

The photometric approach

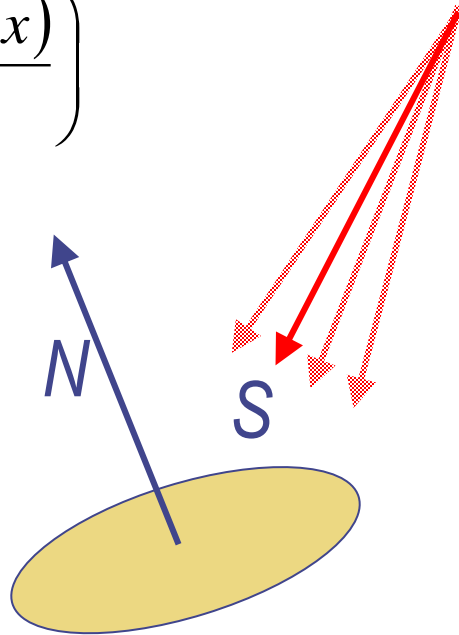
- Artificial images

Standard nearby point source model

- N is the surface normal, ρ is diffuse albedo, S is source vector whose length is the intensity term

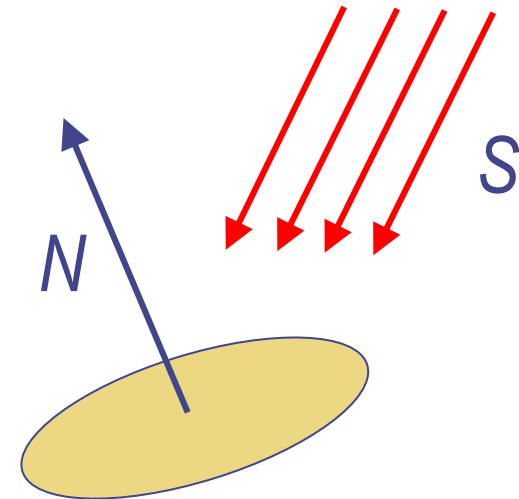
Nearby point source model

$$\rho_d(x) \left(\frac{N(x) \cdot S(x)}{r(x)^2} \right)$$

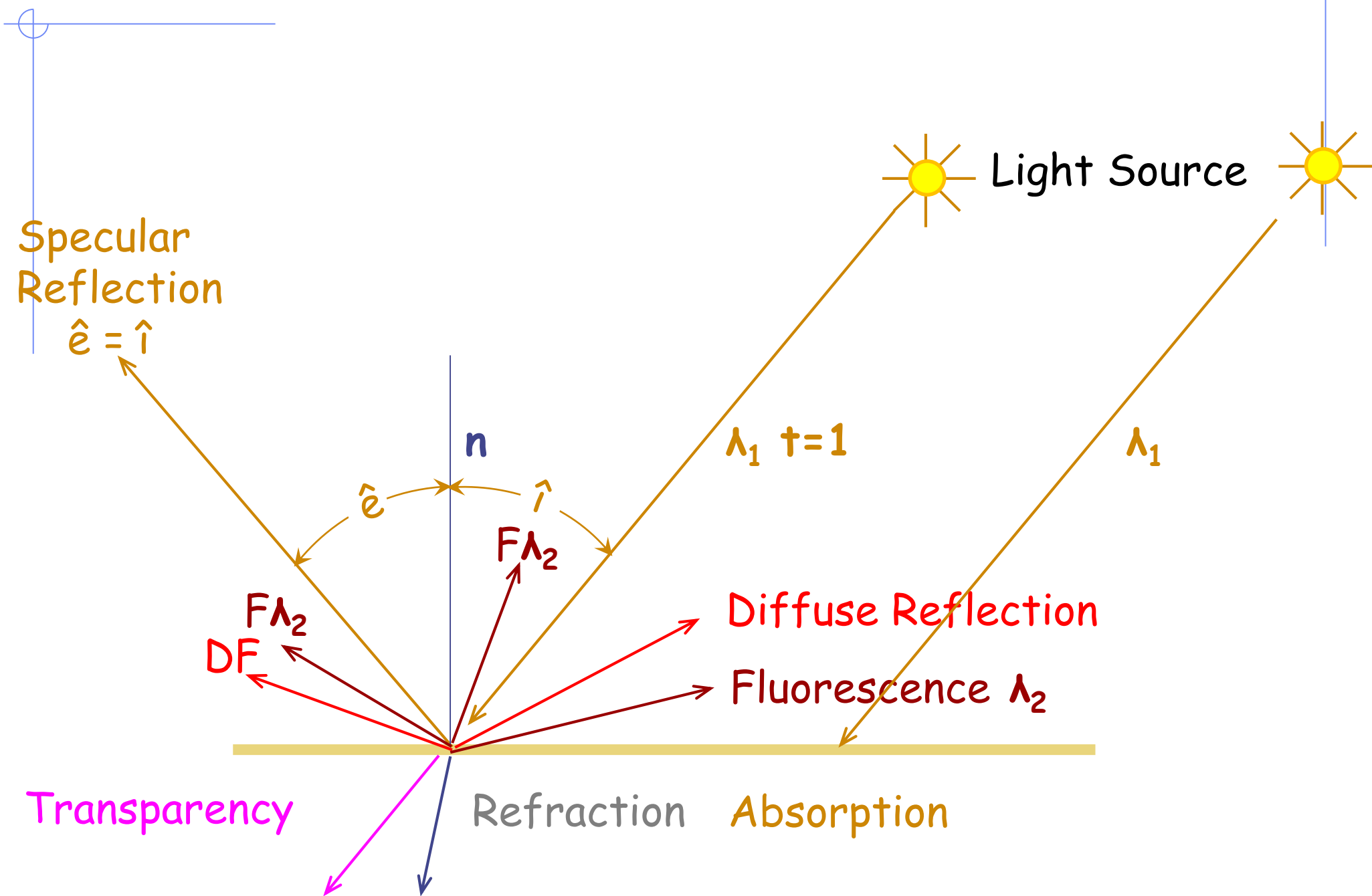


Distant point source model

$$\rho_d(x) (N(x) \cdot S_d(x))$$

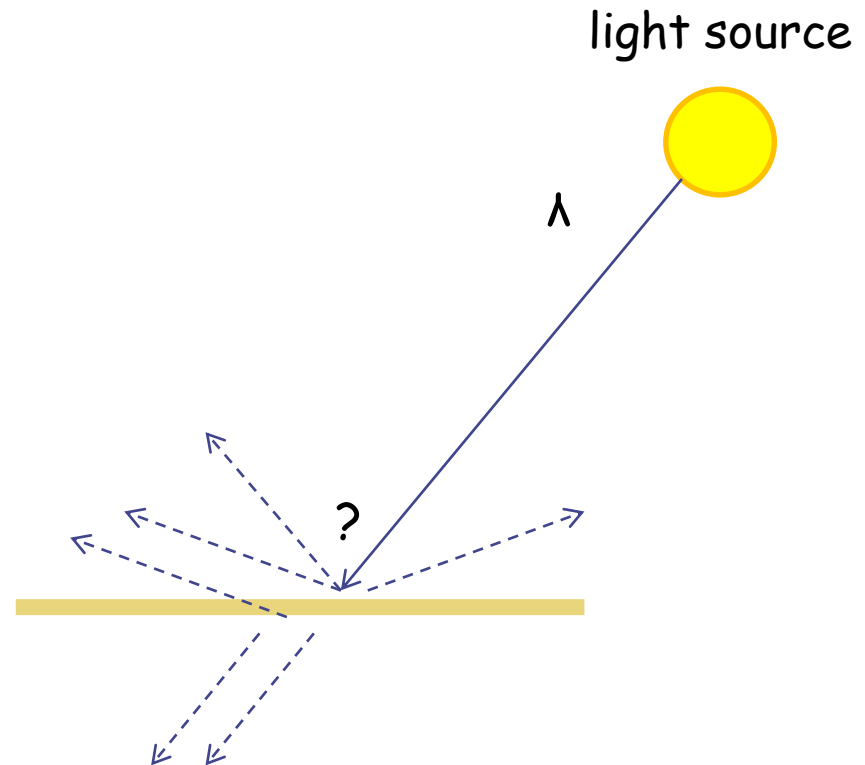


