COMPUTER VISION
Robust estimation

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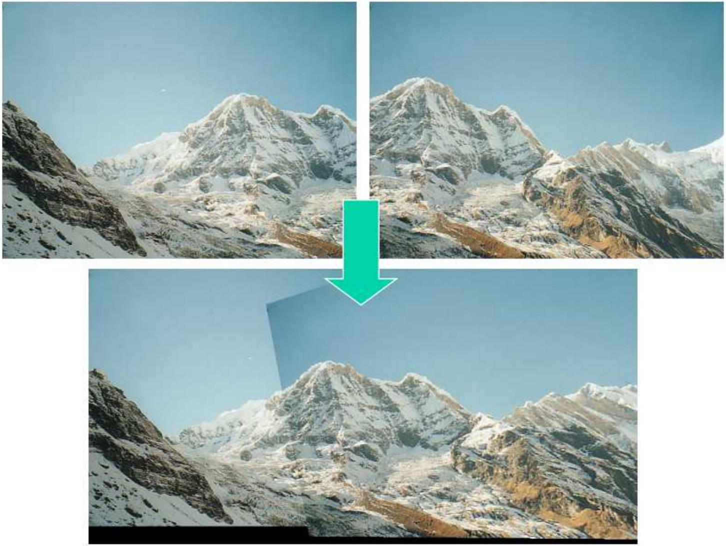
Back to our simple motivator



Objective of the procedure

Panoramic reconstruction

Problem

- Corner detection and association
- Observation (x, y, x', y') : the corner (x, y) in the first image is associated to the corner (x', y') in the second image
- if pure camera rotation pure between the two images $\tilde{\mathbf{x}}' = \mathbf{H}\tilde{\mathbf{x}}$ where :

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- by developping, we get :

$$\begin{cases} x' &= \frac{h_{00}x + h_{01}y + h_{02}}{h_{20}x + h_{21}y + h_{22}} \\ y' &= \frac{h_{10}x + h_{11}y + h_{12}}{h_{20}x + h_{21}y + h_{22}} \end{cases}$$

Panoramic reconstruction

Problem

- the unknowns are the different h_{ij}

$$\begin{cases} x'(h_{20}x + h_{21}y + h_{22}) & = & h_{00}x + h_{01}y + h_{02} \\ y'(h_{20}x + h_{21}y + h_{22}) & = & h_{10}x + h_{11}y + h_{12} \end{cases}$$

$$\begin{bmatrix} x & y & 1 & 0 & 0 & 0 & -x'x & -x'y & -x' \\ 0 & 0 & 0 & x & y & 1 & -y'x & -y'y & -y' \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Panoramic reconstruction

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1x_1 & -x'_1y_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1x_1 & -y'_1y_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n & y_n & 1 & 0 & 0 & 0 & -x'_nx_n & -x'_ny_n \\ 0 & 0 & 0 & x_n & y_n & 1 & -y'_nx_n & -y'_ny_n \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \end{bmatrix} = \begin{bmatrix} x'_1 \\ y'_1 \\ \vdots \\ x'_n \\ y'_n \end{bmatrix}$$

H is determined modulo a multiplicative factor, thus we can set h_{22} to 1.
We note that in order to estimate the homography we need $n = 4$ observations.
We must solve $\mathbf{Ah} = \mathbf{b}$ - easy !

Panoramic reconstruction

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1'x_1 & -x_1'y_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y_1'x_1 & -y_1'y_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n & y_n & 1 & 0 & 0 & 0 & -x_n'x_n & -x_n'y_n \\ 0 & 0 & 0 & x_n & y_n & 1 & -y_n'x_n & -y_n'y_n \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \end{bmatrix} = \begin{bmatrix} x_1' \\ y_1' \\ \vdots \\ x_n' \\ y_n' \end{bmatrix}$$

If $n > 4$, then the system is overdetermined. In order to find the least square solution for $\mathbf{Ah} = \mathbf{b}$, one has to :

1. compute the Singular Value Decomposition (the SVD) of $\mathbf{A}$: $\mathbf{A} = \mathbf{UDV}^T$
2. compute $\mathbf{b}' = \mathbf{U}^T\mathbf{b}$
3. find $\mathbf{y}$ defined as $y_i = b_i'/d_i$
4. the solution is $\mathbf{h} = \mathbf{Vy}$

