

Fundamentals of GPS Navigation

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INSIDE GNSS NEWS



ESA General Director Jan Woerner at media briefing Wednesday (January 18, 2017). Photo by Peter Gutierrez (Click image to enlarge.)



At the traditional January media briefing in Paris (January 18, 2017), **European Space Agency (ESA) General Director Jan Woerner explained the** knowns and unknowns about the failed rubidium and hydrogen maser clocks onboard orbiting Galileo satellites, clocks that are absolutely crucial for accurate positioning.



Each Galileo satellite is equipped with four clocks - two rubidium and two passive hydrogen masers in order to have redundancy. It was thought that the new type of hydrogen maser will improve accuracy. But unfortunately many rubidium clocks as well as hydrogen maser clocks failed. In all, nine of the 72 orbiting clocks have failed three rubidium clocks and six hydrogen masers. **One additional hydrogen maser clock failed but it** has been successfully restarted later.



According to ESA, the Italian-produced hydrogen maser clock is the master clock on the Galileo satellite's payload. Its extremely good performance makes it the most stable of all clocks currently in space, better than one nanosecond per day, according to the agency. Some other features: 18 kilograms of mass, 28 liters of volume, and 20 years expected lifetime.







Passive hydrogen maser atomic clock of the type flown on Galileo, accurate to one second in three million years.





One reference point





The difference between two way positioning and one-way











Network of points in the space





Global Navigation Satellite Systems

GNSS is a technology for outdoor navigation. The most popular example is GPS, which is a constellation of satellites that transmit encoded radio frequency (RF) signals. By means of trilateration, ground receivers can calculate their position using the travel time of the satellites's signals and information about their current location, this being included in the transmitted signal.



GPS or GNSS?

The Global Positioning System (GPS) was once the only global navigation satellite system in operation. Today, there are numerous satellite systems dedicated to providing time and location signals for both military and civilian use. They include Russia's **GLONASS, China's BeiDou and Europe's Galileo, as** well as a number of regional systems. Collectively, these systems are known as Global Navigation Satellite Systems (GNSS).







Country of originry	of origin Unit
Operator(s)	AFS
Туре	Mili
Status	Ope
Coverage	Glob
Precision	5 me
	Constellation
Total satellites	32
Satellites in orbit	31
First launch Total launches	Febr ago 72
	Orbital characte
Regime(s)	6x N
Orbital height	20,18

ted States **SPC** tary, civilian rational bal eters size ruary 1978; 39 years (1978-02) eristics **IEO planes**

20,180 km (12,540 mi)











Glonass

Country of origin	Russia		
Operator(s)	VKO		
Туре	Military, civilian		
Status	Operational		
Coverage	Global		
Precision	4.5–7.4 meters		
Constellation size			
Total satellites	27		
Satellites in orbit	24		
First launch	October 1982		
Last launch	May 28, 2016		
Orbital characteristics			
Regime(s)	3x MEO		
Orbital height	19,130 km		











Country of origin Operator(s) Type

Type

Status

Coverage

Precision

Total satellites

Satellites in orbit

First launch

Galileo

European Union GSA, ESA Civilian, commercial Operational Global 1 metre (public) 1 cm (encrypted) Constellation size 30

11 operational + 4 under commissioning and 3 for testing or not available (January 2017)

2011

Orbital characteristics

Regime(s) Orbital height 3x MEO planes 23,222 km (14,429 mi)











Country of origin Operator(s)

operator

Type

Status

Coverage

Precision

BeiDou - 2

China

Civilian, military Operational Partial 10 metre (public) 10 cm (encrypted)

Constellation size





Beidou-2G - Image: CAST

Total satellites

Satellites in orbit

First launch

30 non stationary; 5 stationary 15 operational + 4 under commissioning and 3 for testing or not available (January 2017) 2007

Orbital characteristics

Regime(s) Orbital height 6 GEO + 5 IGSO + 4 MEO 21,150 km









GPS accuracy

The government is committed to providing GPS at the accuracy levels specified in the GPS Standard Positioning Service (SPS) Performance Standard. The accuracy commitments is applied to the signals transmitted in space. For example, the government commits to broadcasting the GPS signal in space with a global average user range error of ≤ 7.8 m, with 95% probability. Actual performance exceeds the specification. On May 11, 2016, the global average URE was ≤ 0.715 m, 95% of the time. In May 2000 the U.S. government ended the use of Selective Availability in order to make GPS more responsive to civil and commercial users worldwide. The United States has no intent to ever use Selective Availability again.