Assume that all points in the model are close to each other with respect to the distance to the source. Then the source vector doesn't vary much, and the distance doesn't vary much either, the computation are simplified.



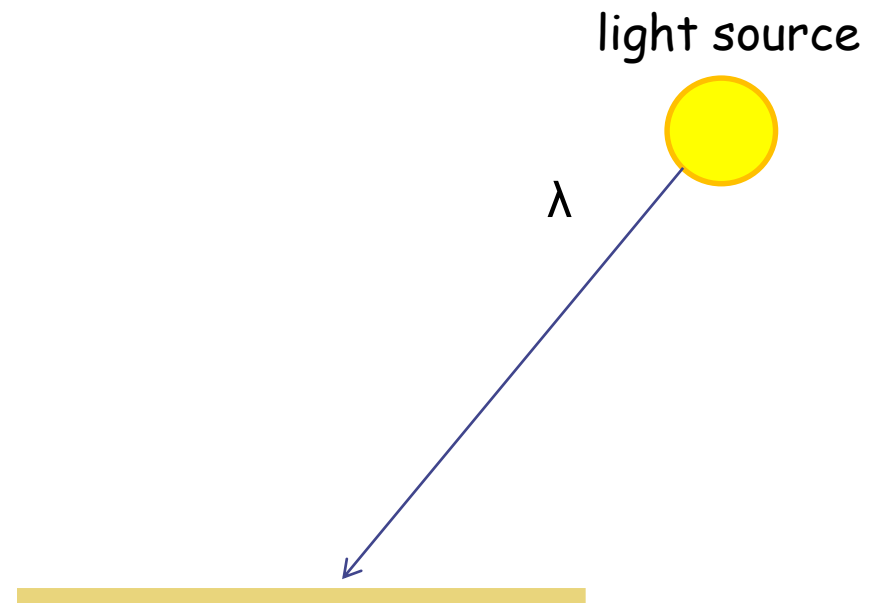
A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence



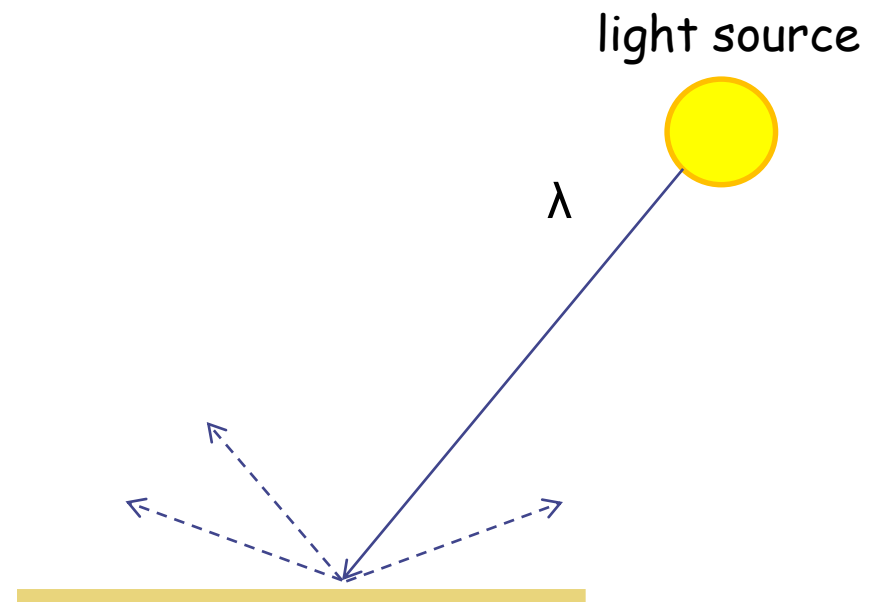
A photon's life choices

- **Absorption**
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence



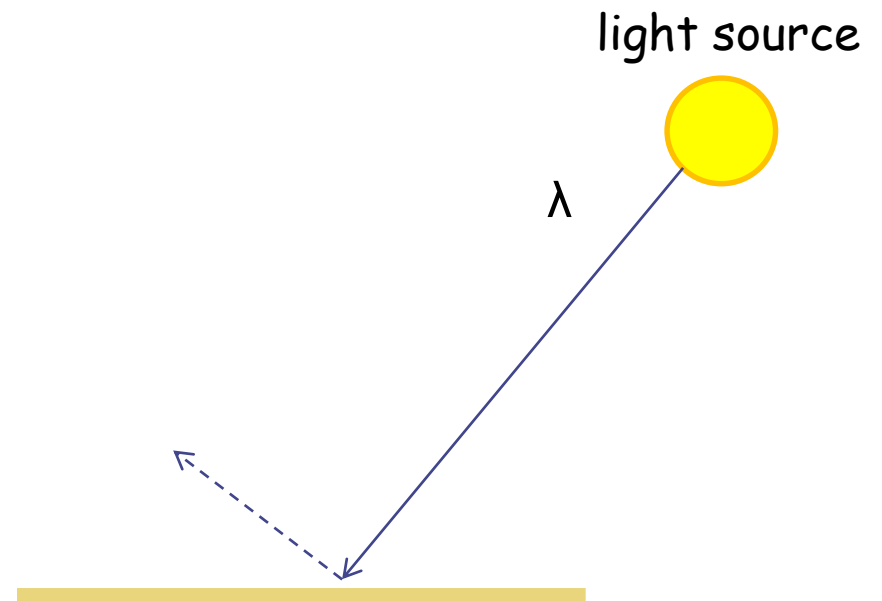
A photon's life choices

- Absorption
- **Diffuse Reflection**
- Reflection
- Transparency
- Refraction
- Fluorescence



