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Tecniche e Strumenti per l'Elaborazione di Immagini ed il Riconoscimento di Forme

Parte II: Livello Alto e Strumenti Disponibili

6.

22 Novembre 1989
c/o SPERONI Spa, Spessa Po

V. Cantoni:

Riconoscimento di Forme:
Accoppiamento diretto ed
Approccio Statistico.

Dipartimento di Informatica e Sistemistica, Università degli Studi di Pavia

SHAPE :

"SHAPE IS A PROPERTY THAT CHARACTERIZES AN OBJECT'S EXTERIOR IN SPATIAL SENSE"

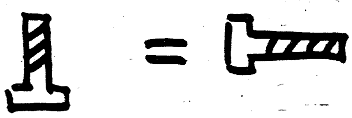
IT IS AN IMPORTANT PROPERTY THAT PERMITS TO DISTINGUISH ONE OBJECT FROM ANOTHER ON THE BASIS OF THE FORM WITHOUT REGARD TO SIZE, POSITION AND ORIENTATION.

DEPENDING ON THE PARTICULAR APPLICATION THE RECOGNITION OF THE SHAPE MAY BE DESIRED WITH OR WITHOUT REGARD TO :

RELATIVE SIZE

ORIENTATION

ex. $N \neq Z$ CHARACTER RECOGNITION



OBJECT RECOGNITION

MEANING OF SHAPE

DEPENDING ON THE APPLICATION A SHAPE DEFINITION MAY FOLLOW DIFFERENT BASES :

- GIVEN A SET OF POINTS C^* WHICH DEFINES A MODEL OF ARBITRARY SHAPES, WE ASSUME THAT ALL THE SET OF POINTS C_i WHICH CAN BE PRODUCED BY TRANSLATING, ROTATING AND SCALING THE MODEL C^* HAVE THE SAME "SHAPE", THAT IS BELONG TO THE FAMILY C. (RIGID MOTION)

  APPLICATIONS : RECOGNITION OF MECHANICAL PARTS,

- OBJECTS HAVING THE SAME SHAPES POSSESS THE SAME SEMANTIC ATTRIBUTES OR STRUCTURAL PRIMITIVES

  APPLICATIONS : CROMOSOME CLASSIFICATIONS , ETC

THE RECOGNITION PROCESS

OBJECT RECOGNITION IS A PROCESS THAT CLASSIFIES A PART OR A FEATURE AS DIFFERENT FROM THE OTHERS IN A SET.

MOST INDUSTRIAL TASKS INVOLVING RECOGNITION, PRESENT PARTS IN AN INFIXTURED STATE, BUT OFTEN ALLOWS THE APPEARANCE OF OBJECTS OR FEATURES TO BE HIGHLY CONTROLLED THROUGH STRUCTURED LIGHTING.

. THE STEPS OF A RECOGNITION PROCESS :

- DETECTION (YES/NO)
- LOCATION (X_0, Y_0)
- ORIENTATION (θ)

STRA[illegible] [illegible] MANUFACTURE[illegible]

- THE OBJECTS UNDER SCRUTINY HAVE A SMALL NUMBER OF STABLE STATES

- THE OBJECT UNDER SCRUTINY CAN BE CONTROLLED BOTH IN POSITION AND APPEARANCE

  MATERIAL THICKNESS GAUGING, FEATURE PRESENCE CHECKING, ETC

- THE OBJECT UNDER SCRUTINY IS UNCONTROLLABLE IN EITHER POSITION OR APPEARANCE, BUT NOT SIMULTANEOUSLY

  OBJECT LOCATION FOR ROBOT GUIDANCE, PART RECOGNITION AND SORTING, ETC

- PROBLEMS INVOLVE SIMULTANEOUSLY POSITION AND APPEARANCE UNCERTAINTIES

  CHARACTER READING, INTEGRATED CIRCUIT POSITIONING, ETC

J. K. WEST
Computer Design [illegible]

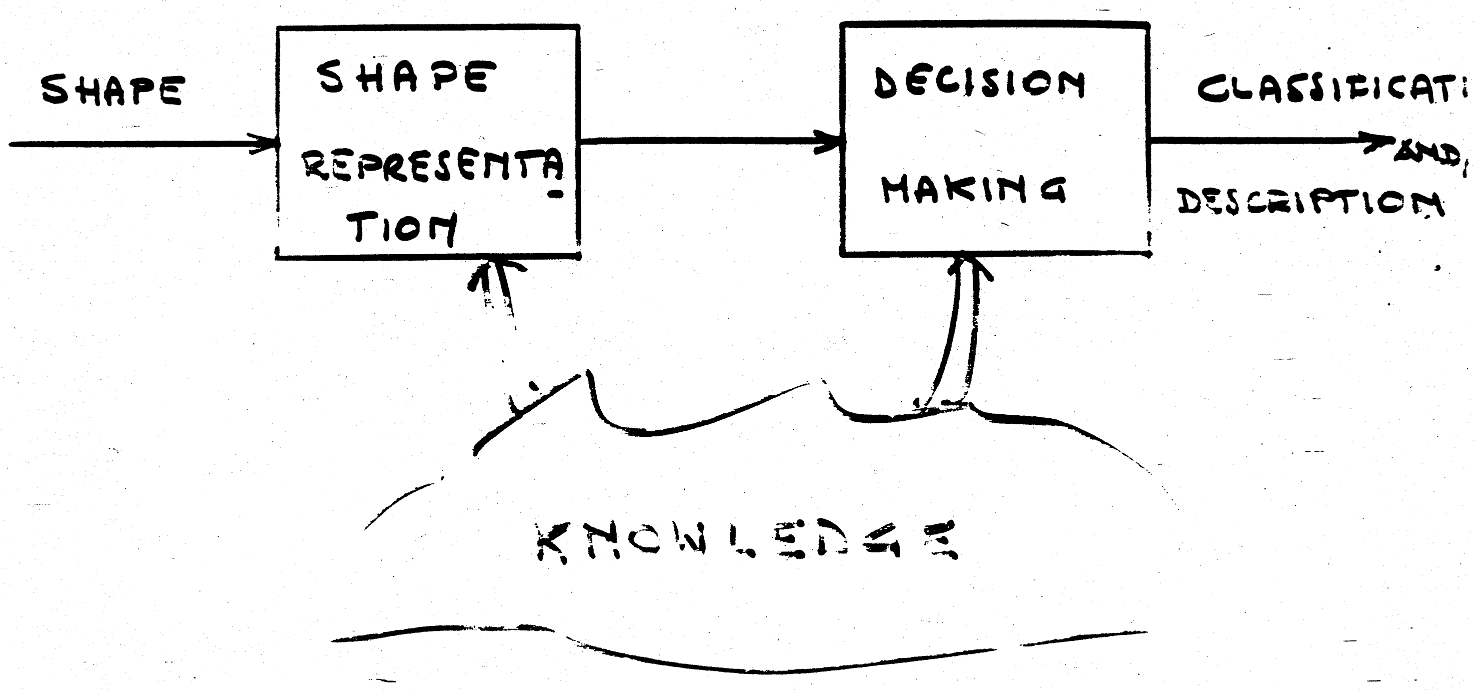
OBJECT RECOGNITION FROM SINGLE PERSPECTIVE VIEWS