Точность навигационных определений ГЛОНАСС

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Выбор даты:

Смотреть

Дата: 2017-03-06

Crawing	Ошибка навигационных определений (p=0,95)		лений (p=0,95)	C
Станция	по широте (м.)	по долготе (м.)	по высоте (м.)	Среднее кол-во ка в навит. определениях
Арти	5.92	4.46	13.63	9
Геленджик	5.79	5.75	13.49	8
Иркутск	4.88	4.30	11.32	9
Магадан	5.10	4.32	12.12	9
Менделеево	5.78	5.03	13.40	9
Новосибирск	5.02	4.45	11.85	9
Ноябрьск	4.96	4.39	13.27	9
Прогресс	7.10	7.28	17.64	8
РНИИ	6.13	7.47	13.86	9
Светлое	5.75	4.69	13.22	9

Lat.

Long.

Height Number of satellites





Точность навигационных определений GPS

Выбор даты:

. Смотреть

Дата: 2017-03-06

Ground	Ошибка навигационных определений (р=0,95)		лений (p=0,95)	
Станция	по широте (м.)	по долготе (м.)	по высоте (м.)	среднее кол-во ка в навит. определениях
Арти	4.81	4.11	12.60	11
Геленджик	6.04	4.14	13.58	10
Иркутск	4.79	3.68	12.26	10
Магадан	4.86	3.91	12.83	11
Менделеево	4.70	4.30	11.55	11
Новосибирск	4.98	4.06	12.55	11
Ноябрьск	4.08	4.30	12.70	11
Прогресс	3.95	4.07	14.56	11
РНИИ	4.59	6.62	11.72	11
Светлое	4.76	4.13	13.30	11

Lat.

Long.

Height Number of satellites





GPS Structure

- Space Segment
- Control Segment
- User Segment







Space segment

A constellation of at least 24 satellites orbiting the Earth in nearly circular orbits at an altitude of about 20,000 km. They occupy six orbits inclined at 55 to the equator, each with four primary satellites which are unevenly distributed. The orbital period is about 12 h.







Control segment

The control segment is responsible for the overall control and maintenance of the system, and its functionalities are:

- Monitoring and maintaining the orbit of each satellite in the constellation by manoeuvring and relocation (if needed).
- Ensuring the performance of the satellites.
- Maintenance of the GPS system time.
- Prediction of the ephemerides and clock parameters of each satellite, and periodic uploading of this information to keep the navigation message up to date.







User segment

This segment consists of receivers which receive the radio frequency signals from GPS satellites and estimate their position, velocity and time. Users of GPS can be classified into civilian and military.



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How GNSS work?

Each satellite transmits a radio signal that contains a pseudo-random noise (PRN) code and a navigation message. The PRN code is used by the receiver to find the transit time, as a preliminary to calculating the range (called pseudo-range) from the satellite to the receiver by multiplying this by the speed of light. It also calculates the satellite's position from the information in the navigation message.



With the information from at least three satellites the receiver can use the process of trilateration to calculate its own position in terms of latitude, longitude and altitude. The signal from the fourth satellite is needed to cancel the receiver's clock bias.





GPS Signals

GPS satellites transmit signals on two frequencies L1 and L2. Each frequency is modulated by pseudo-random noise (PRN) sequence for precise range measurements. A coarse acquisition (C/A) code is associated with the standard positioning service (SPS) for civilian use. The precise code (P-code) is associated with the precise positioning service (PPS). This is further encrypted to the P(Y)-code for authorized military users. L1 is modulated by both C/A and P-codes, whereas L2 is modulated by P-code only.





GPS Signals

With the removal of selective availability (SA) both the codes have equal accuracy, which is around 5 to 30 m for single GPS receivers. The receiver must determine the position of the satellite in order to use the navigation message to convert the range measurements into the position and velocity of the user. The navigation message is superimposed on both the L1 and L2 carriers.