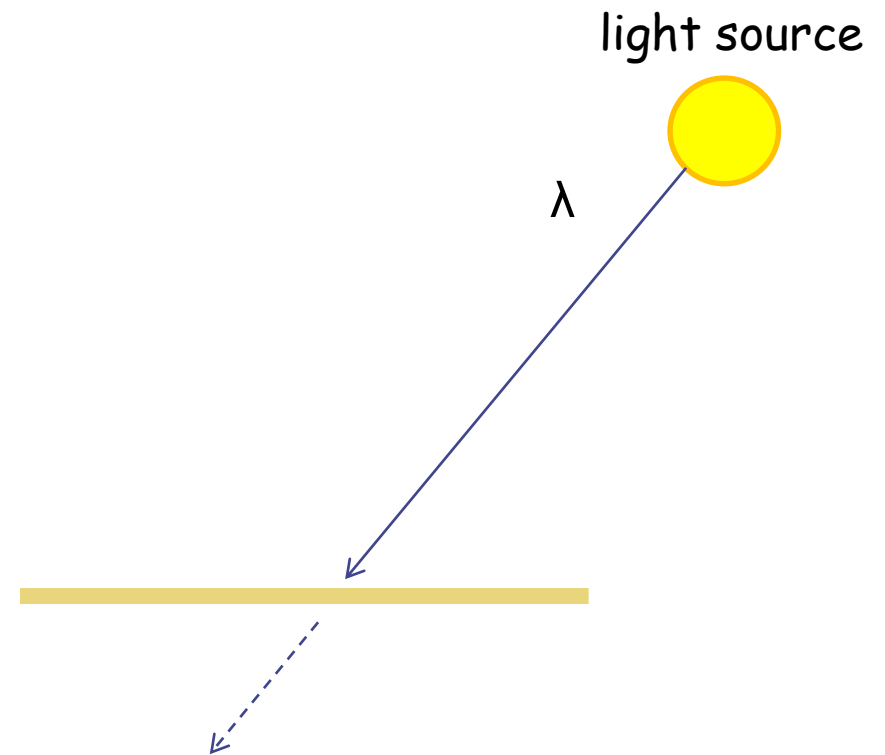
A photon's life choices

- Absorption
- Diffusion
- **Specular Reflection**
- Transparency
- Refraction
- Fluorescence



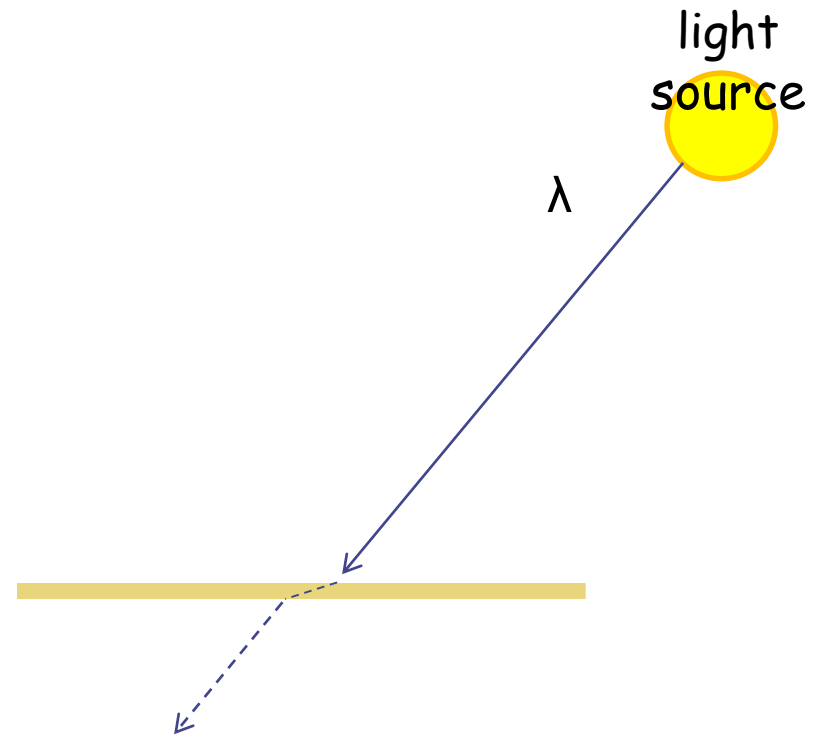
A photon's life choices

- Absorption
- Diffusion
- Reflection
- **Transparency**
- Refraction
- Fluorescence



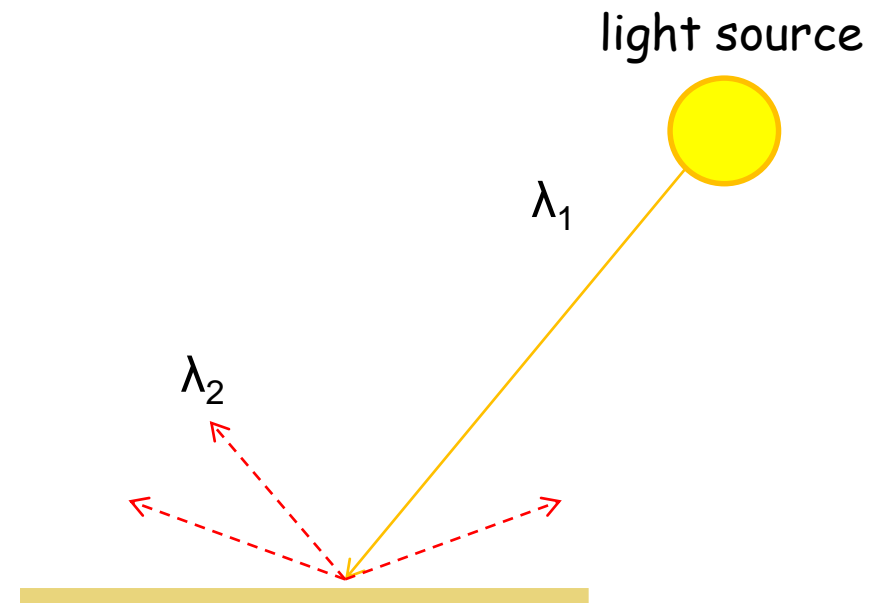
A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- **Refraction**
- Fluorescence