HUMAN BEINGS HAVE THE CAPACITY TO IDENTIFY AND RECOGNIZE COMPLEX THREE-DIMENSIONAL OBJECTS FROM SINGLE PHOTOGRAPHS

- WE POSSESS A REMARKABLE GEOMETRIC REASONING POWER THAT ALLOWS US TO INFER THE INVERSE OF THREE DIMENSIONAL PROJECTIONS

- WE WORK WITH CERTAIN EXPECTATIONS ABOUT THE SCENE AND WE HAVE A DATABASE OF POSSIBLE OBJECTS FROM WHICH WE CHOOSE THE MOST LIKELY CANDIDATE

FRAME WORK OF SHAPE RECOGNITION



SHAPE RECOGNITION APPROACHES

APPROACHES	SHAPE REPRESENTATION	DECISION MAKING
TEMPLATE MATCHING	SEGMENT (RAW DATA)	DIRECT MATCHING
DECISION THEORETIC	FEATURE VECTOR	DISCRIMINANT FUNCTIO MINIMUM DISTANCE. MAXIMUM LIKEHOOD MINIMUM BAYES RISK
SYNTACTIC	STRING TREE GRAPH	PARSING GRAPH MATCHING ERROR CORRECTING – , –
STRUCTURAL	TABLE	STATISTICAL ANALYSIS
HYBRID	MIXED IN VARIOUS WAY	

TEMPLATE MATCHING APPROACH: PRELIMINARY CONSIDERATIONS

- A SET OF TEMPLATES OR PROTOTYPES, ONE FOR EACH CLASS, IS STORED IN THE MACHINE. THE INPUT SEGMENT WITH UNKNOWN CLASSIFICATION IS MATCHED OR COMPARED WITH THE TEMPLATE OF EACH CLASS AND THE CLASSIFICATION IS BASED ON A PRESELECTED MATCHING CRITERION OR SIMILARITY MEASURE (E.G. CORRELATION)

- USUALLY, FOR SEMPLICITY OF THE MACHINE INPUT SEGMENTS AND TEMPLATES ARE REPRESENTED IN THEIR RAW-DATA FORM.

- THE DISADVANTAGE OF THIS APPROACH IS THAT IT IS SOMETIMES DIFFICULT TO SELECT A GOOD TEMPLATE FOR EACH CLASS AND TO DEFINE AN APPROPRIATE MATCHING CRITERION. THIS DIFFICULTY IS ESPECIALLY REMARKABLE WHEN LARGE VARIATIONS AND DISTORTIONS ARE EXPECTED IN THE SEGMENT UNDER STUDY.
  (FLEXIBLE TEMPLATE MATCHING OR "RUBBER MASK" TECHNIQUES HAS BEEN PROPOSED)

THE MATCHING PROBLEM

WE WISH TO RECOGNIZE COMPLETE, ISOLATED, RIGID SHAPE IN THE PLANE, KNOWN TO BE INSTANCES OF A SMALL SET OF PROTOTYPE (OR TEMPLATE) SHAPES, BUT PERTURBED BY LOCATION NOISE AND ARBITRARILY TRANSLATED, ROTATED AND SCALED

WE CAN CONSIDER A FEATURE EXTRACTOR YIELDING SHAPE DESCRIPTIONS IN THE FORM OF SETS OF POINTS IN THE PLANE, EACH POINT REPRESENTING THE ESTIMATED LOCATION OF A FEATURE OF THE SHAPE

WE ALSO REQUIRE THAT THE FEATURE EXTRACTOR DOES NOT OMIT GOOD FEATURE POINTS OR INVENT SPURIOUS ONES (THE PROTOTYPE AND INSTANCE POINT SET HAVE THE SAME CARDINALITY)

THE MATCHING PROBLEM IS:

GIVEN	A PAIR OF POINT SETS P AND Q
FIND	TOTAL MATCHING M AND REGISTRATION R
MINIMIZING	ERROR E

GEOMETRICAL AND COMBINATORIAL SUBPROBLEMS

- THE MATCHING M IS A ONE TO ONE MAPPING FROM P INTO Q (BIJECTION)

- THE REGISTRATION R IS A ONE TO ONE MAPPING FROM THE PLANE ONTO ITSELF, CONSISTING OF A COMPOSITION OF TRANSLATION (x_3, x_4), ROTATION (x_2), AN POSITIVE (NON ZERO (x_1) SCALE