GPS Signals

Fundamental frequency $f_0=10.23$ MHz L1 = 154. $f_0=1575.42$ MHz L2 = 120. $f_0=1227.60$ MHz C/A code 1ms long sequence of +-1 step at $f_0/10$ P code 267 day long sequence of +-1 step at f_0 Message 1500 bit sequence of +-1 step at 50 bps Phase modulation technique – 180^o Phase Shift



A GPS receiver can make basically only two kinds of measurements: pseudo-range, and carrier beat phase. **Pseudo-range** is the time shift required to line up a replica of the code generated in the receiver with the received code from the satellite multiplied by the speed of light. Ideally the time shift is the difference between the time of signal reception and the time of emission (measured in the satellite time frame). In fact, the two time frames will be different, which introduces a bias into the measurement. These biased time delay measurements are thus referred to as pseudo-ranges.



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The precision of pseudo-range measurements is 1% of the period between successive code epochs. For the C/A-code the range measurement precision is 3 metres.







For the P-code, the numbers are ten times more precisely in comparison with C/A code, successive epochs are 0.1 microsecond apart, implying a measurement precision of 1 nanosecond. When multiplied by the speed of light, this implies a range measurement precision of 30 centimetres.



Carrier beat phase is the phase of the signal which remains when the incoming Doppler-shifted satellite carrier is differenced (beat) with the constant frequency generated in the receiver. It can be obtained as a byproduct of the correlation channel or from a squaring channel. A squaring channel multiplies the received signal by itself to obtain a second harmonic of the carrier, which does not contain the code modulation.

$$y^2 = A^2 \cos^2(\omega t + \phi) = A^2 [1 + \cos(2\omega t + 2\phi)]/2$$

Since $A^2=1$ he resulting signal, y^2 is then pure carrier, but at twice the original frequency.



Because the wavelength of the carrier is much shorter than the wavelength of either of the codes, the precision of carrier beat phase measurements is much higher than the precision of code pseudo-ranges. For the GPS L1 carrier signal, the wavelength is about 20 cm. The phase measurements can be made to about 1% of the wavelength, this implies a precision of 2 mm.



Two disadvantages of carrier beat phase measurements have to be resolved:

- Obtaining the initial number of integer cycles of the carrier between the satellite and the receiver is very difficult.
- Maintaining an integer cycle count as the satellite-toreceiver range changes with time is something most good quality GPS receivers can do most of the time. However, due to noisy signal or a shaded antenna, any receiver could suffer cycle slips, or the loss of a coherent integer cycle count. In many cases painstaking postprocessing permits the detection and correction of cycle slips.


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GPS Error Sources

• Satellite Clock Error

Although fairly accurate, over time satellite clocks drift away from GPS system time. Based on the observed satellite clock data provided by monitoring stations, the control segment estimates the correction parameters for the satellite clocks and uploads them to the satellites, which broadcast these parameters in the navigation message to enable a receiver to correct for satellite clock error in a measured range.





• Receiver Clock Error

Receiver clocks are meant to be inexpensive for affordability. Consequently they are much less accurate than the satellite clocks and contain a bias. This clock bias error affects all measurements in same manner. Therefore if four pseudo-range measurements are available, the clock bias can be estimated along with the three components required to determine the position of the user. This is usually done by a Kalman filter.



Ionospheric and tropospheric effects on GPS signals for various elevation angles





• Ionosphere Delay

The ionosphere is the layer of the atmosphere with ionized gases approximately from 60 to 1,000 km above the Earth's surface. The ionization level of this layer changes with solar activity, affecting the refractive indices of the various layers of the ionosphere and, as a result, changing the transit time of a GPS signal. Satellite elevation also adds to the variability of this error, because signals from low elevation satellites pass a greater slant-range distance through the ionosphere than those at higher elevations. The ionospheric delay in pseudo-range and carrier phase is equal but opposite in sign and is expressed as:



$$I_{\phi} = -I_{\rho} = \frac{40.3TEC}{f^2}$$
$$I_1 = \frac{40.3TEC}{f_1^2}; I_2 = \frac{40.3TEC}{f_2^2}$$
$$I_1 = \frac{f_2^2}{f_1^2} I_2 \Rightarrow I_1 f_1^2 = f_2^2 I_2$$

Dual frequency GPS receivers equipped with both L1 and L2 are able to calculate ionospheric delay much more accurately. A single frequency receiver relies on the Klobuchar model, whose parameters are broadcasted by the satellites.