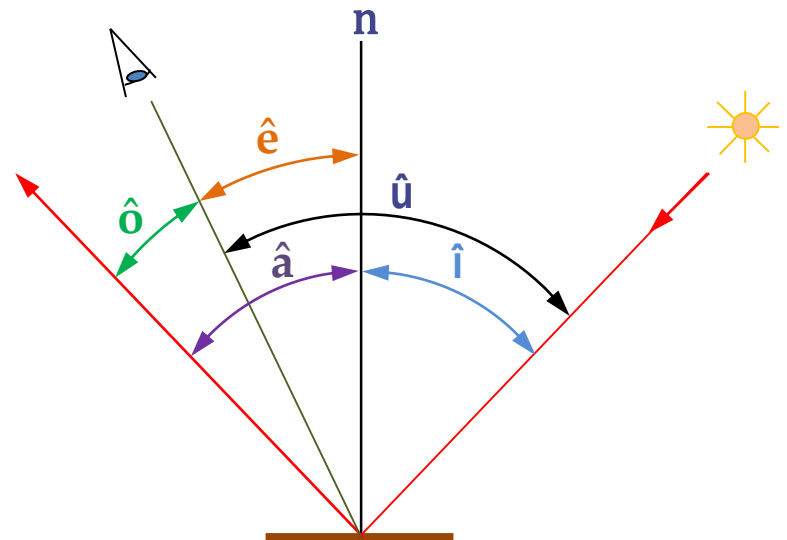
A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- **Fluorescence**

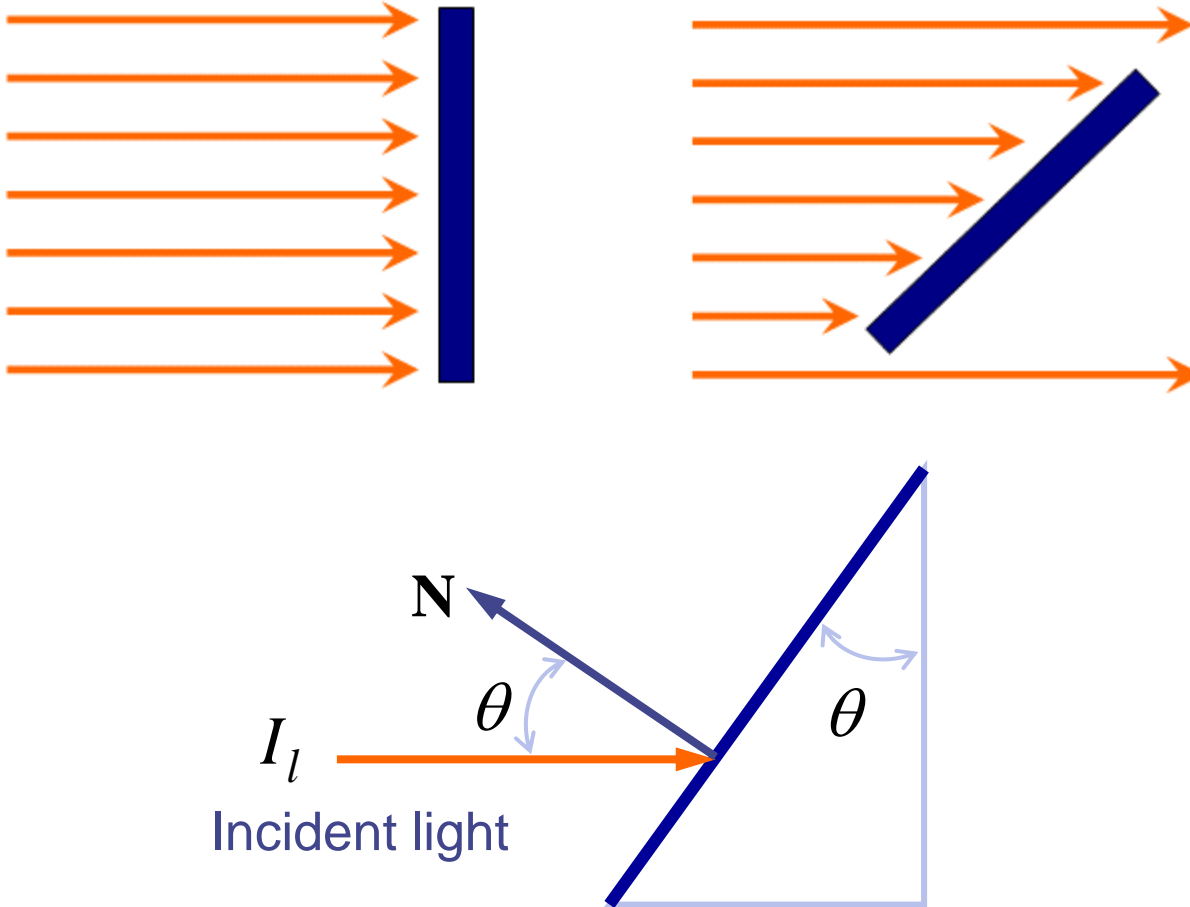


Local rendering

- Optical geometry of the light-source/eye-receiver system
- Notation:
 - n is the normal to the surface at the incidence point
 - \hat{i} corresponds to the incidence angle
 - $\hat{a} = \hat{i}$ is the reflectance angle
 - \hat{o} is the mirrored emergence angle
 - \hat{u} is the phase angle
 - \hat{e} is the emergence angle.

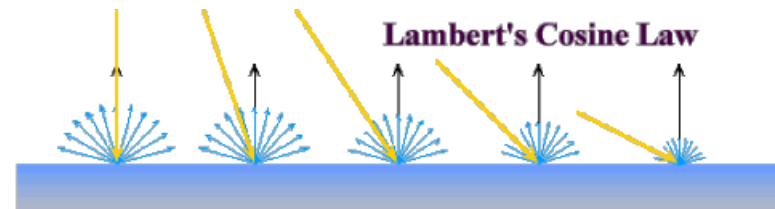
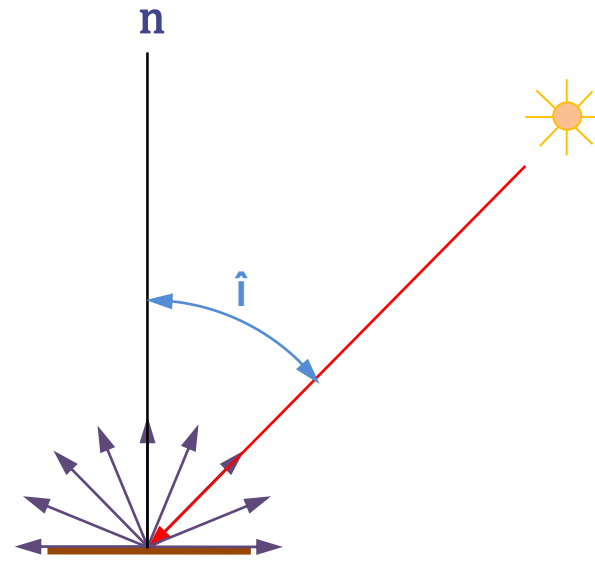


The amount of incident light depends on the orientation of the surface relative to the light source direction.