$p = \langle x, y \rangle$

$$R(p) = x_1 \cdot \begin{bmatrix} \cos x_2 & \sin x_2 \\ -\sin x_2 & \cos x_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}.$$

- THE ERROR E OF A MATCHING M AND A REGISTRATION R FOR POINT SETS P AND Q CAN BE DEFINED FOR EXAMPLE AS THE SUM OF SQUARES OF LOCATION ERRORS BETWEEN MATCHED PAIRS OF POINTS FROM P AND $R(Q$

$$E = \sum_{p_i \in P} \left\| R(M(p_i)) - p_i \right\|^2$$

PRIOR CONSTRAINTS

CONSTRAINED TRANSLATION

A STRAIGHTFORWARD AND GENERAL METHOD OF CONSTRAINED TRANSLATION IS TO "MATCH" THE CENTROID OF $\bar{P}$ AND $\bar{Q}$:

$$\bar{p} = \frac{1}{n} \sum_{i=1}^{n} p_i \qquad \bar{q} = \frac{1}{n} \sum_{i=1}^{n} q_i$$

CONSTRAINED SCALE

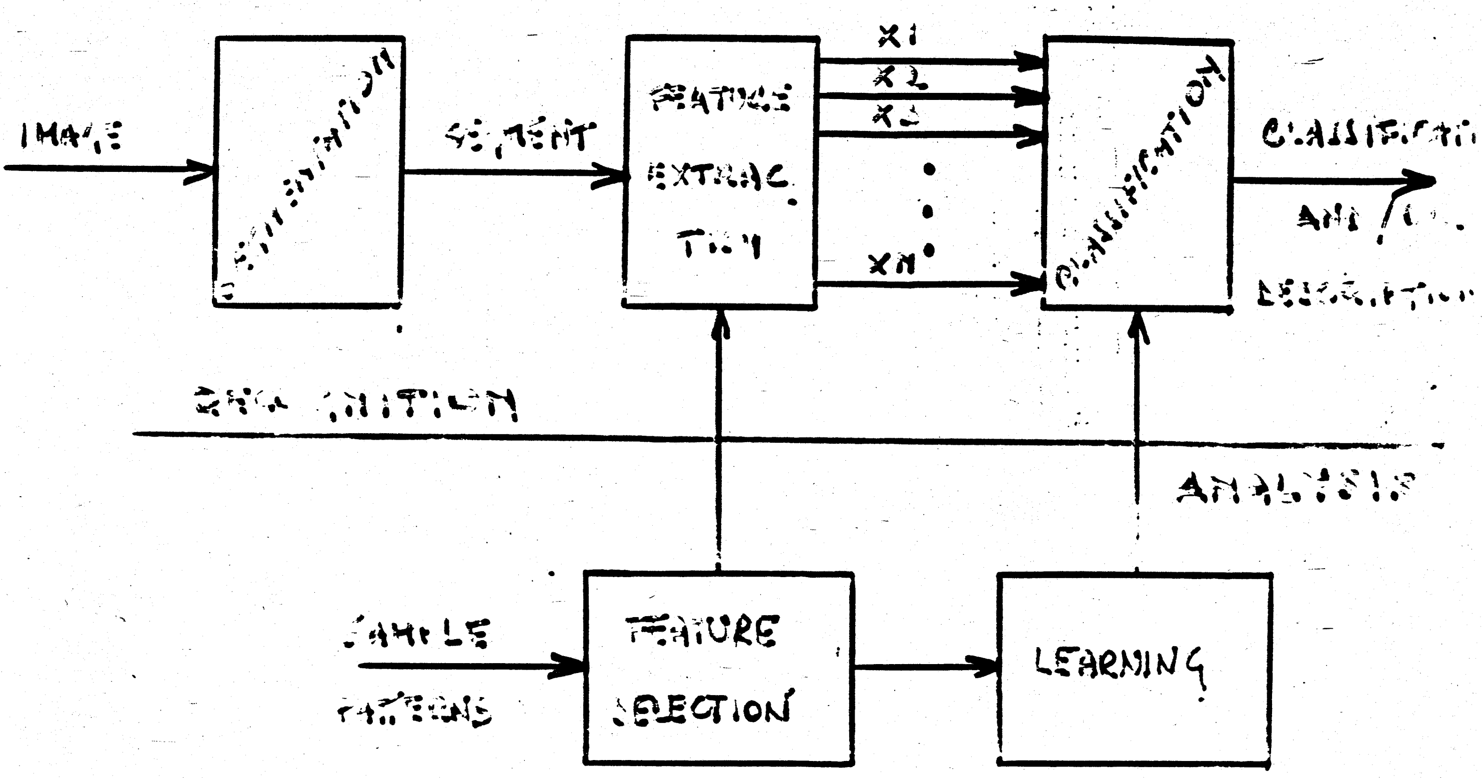
IN MANY CASES SCALE IS CONSTANT OR, IF VARIABLE, CAN BE BOUNDED FIXING, FOR EXAMPLE, THE MAXIMUM DISTANCE OF OBJECTS FROM THE CAMERA. IN ANY CASE AN EXTIMATION CAN BE GIVEN BY:

$$\frac{\sum_{i=1}^{n} \| p_i - \bar{p} \|}{\sum_{i=1}^{n} \| q_i - \bar{q} \|}$$

CONSTRAINED ROTATION

IT SEEMS NOT STRAIGHTFORWARD IN GENERAL, GIVEN $\bar{P}$ AND $\bar{Q}$ TO DERIVE INTRINSIC CONSTRAINTS ON ROTATION HOWEVER THE FEATURE EXTRACTOR MAY PROVIDE, FOR SOME OR ALL FEATURE POINTS, AN ESTIMATE OF ORIENTATION ANGLE AND WORST-CASE BOUNDS OF THIS ESTIMATE

BLOCK DIAGRAM OF THE DECISION THEORETIC APPROACH



AN APPROACH IS TO ESTABLISH M **DECISION FUNCTIONS** $d_1(x), d_2(x), \ldots d_M(x)$ WITH THE PROPERTY THAT IF A PATTERN X BELONGS TO THE CLASS ω_i THAN:

$$d_i(x) > d_j(x), \quad j = 1, 2, \ldots, M \quad j \neq i$$

WHERE M IS THE NUMBER OF CLASSES.