Tropospheric Delay

The troposphere is extending from 8 to 40 km above the Earth's surface and it is mainly composed of dry gases $(N_2 \text{ and } O_2)$ and water vapor. Unlike the ionosphere, the troposphere, being electrically neutral, is non-dispersive for GPS frequencies but since it is refractive it causes a decrease in speed relative to free space. Therefore apparent ranges appear longer by 2.5–25 m depending upon the elevation angle of the satellite. Tropospheric errors are consistent between L1 and L2 carriers. Tropospheric delay has a dry component responsible for 90 % of the delay and a wet component for the rest 10 %.



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Multipath Errors



Multipath is a major error source in urban environments where the GPS signal is able to reach the receiver by several different paths. These paths include direct line of sight and reflected signals from other objects around the receiving antenna. The signal that arrives

indirectly is delayed and has a lower signal to noise ratio. Multipath distorts the original signal because of interference with the reflected signals at the receiving antenna. This can cause a position error in excess of 10 m. The multipath error is two orders of magnitude lower for carrier phase measurements than for pseudo-range measurements.



Ground Reflection



when Θ≈45⁰

δp≈2h sin⊖



Why always the error in height determination is bigger?



Vertical datum is a surfice, that best fit mean sea level. h_t is orthometric height (topographic). The difference Hh is not more than 100 m!



Satellite Orbital Errors

Satellite orbital errors are caused by difference between the actual position of a satellite in space and the position of the satellite calculated by the receiver using ephemeris data. Depending on the previous motion of the satellite and knowledge of Earth's gravity, the orbital errors are predicted by the control segment and uploaded to the satellites for broadcast to the users as ephemeris data. Since the ephemeris model is a curve fit to the measured orbit, it will include time varying residual errors relative to the actual orbit. Typically, this error is between 2 and 5 m.



The idealized satellite motion obeys to Kepler laws: 1. The satellite motion occurs in a stationary plane which contains the centre of mass of the earth. The orbit is a conic with one of the foci located at the earth's centre of mass (the geocentre).

2. The satellite radius vector (i.e., the line joining the earth's centre to the satellite) sweeps out equal areas in

equal time.





- Non-central part of the earth's gravitational attraction. The density distribution within the earth departs considerably from radial symmetry.
- Third-body effects.
- Tides. The gravitational attractions of the moon and the sun also have an indirect effect on satellite motion.
- Solar radiation pressure.
- Atmospheric drag.



Determination of Satellite position

One of the most precise positioning techniques is laser ranging. A short pulse of intense laser light is directed to a satellite equiped with light reflector. A part of reflected light is collected by a telescope with sensitive photo receiver (from a single photon and more). The achieved accuracy of ranging $\sim 2 \text{ cm RMS}$. The accuracy of determination of the baseline -3 - 5 cm. The stations are static and mobile ones.



The prediction of the position of the satellites in time is a rather difficult task. It is possible for the user of the system to improve the satellite position estimates in two different modes: by using additional tracking stations in real time or by using optimal tracking algorithms over past (old) data (forward - backward). The enhancement could reach more than an order.



Receiver Noise

This is a random measurement noise intrinsic to the electronics of a GPS receiver. It is caused by the cumulative effects of antenna circuitry, cables, thermal noise, RF signal interference, signal quantization and sampling. Since it is a function of the signal to noise ratio, it varies with the satellite elevation angle. It gives rise to an incorrect measurement of the transit time of the GPS signal. As in the case of the multipath effect, receiver noise is two orders of magnitude lower for carrier phase measurements than for pseudo-range measurements.



GPS GEOMETRY AND ACCURACY

The accuracy depends on two factors: the satellite configuration geometry, and the measurement accuracy. The effect of satellite configuration geometry is expressed by the dilution of precision (DOP) factor, which is the ratio of the positioning accuracy to the measurement accuracy.