The Lambertian model

$$\Phi = \begin{cases} \cos \hat{i} & -\frac{\pi}{2} \leq \hat{i} \leq \frac{\pi}{2} \\ 0 & \text{elsewhere} \end{cases}$$



$$\cos(\theta) = \frac{\vec{O} \cdot \vec{R}}{|\vec{O}| |\vec{R}|}$$

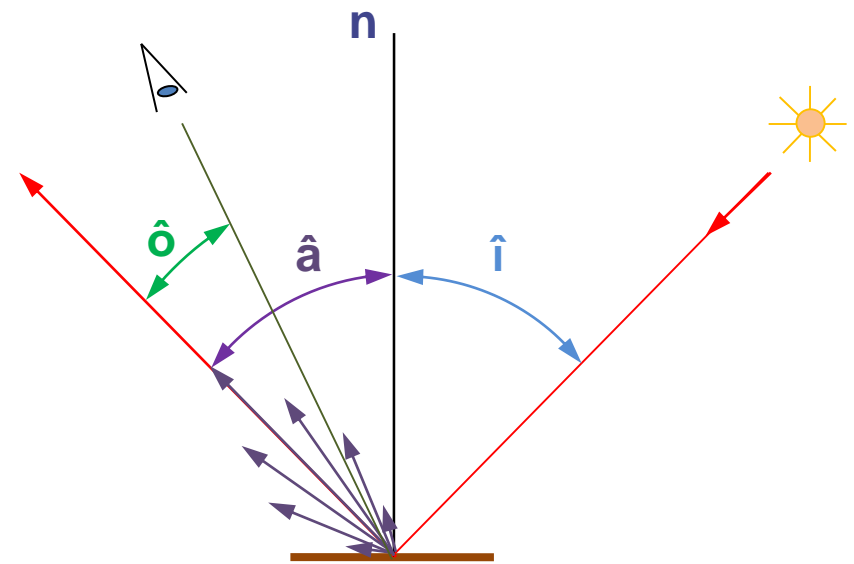
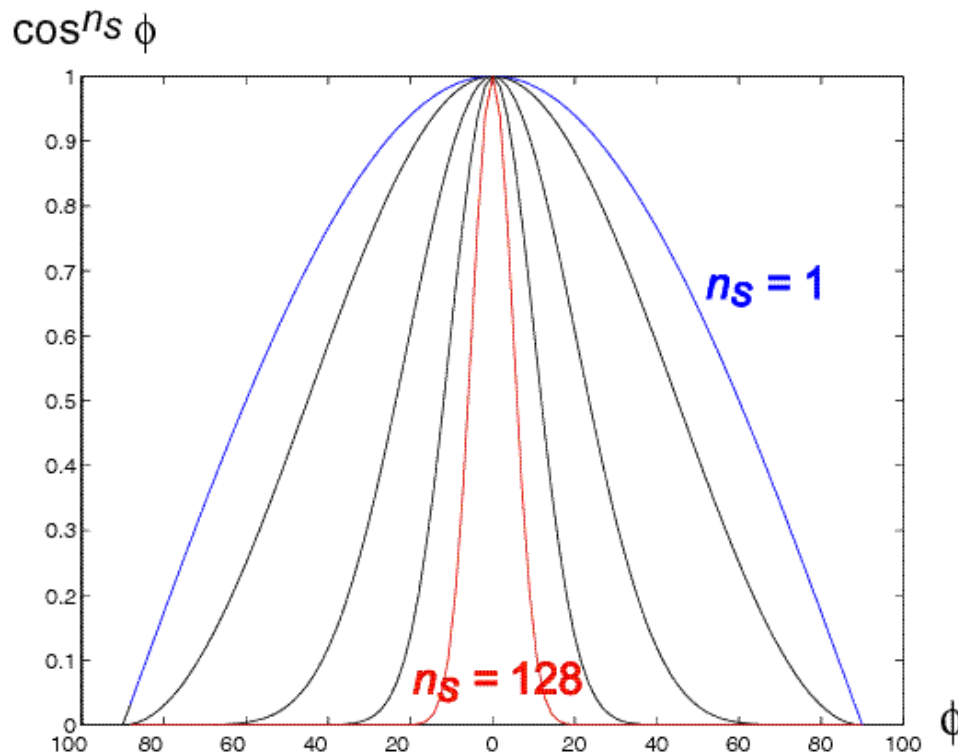
$$\vec{r}\vec{o} = 2(\vec{i}\vec{n})(\vec{n}\vec{o}) - (\vec{i}\vec{o})$$

$$\cos(\hat{\mathbf{i}}) = \vec{i} \cdot \vec{n}$$

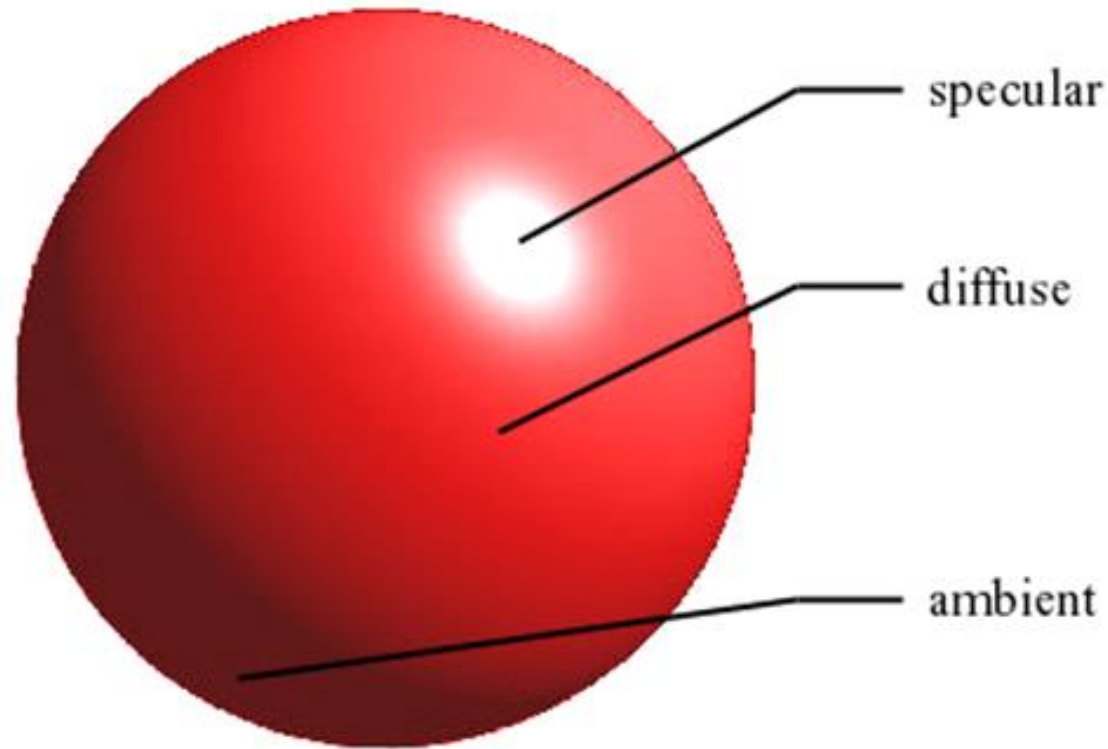
$$\cos(o) = 2 \cos(i) \cos(e) - \cos(u)$$