THE DECISION THEORETIC DEALS WITH ALGORITHMS FOR ESTIMATING THE PARAMETERS OF THE DISCRIMINANT FUNTIONS USING SAMPLE PATTERNS IN A TRAINING PROCESS

THEORY OF MOMENTS FOR ... SHAPE ANALYSIS

THE CONVENTIONAL DEFINITION OF THE TWO-DIMENSIONAL MOMENT OF ORDER $(p+q)$ OF A FUNCTION $f(x,y)$ IS:

$$M_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q f(x,y)\, dx\, dy$$

$$p, q = 0, 1, 2, \dots$$

THE DOUBLE MOMENT SEQUENCE $\{M_{pq}\}$ IS UNIQUELY DETERMINED BY $f(x,y)$; AND CONVERSELY $f(x,y)$ IS UNIQUELY DETERMINED BY $\{M_{pq}\}$.

(THE CHARACTERISTIC FUNCTION MAY BE EXPRESSED BY:

$$\Phi(u,v) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} \frac{(iu)^p}{p!}\frac{(iv)^q}{q!} M_{pq} \;)$$

A TRUNCATED SET OF MOMENTS MAY OFFER A MORE CONVENIENT AND ECONOMICAL REPRESENTATION OF THE ESSENTIAL SHAPE CHARACTERISTICS OF AN IMAGE SEGMENT THAN A PIXEL BASED REPRESENTATION

- OPERATIONS SUCH AS ROTATION, TRANSLATION AND SCALE CHANGE MAY BE MORE EASILY ACHIEVED IN THE MOMENT DOMAIN THAN THE ORIGINAL PIXEL DOMAIN

TRANSFORMATIONS OF CONVENTIONAL MOMENTS

SCALE CHANGE λ

$$M'_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q \, f(x/\lambda, y/\lambda) \, dx \, dy$$

$$M'_{pq} = \lambda^{2+p+q} M_{pq}$$

TRANSLATION (a,b)

$$M'_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x+a)^p (y+b)^q \, f(x,y) \, dx \, dy$$

$$M'_{pq} = \sum_{r=0}^{p} \sum_{s=0}^{q} \binom{p}{r} \binom{q}{s} a^{p-r} b^{q-s} M_{rs}$$

ROTATION θ

$$M'_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x\cos\theta + y\sin\theta)^p (y\cos\theta - x\sin\theta)^q \, f(x,y) \, dx \, dy$$

$$M'_{pq} = \sum_{r=0}^{p} \sum_{s=0}^{q} \binom{p}{r} \binom{q}{s} (-1)^{q-s} (\cos\theta)^{p-r+s} (\sin)^{q+r-s} M_{p+q-r-s,}$$

REFLECTION (y)

$$M'_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q \, f(x,-y) \, dx \, dy$$

$$M'_{pq} = (-1)^q M_{pq}$$

COMPLETE MOMENT SET

FOR IMAGE ANALYSIS PURPOSES AN IMAGE SEGMENT IS REPRESENTED BY A TRUNCATED SET OF MOMENTS.

A COMPLETE MOMENT SET (CMS) OF ORDER n IS DEFINED TO BE A SET OF ALL MOMENTS OF ORDER n AND LOWER

OPERATIONS WHICH CORRESPOND TO TRANSLATION, ROTATION AND SCALE CHANGE ARE CLOSED WITH RESPECT TO CMS OF $f(x,y)$

CMS OF ORDER n

$$
\begin{array}{lllll}
M_{00} & M_{01} & M_{02} & \cdots & M_{0n} \\
M_{10} & & & & \\
M_{20} & & & & \\
\vdots & & & \iddots & \\
M_{n0} & & & &
\end{array}
$$

STANDARD MOMENT SET

SCALE NORMALIZATION MAY BE ACHIEVED BY SETTING M_{00} TO 1. (FOR BINARY IMAGE CORRESPONDS TO STANDARDIZING THE AREA OF THE IMAGE SEGMENT TO 1)

$$\lambda_M = 1/\sqrt{M_{00}}$$

TRANSLATION NORMALIZATION IS USUALLY ACHIEVED BY TRANSLATING THE ORIGIN TO THE CENTER OF GRAVITY OF THE IMAGE.

$$\bar{x} = M_{10}/M_{00} \qquad \bar{y} = M_{01}/M_{00}$$

THE CENTRAL MOMENT μ_{pq}

$$\mu_{pq} = \sum_{r=0}^{p} \sum_{s=0}^{q} \binom{p}{r} \binom{q}{s} (-\bar{x})^{p-r} (-\bar{y})^{q-s} M_{rs}$$

THE PRINCIPAL AXIS MOMENTS ARE OBTAINED BY ROTATING THE AXIS OF THE CENTRAL MOMENTS UNTIL μ_{11} IS ZERO

$$\tan 2\theta = \frac{2\mu_{11}}{\mu_{20} - \mu_{02}}$$

AS AN EXAMPLE THE MOMENTS OF A PROJECTION ONTO THE MAJOR PRINCIPAL AXIS (i.e. $\{M_{p0}\}$ FOR STANDARD MOMENTS) MAY BE USED TO COMPARE AN IMAGE WITH SOME FUNDAMENTAL OBJECT SHAPES.