The more common DOPs are:

- VDOP the standard deviation in height (Vertical)
- HDOP the accuracy in 2D Horizontal position
- PDOP the accuracy in 3D Position
- TDOP the standard deviation in Time
- HTDOP · the accuracy in Horizontal position and Time
- GDOP · the accuracy in 3D position, and time (Geometrical)

$$\sigma_{posit} = DOP. \sigma_{meas}$$







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Differential GPS (DGPS)

DGPS is broadly categorized into two techniques based on the area in which they can mitigate GPS errors, the local area DGPS (LADGPS) and the wide area DGPS (WADGPS).



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Local Area DGPS

In LADGPS, a single reference station (RS) calculates the pseudo-ranges with code-phase measurements and, based on accurate knowledge of its position, determines the bias in the measurements for all the visible satellites. These differential corrections are transmitted to the users in the area on a radio link for real-time applications. The errors include ephemeris prediction errors, uncorrelated satellite perturbations, and errors introduced by the atmosphere. These corrections are more accurate for users who are closer to the RS.





Local Area DGPS







Wide Area DGPS

As the GPS receiver moves farther away from the associated RS, the correlation between errors reduces and they become spatially correlated, with the result that the errors estimated at the RS can become different from those experienced by the user. This situation can be ameliorated by expanding the coverage by adding more reference stations along the perimeter of the area covered by the single RS. The receiver weights the corrections based on its proximity to each individual RS. This method of differential correction, called wide area DGPS (WADGPS)



Signal acquisition

The first thing any GPS receiver must do is to acquire the satellite signals. If nothing is known the receiver must perform a 'cold start' and do an 'all sky search.' This involves selecting each of the 32 possible C/A-codes, and searching over all possible Doppler shifted frequencies for each signal, attempting to lock on to any one of them. This search may take as long as 30 minutes. This time can be reduced considerably if the receiver knows which satellites are visible, which limits the search to only a few of the 32 possible codes. If the satellite positions are known as well, then estimates of the **Doppler shift can be predicted, limiting the frequency band over** which the search must be performed. In this way, the search can be reduced to a few minutes.





Signal Wavelengths

Signal	Frequency	Wavelength
L1	1575.42 MHz	20 cm
L2	1227.60 MHz	24 cm
C/A code	1 MHz	300 m
P code	10 MHz	30 m



From ephemeridis known From time of flight measured



neasured
$$\rho_1, \rho_2, \rho_3, \rho_4$$

 $R = ?$
 $\vec{S}_i - \vec{R} = \vec{\rho}_i + c\Delta t, i = \overline{1,4}$

 S_1, S_2, S_3, S_4

Here ∆t is receiver clock shift. If i>4 the system is overdetermined. The system is solved minimising RMS error.



The L1 and L2 pseudorange measurements for the i-th satellite at time t can be modeled as:

$$\tilde{\rho}_{r_1}^i(t) = \|\boldsymbol{p}_r(t) - \boldsymbol{p}^i(t)\|_2 + c\delta t_r(t) + \frac{f_2}{f_1}I_r^i(t) + T_r^i(t) + M_{\rho_1}^i(t) + n_{\rho_1}^i(t)$$
$$\tilde{\rho}_{r_2}^i(t) = \|\boldsymbol{p}_r(t) - \boldsymbol{p}^i(t)\|_2 + c\delta t_r(t) + \frac{f_1}{f_2}I_r^i(t) + T_r^i(t) + M_{\rho_2}^i(t) + n_{\rho_2}^i(t)$$

 $\|p_r - p^i\|_2$ is the geometric distance between the receiver position and the i-th satellite position, c is the light speed and δt_r is the receiver clock bias, identical for all channels of the receiver, $f_1 = 1575.42MHz$ and $f_2 = 1227.60MHz$ are the frequencies for L1 and L2 carrier,



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I_r^i is the Ionospheric delay caused by the ionization effect in the Ionosphere,

 T_r^i is the Tropospheric delay caused by the block from the Troposphere for the L-Band signal,

 $M_{\rho 1}{}^{i}$ and $M_{\rho 2}{}^{i}$ are multipath errors caused by signal reflections,

 $n_{\rho 1}{}^{i}$, $n_{\rho 1}{}^{i} \sim N(0, (\sigma_{\rho}{}^{i})^{2})$ is the (non-common mode) measurement noise.