The specular model

$$\Phi = \begin{cases} \cos^m \hat{\theta} & -\frac{\pi}{2} \leq \hat{\theta} \leq \frac{\pi}{2} \\ 0 & \text{elsewhere} \end{cases}$$



Components of Reflections



Ambient - surface exposed to **indirect light** reflected from nearby objects.

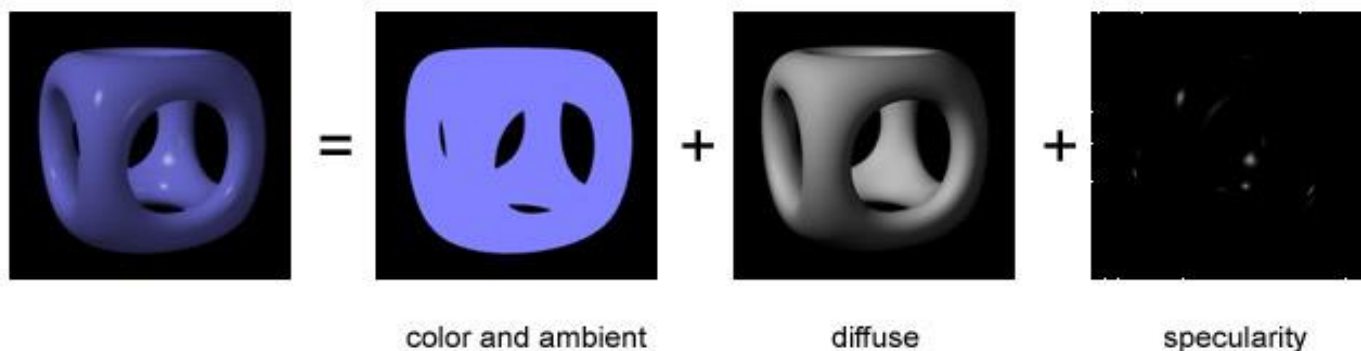
Diffuse - reflection from **incident** light with equal intensity in all directions. Depends on surface properties.

Specular - near **total** of the incident light around reflection angle.

The Phong model

- **a** is the amount of incident light diffused according to a Lambertian model (isotropic) independent from the receiver's position
- **b** is the amount of incident light specularly reflected by the object, which depends on the phase angle, and m being the exponential specular reflection coefficient
- **c** accounts for the background illumination

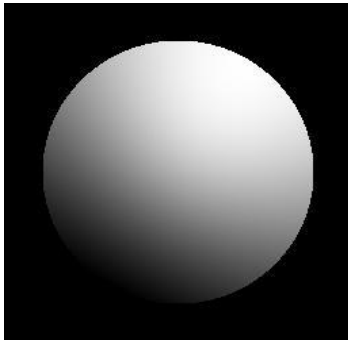
$$\Phi = a \cos \hat{i} + b \cos^m \hat{o} + c$$



Bùi Tường Phong, Illumination for computer generated images, Commun. ACM 18 (6) (1975) 311317.

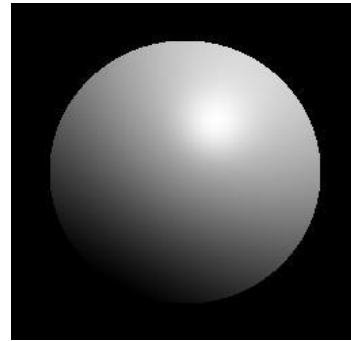
Sphere ($m=10$, $c=0$)

$a=1.0$



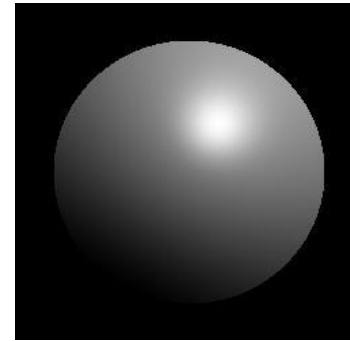
$b=0.0$

0.8



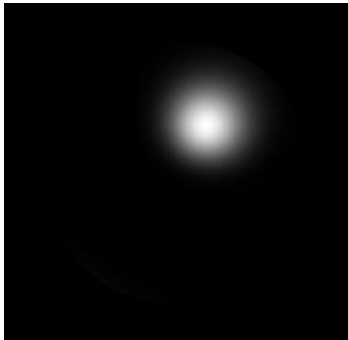
0.2

0.6



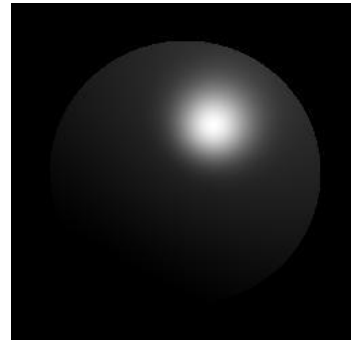
0.4

0.0



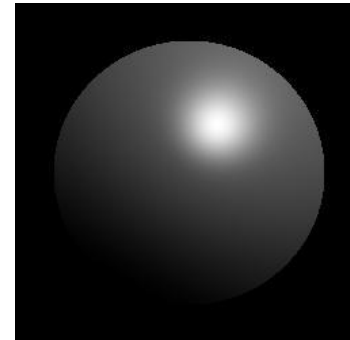
1.0

0.2



0.8

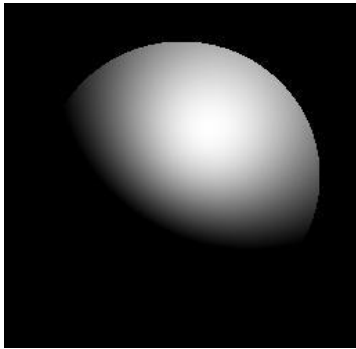
0.4



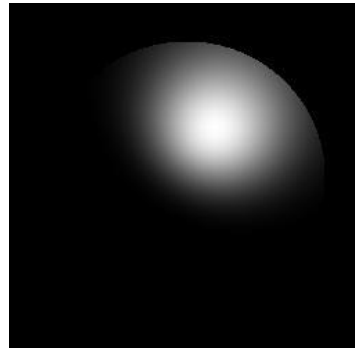
0.6

Specular sphere ($b=0.9$, $c=0.1$)