MEASURES THAT ARE ADEQUATE TO DESCRIBE THE SHAPE TYPE FROM SILHOUETTE MOMENTS OF SIMPLE BLOB-LIKE IMAGE SEGMENTS ARE THE FOLLOWING:

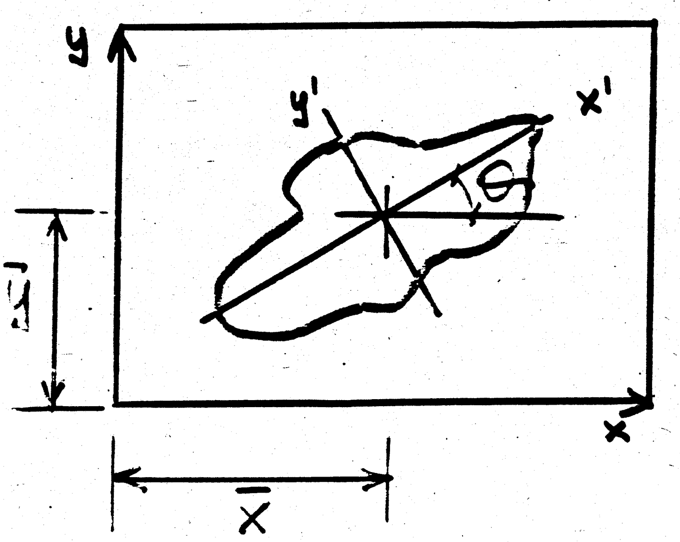
[illegible] =

[illegible] = [illegible]

[illegible] = [illegible]

[illegible] = [illegible]

STANDARD MOMENT SET

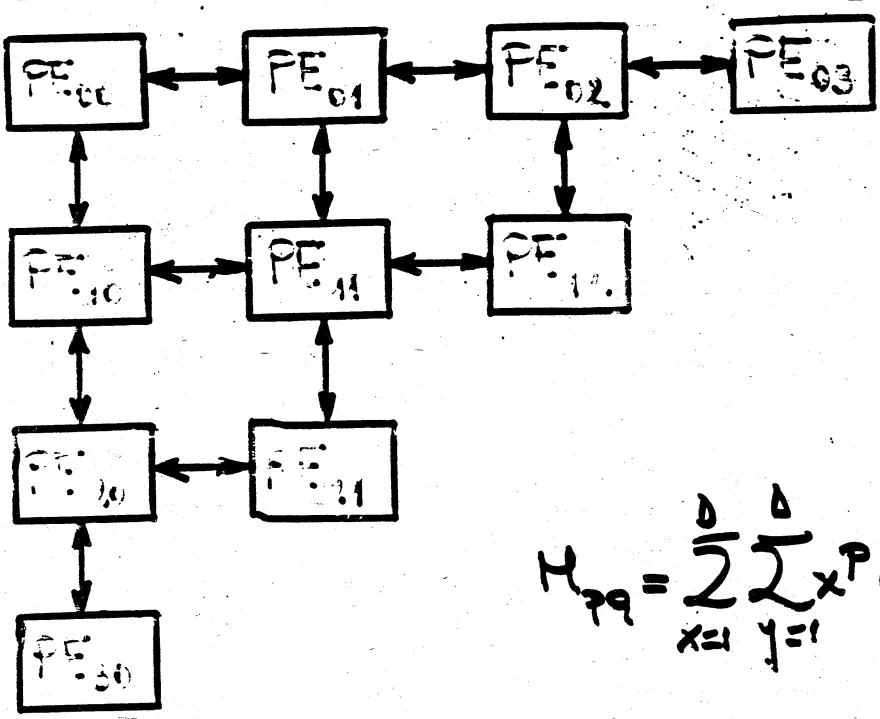


CMS OF ORDER n

$$
\begin{array}{cccc}
1 & 0 & M_{02} & \cdots\ M_{0n} \\
0 & 0 & & \\
M_{20} & & & \\
\vdots & & \iddots & \\
M_{n0} & & &
\end{array}
$$