The L1 and L2 carrier phase measurements $\tilde{\varphi}_{r1}^i$ and $\tilde{\varphi}_{r2}^i$ for the *i*-th satellite at time t can be modeled as: $\lambda_1 \tilde{\varphi}_{r1}^i(t) = \| \boldsymbol{p}_r(t) - \hat{\boldsymbol{p}}^i(t) \|_2 + c \delta t_r(t) + \lambda_1 N_1^i(t) + E_{cm3}^i(t) + M_{\omega_1}^i(t) + n_{\omega_1}^i(t)$ $\lambda_2 \tilde{\varphi}_{r2}^i(t) = \|\boldsymbol{p}_r(t) - \hat{\boldsymbol{p}}^i(t)\|_2 + c\delta t_r(t) + \lambda_2 N_2^i(t) + E_{cm4}^i(t) + M_{\omega_2}^i(t) + n_{\omega_2}^i(t)$ λ_1 and λ_2 are the wavelength of the corresponding carrier signals, N^{i} is the ambiguous integers representing the unknown number of whole cycles,



$$E_{cm3}^{i} = E^{i} + c\delta t^{i} - \frac{f_{2}}{f_{1}}I_{r}^{i} + T_{r}^{i},$$
$$E_{cm4}^{i} = E^{i} + c\delta t^{i} - \frac{f_{1}}{f_{2}}I_{r}^{i} + T_{r}^{i},$$

 $M_{\varphi_1}^i, n_{\varphi_1}^i, M_{\varphi_2}^i, n_{\varphi_2}^i$ are non-common mode errors similar to those of pseudorange measurements but with magnitude typically less of 1% of the respective errors in pseudorange measurements.

Thus, carrier phase measurements have much lower noise level but are biased by the unknown integer ambiguity $\{N^i\}$.



Common Mode Errors	L1, σ , meters
Ionosphere	7-10
Troposphere	1
Sv Clock	2
Sv Ephemeris	2
Non-common Mode Errors	
Multipath	0.1-3.0
Receiver Noise	0.1 - 0.7



The choice of L band carriers represents a trade-off between the limited available bandwidth in UHF and the excessive atmospheric absorption, called space loss, of C band. The L band frequency is high enough to accommodate the 20 MHz bandwidth required and the centimeter wavelength enables precision velocity

measurement.

	C/A	Р	NAV
L1	\checkmark	\checkmark	\checkmark
L2		\checkmark	\checkmark



Navigation Message

In addition to the PRN codes, a second block of information is digitally broadcast on both carriers. This navigation message is transmitted at the very slow rate of 50 bits per second, and is updated and repeated every 12.5 minutes. The navigation message contains a large amount of information provided to assist receivers in their operation. The system time of the week, known as the handover word, is provided to assist in acquiring the P code after the C/A code is acquired. The transmitting satellite provides precise ephemeris data on its own orbit for use in the navigation solution.



Also, less accurate ephemeris data for all other satellites, known as the almanac, is transmitted to permit receivers to predict when new satellites will rise above the horizon and become available. The ephemeris data is described by the classical ellipse parameters discovered and defined by Kepler.

Atmospheric delay model coefficients are provided since the magnitude of the delay varies significantly throughout the day. Satellites also provide information on their own health and the precision of range measurements that can be expected if their signals are used. In this way, receivers can pick an optimal set of satellites in view for their computations.



The message consists of binary coded data with information about the satellite's health, ephemeris (its position and velocity), clock bias parameters, and an almanac that is a less precise version of the ephemeris data but for all the satellites of the constellation. The ephemeris and clock parameters are repeated every 30 s. The chipping rates of the P-code and C/A code are 10.23 and 1.023 M/sec respectively. The navigation message is broadcasted at the much lower rate of 50 bps and therefore takes 12.5 min to transmit all of the information.












Errors

Error Source	Nominal Value (rms)
Selective Availability	8 meters
Atmospheric Delays	4 meters
Satellite Clock & Ephemeris	3 meters
Multipath	1 - 3 meters
Reciever Electronics & Vehicle Dynamics	1.5 meters
TOTAL	10 meters



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