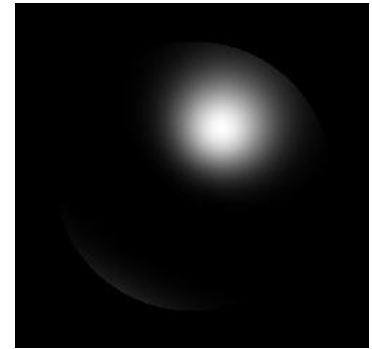
$m=1$



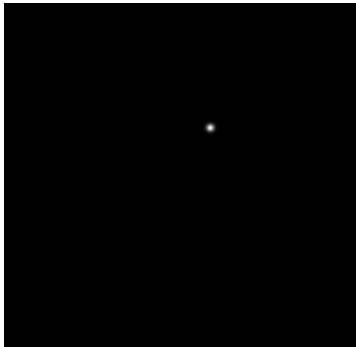
$m=3$



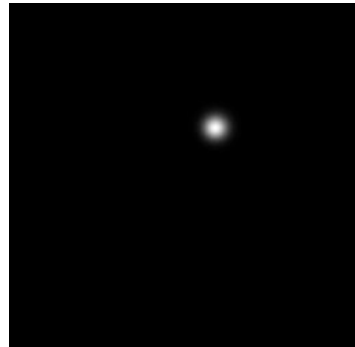
$m=6$



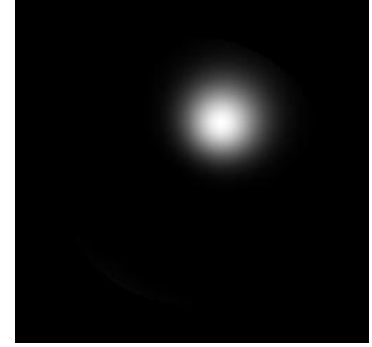
$m=1000$



$m=100$

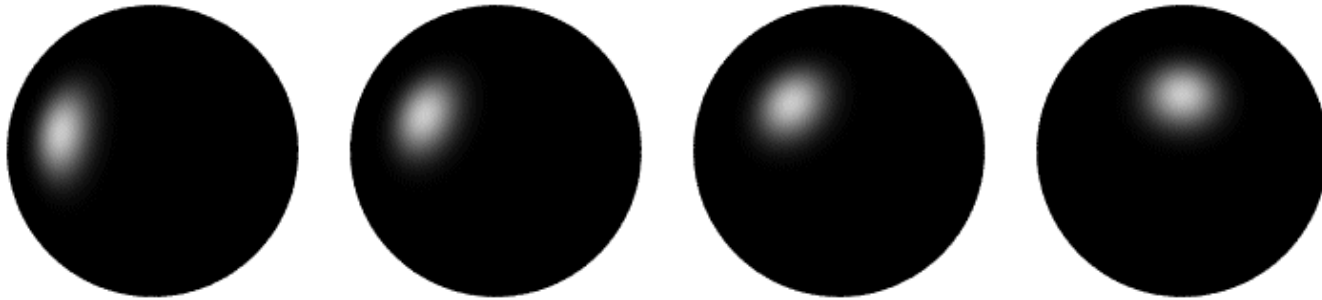


$m=10$

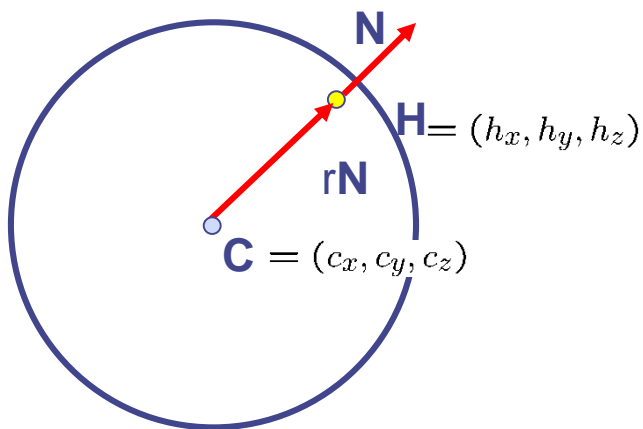


Computing light source directions

- Trick: place a specular sphere in the scene



- the location of the highlight tells you where the light source is



$$\|H - C\| = r$$

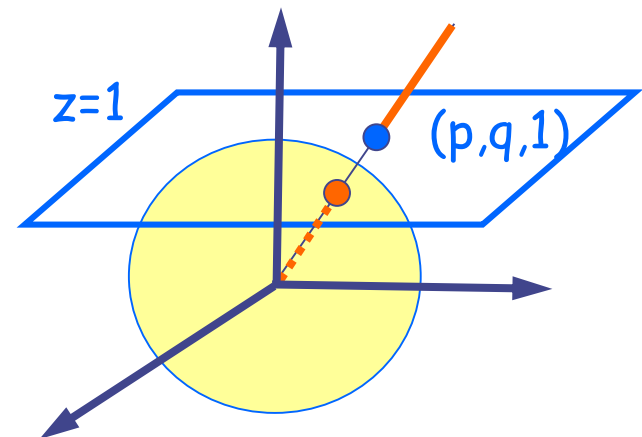
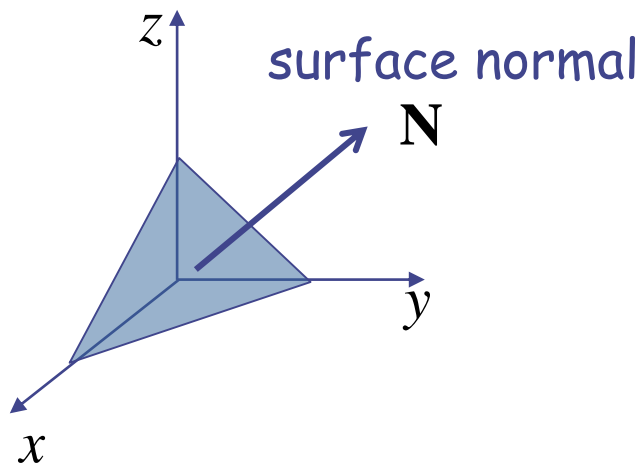
Gradient space