PARALLEL MOMENT COMPUTERS

PROCESSOR ELEMENT ORGANIZATION



$$M_{pq} = \sum_{x=1}^{N} \sum_{y=1}^{N} x^p y^q f(x,y)$$

SIMD MACHINE: ONLY ONE CONTROL UNIT AND ALL PEs PERFORM THE SAME OPERATION ON DIFFERENT DATA

GENERATION - SCALE - TRANSLATION - ROTATION

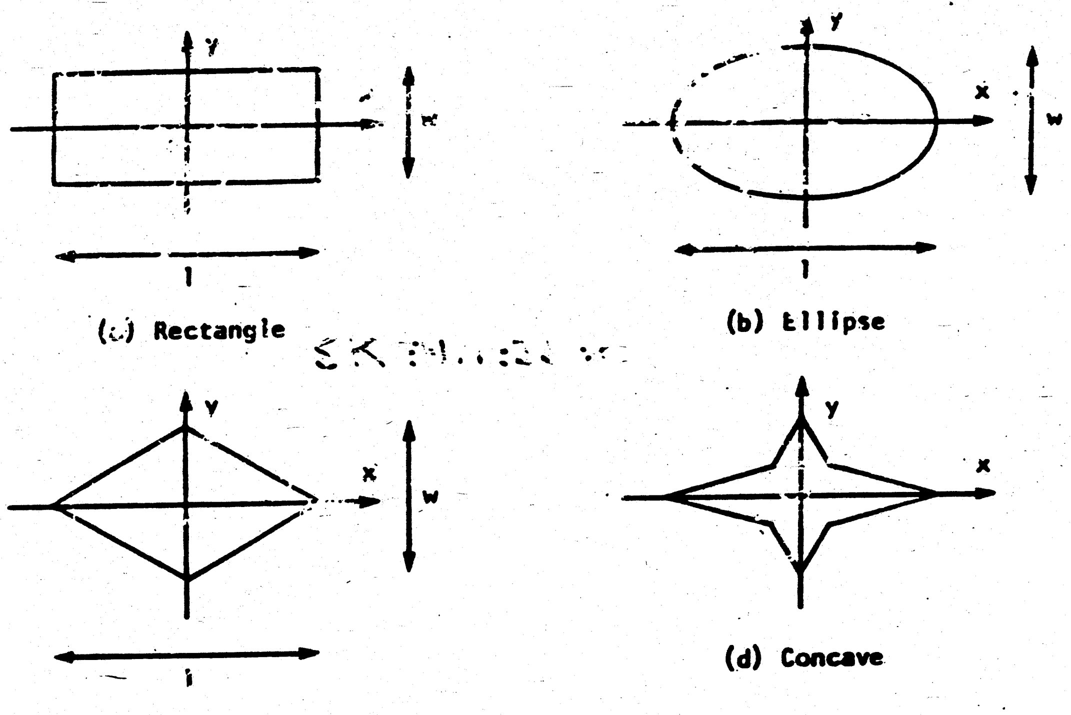
SS SS S ~

GENERIC SHAFT ANALYSIS

$$\text{SKEWNESS} = M_{30}/(M_{20})^{3/2}$$

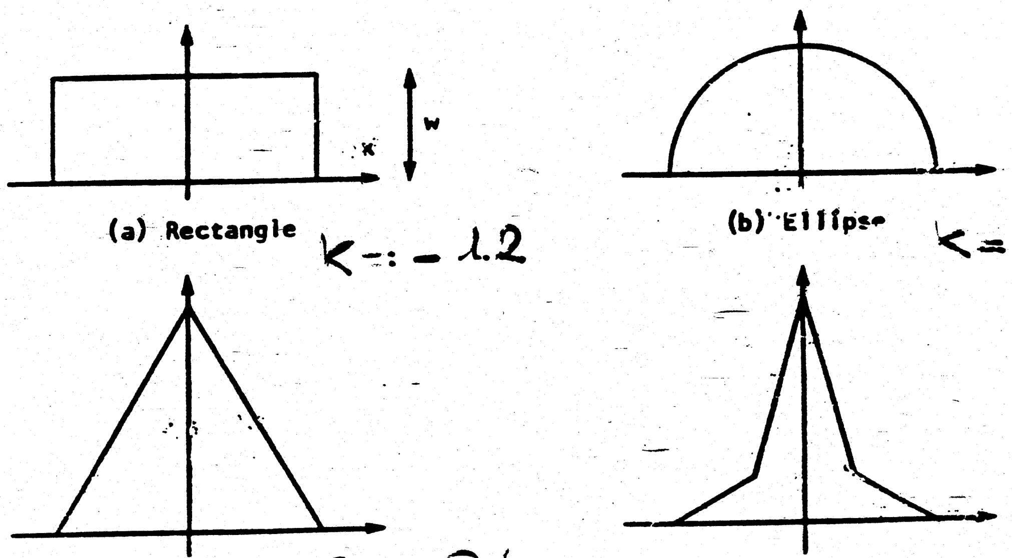
$$\text{KURTOSIS} = M_{40}/(M_{20})^{2} - 3$$