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- underlying idea : outliers participate to "strange" solutions

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Problem framework :

- observations provided by images
  - interest points (but sometimes contours, regions etc.)
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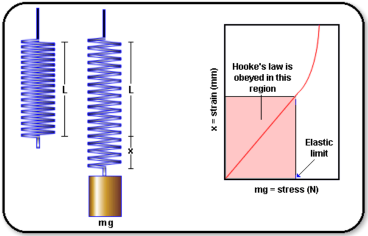
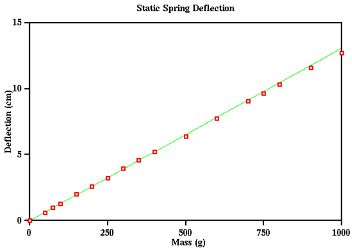
- presence of outliers which do not respect the model

Toy example

The elastic constant of a string

- Hooke's law : $F = kx$
- Objectve : $\boldsymbol{\theta} = \{k\}$
  - we vary N times the applied force, we measure the deformation
  - N observations $\{(F_i, x_i)\}$
  - minimal set of measures for determining $\boldsymbol{\theta}$: $K = 2$
  - in practice we use the N observations for a least square estimation, as the observations are noisy
- no outliers, all observations are explained by the model



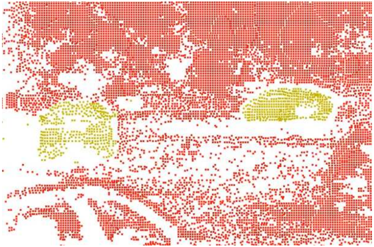


Example in vision

Estimating ego-mouvement

- N observations $\{x_i\}_{1 \leq i \leq N}$ (one obs. per pixel)
- minimal set of size K, $N \gg K$
- objective : $\boldsymbol{\theta} = \{\boldsymbol{R}, \boldsymbol{t}\}$
- an algorithm f which provides $\boldsymbol{\theta} = f(x_1, \ldots, x_K)\}$
- problem : static scene hypothesis
- dynamic elements $\Rightarrow$ observations which do not respect the model $\boldsymbol{\theta}$

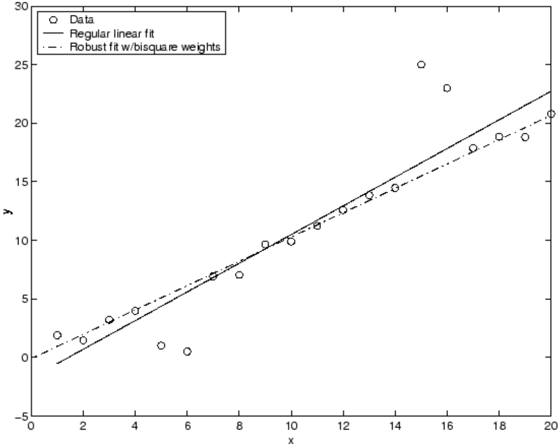
Objective : determine $\boldsymbol{\theta}$ **and** the valid observations

The source of the problem

Influence of outliers

- one may not ignore the outliers and determine the parameters of the model



- the least square based methods are very sensitive to outliers due to the quadratic error function $\rho(r_i) = r_i^2$

Two types of approaches

Analysis of the set of residuals

- Least Median of Squares (LMedS) ; we replace the sum by the median of residuals :

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In any case, we must separate the inliers, and only then we can apply the classical LMS.

