaborate on this matter, and especially on the second part of it.

End of Part 2.2

End of the lecture

Thank you