SYMMETRIC PROTOTYPE SHAPES



(a) Rectangle (b) Ellipse (c) Diamond (d) Concave

PROJECTIONS ALONG THE MINOR AXIS



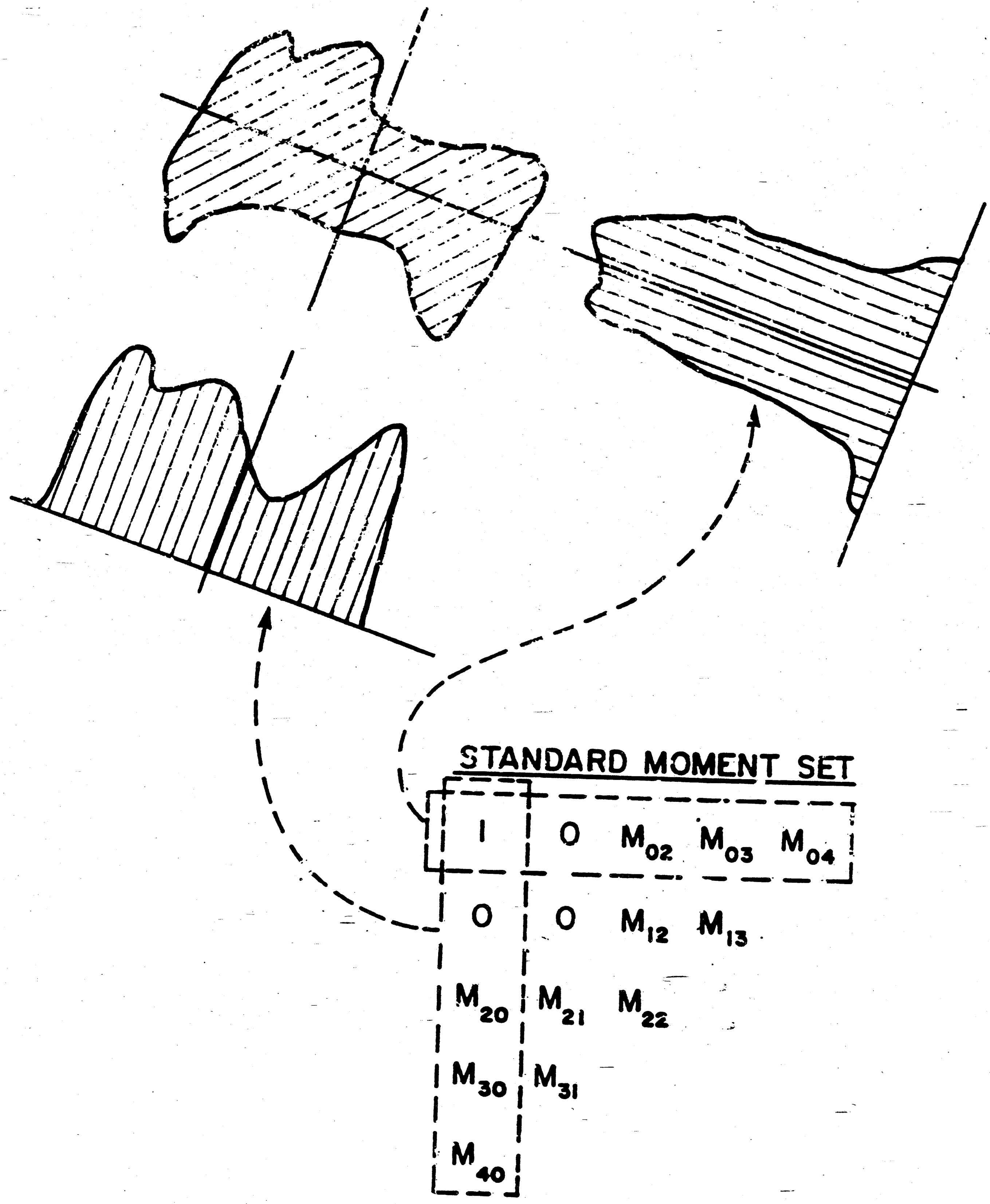
(a) Rectangle $k = -1.2$

(b) Ellipse $k = -1$

(c) Diamond $k = -0.6$

(d) Concava

PRINCIPAL AXIS PROJECTIONS AND STANDARD MOMENTS



ALTERNATIVE MOMENT REPRESENTATION

- ROTATIONAL MOMENTS

$$F_{nl} = \int_{-\pi}^{\pi}\int_{0}^{\infty} r^n \exp(il\theta) f(x,y)\, dr\, d\theta$$

WHERE $r = \sqrt{x^2+y^2}$, $\theta = tg^{-1}\left(\frac{y}{x}\right)$, n order of the moment
$0 \leq l \leq n$, $n-l$ is even

$$F_{nl} = \sum_{j=0}^{n} a_j M_{n-j,j}$$

WHERE $\{a_j\}$ are complex coefficient

ROTATIONAL MOMENTS ARE OF INTEREST AS ROTATION TRANSFORMATIONS (θ) FOR THESE MOMENTS ARE VERY SIMPLY SPECIFIED AS FAR AS A REFLECTION ABOUT AN ARBITRARY AXIS (φ) WITH RESPECT TO THE y AXIS

$$F'_{nl} = \exp(il\theta) F_{nl} \qquad F''_{nl} = \exp(-i2l\varphi) F^{*}_{nl}$$

UNFORTUNATELY TRANSLATIONS ARE MUCH MORE DIFFICULT WIT ROTATIONAL MOMENTS THAN CONVENTIONAL MOMENTS

ALTERNATIVE MOMENT REPRESENTATION

- ORTHOGONAL MOMENTS

TWO EXAMPLES: LEGENDRE MOMENTS BASED ON CONVENTIONAL MOMENTS AND ZERNIKE MOMENTS BASED ON ROTATIONAL MOMENS

LEGENDRE MOMENTS

ORTHOGONAL MOMENTS MAY BE DEFINED USING LEGENDRE POLINOMIAL BASIS FUNCTIONS $P_n(x)$ RATHER THAN CONVENTIONAL MONOMIAL

$$L_{pq} = \frac{(2p+1)(2q+1)}{4} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_p(x)\, P_q(y)\, f(x,y)\,dx\,dy$$

WHERE LEGENDRE POLINOMIALS ARE ORTHOGONAL OVER THE RANGE -1, +1, i.e.:

$$\int_{-1}^{+1} P_n(x)\, P_m(x)\, dx = \frac{2}{2n+1}\,\delta_{nm}$$

WHERE δ_{nm} IS THE KRONECKER DELTA.

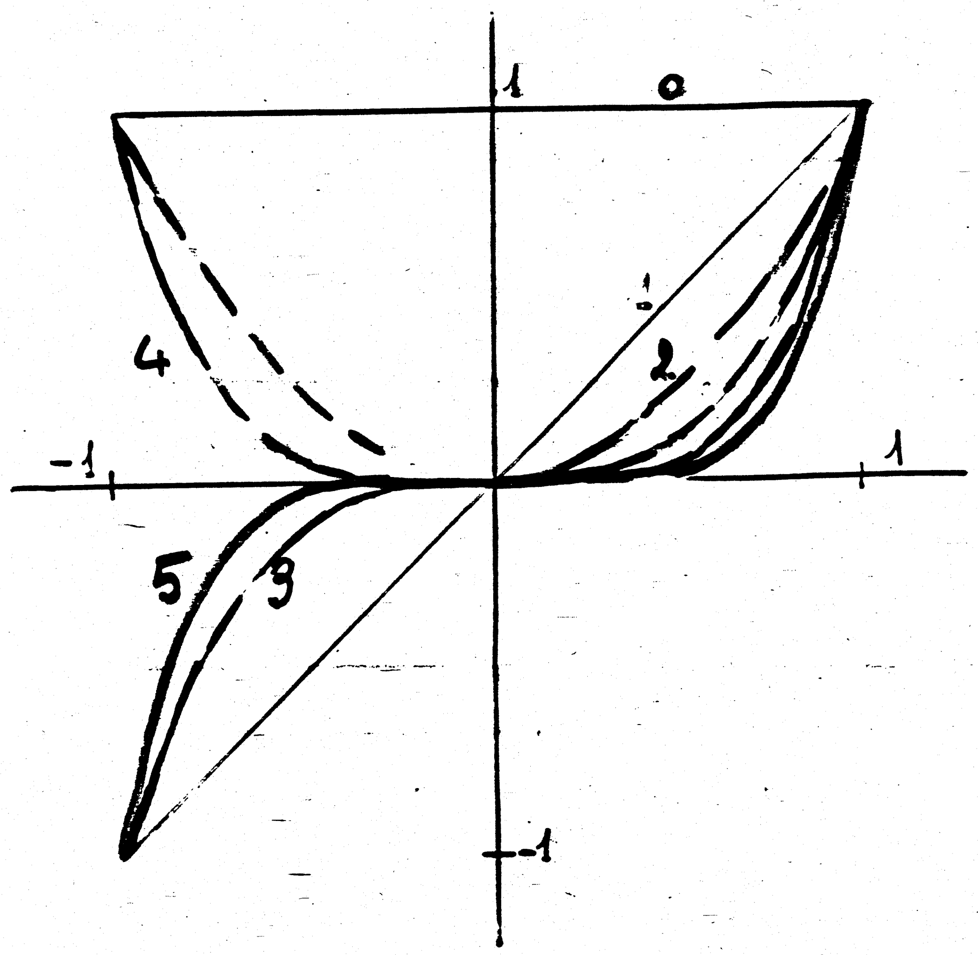
$$L_{pq} = \frac{(2p+1)(2q+1)}{4} \sum_{r=0}^{p}\sum_{s=0}^{q} C_{pr}\, C_{ps}\, M_{rs}$$