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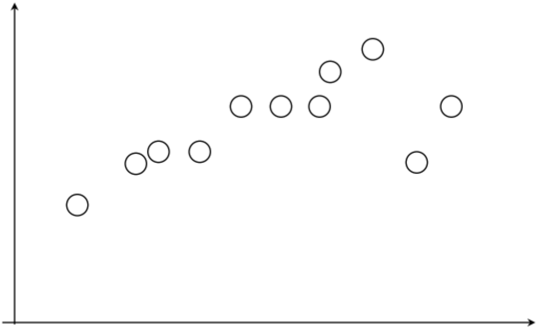
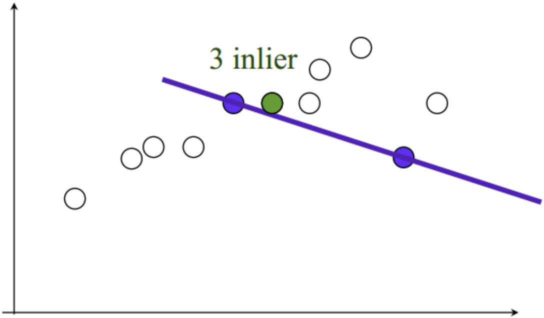
- depending on the application and on the inlier proportion

Example in 2D



Initial set

Example in 2D



Fit line - 3 inliers

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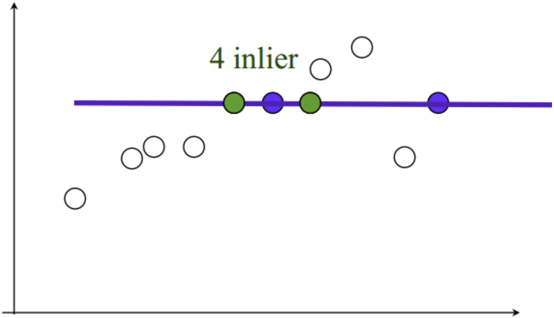

Fit line - 4 inliers

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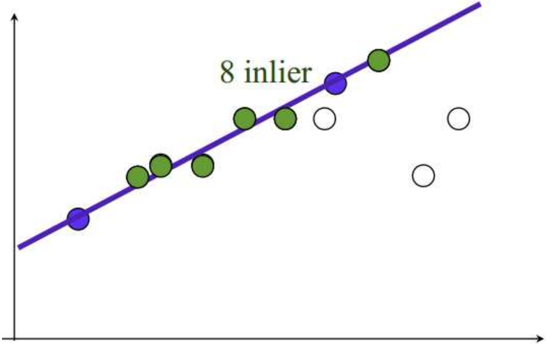

Fit line - 8 inliers

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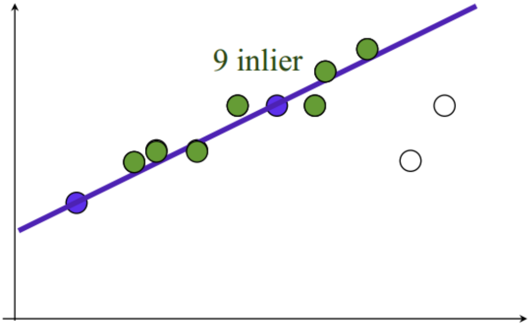
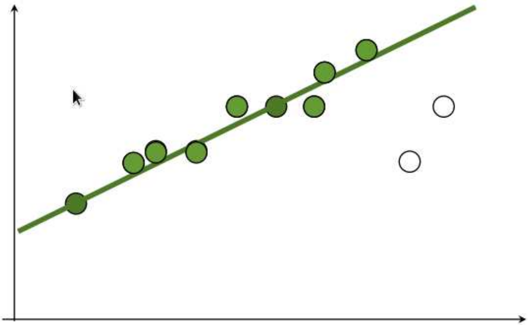


Fit line - 9 inliers

Example in 2D



Final estimation by least squares

RANSAC

Question 1

Let us consider a parameter estimation problem with $\boldsymbol{\theta} \in \mathbb{R}^5$. Assuming that the observations exhibit an outlier percentage $f = 0.4$, what is the number of draws T we should perform in order to recover the correct model parameters with a probability $p = 0.99$?

RANSAC

Question 2

Using a LASER device, a small robot has mapped an empty room. The result is a point cloud, in which 40%, 30% et 20% of the points belong to three walls respectively, and 10% of the points represent outliers. What is the number of draws required in order to recover the largest wall with a probability $p = 0.99$?

RANSAC

Question 3

For the same setting as in Question 2, what is the number of draws required in order to recover any wall with a probability $p = 0.99$?

RANSAC

Question 4

For the same setting as in Question 2, propose an algorithm for extracting all the walls from the point cloud.