AND, DUE TO THE ORTHOGONALITY PROPERTY

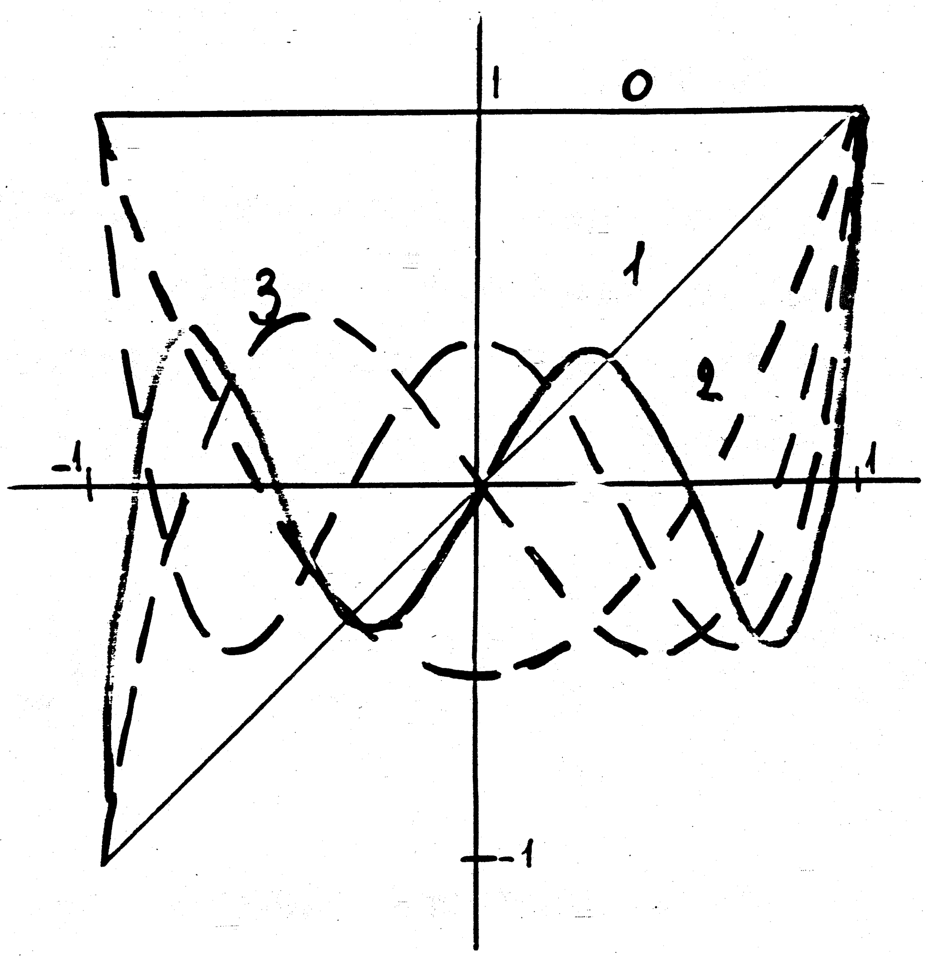
$$f(x,y) = \sum_{r=0}^{n}\sum_{s=0}^{r} P_{r-s}(x)\, P_s(y)\, L_{r-s,s}$$

THE NUMBER RANGE AND THE PRECISION NEEDED IS MUCH

SCALED MONOMIAL FUNCTIONS (ORDERS 0-5)



LEGENDRE POLYNOMIALS OF ORDER 0-5



GRAY LEV. MOMENTS

MOMENTS MAY BE COMPUTED FOR THE GREY LEVEL VALUES OF THE IMAGE AS WELL AS ITS SILUETTE

$$M_{pqr} = \iint_{\text{SEGMENT}} x^p y^q f(x,y)^r \, dx \, dy$$

THE DISTRIBUTION OF GREY LEVELS MAY CONTAIN USEFUL INFORMATION FOR CLASIFYING SEGMENT

THE MOMENTS RESULTING FROM THE ADDITION OF A BIAS OF α OF THE GRAY LEVELS AND BY A SCALE FACTOR β ARE:

$$M'_{pqr} = \sum_{j=0}^{r} \binom{r}{j} \alpha^j M_{p,q,r-j} \qquad M''_{pqr} = \beta^r M_{pqr}$$

STANDARD GREY LEVEL MOMENTS HAVING THE INFORMATION REPRESENTED IN A NORMALIZED FORM ARE OBTAINED FROM THE ORIGINAL ONE CONSIDERING

$$\alpha = - \frac{M_{001}}{M_{000}} \qquad \beta = \sqrt{\frac{M_{002}}{M_{000}}}$$

THE FINAL GLS VALUES ARE THEN

$$M_{001} = 0 \quad \{\text{grey level mean} = 0\}$$

$$M_{002} = 1 \quad \{\text{gray level variance} = 1\}$$