$$z = f(x, y)$$

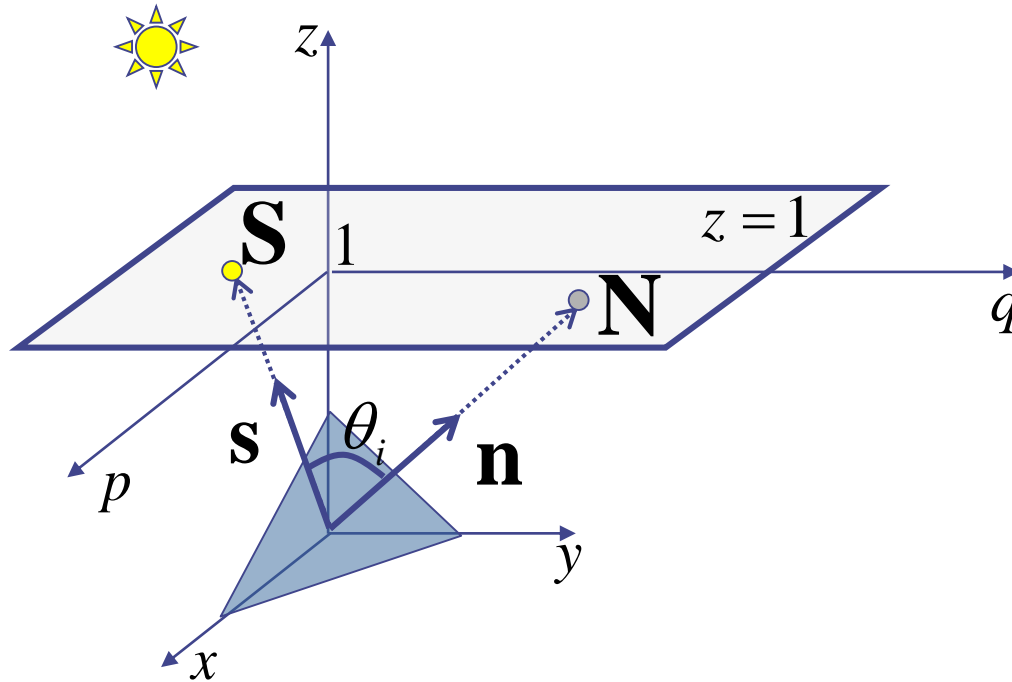
$$p = -\frac{\partial f}{\partial x} \quad q = -\frac{\partial f}{\partial y}$$

$$\mathbf{n} = (p, q, 1)$$

$$\hat{\mathbf{n}} = \frac{1}{\sqrt{p^2 + q^2 + 1}} (p, q, 1)$$



Gradient Space



Normal vector

$$\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = \frac{(p, q, 1)}{\sqrt{p^2 + q^2 + 1}}$$

Source vector

$$\mathbf{s} = \frac{\mathbf{S}}{|\mathbf{S}|} = \frac{(p_s, q_s, 1)}{\sqrt{p_s^2 + q_s^2 + 1}}$$

$$\cos \theta_i = \mathbf{n} \cdot \mathbf{s} = \frac{(pp_s + qq_s + 1)}{\sqrt{p^2 + q^2 + 1} \sqrt{p_s^2 + q_s^2 + 1}}$$

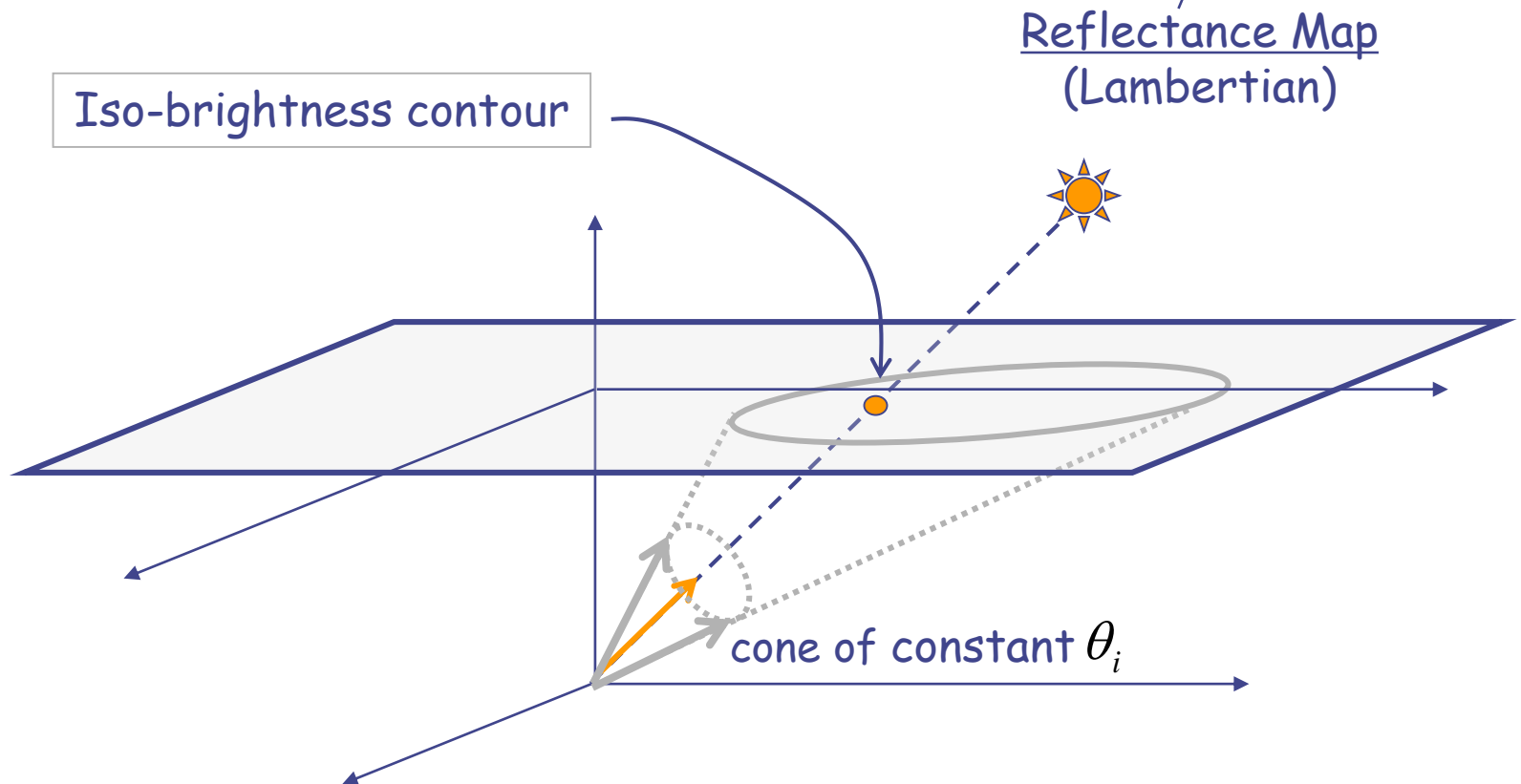
$z = 1$ plane is called the Gradient Space (p q plane)

Every point on it corresponds to a particular surface orientation

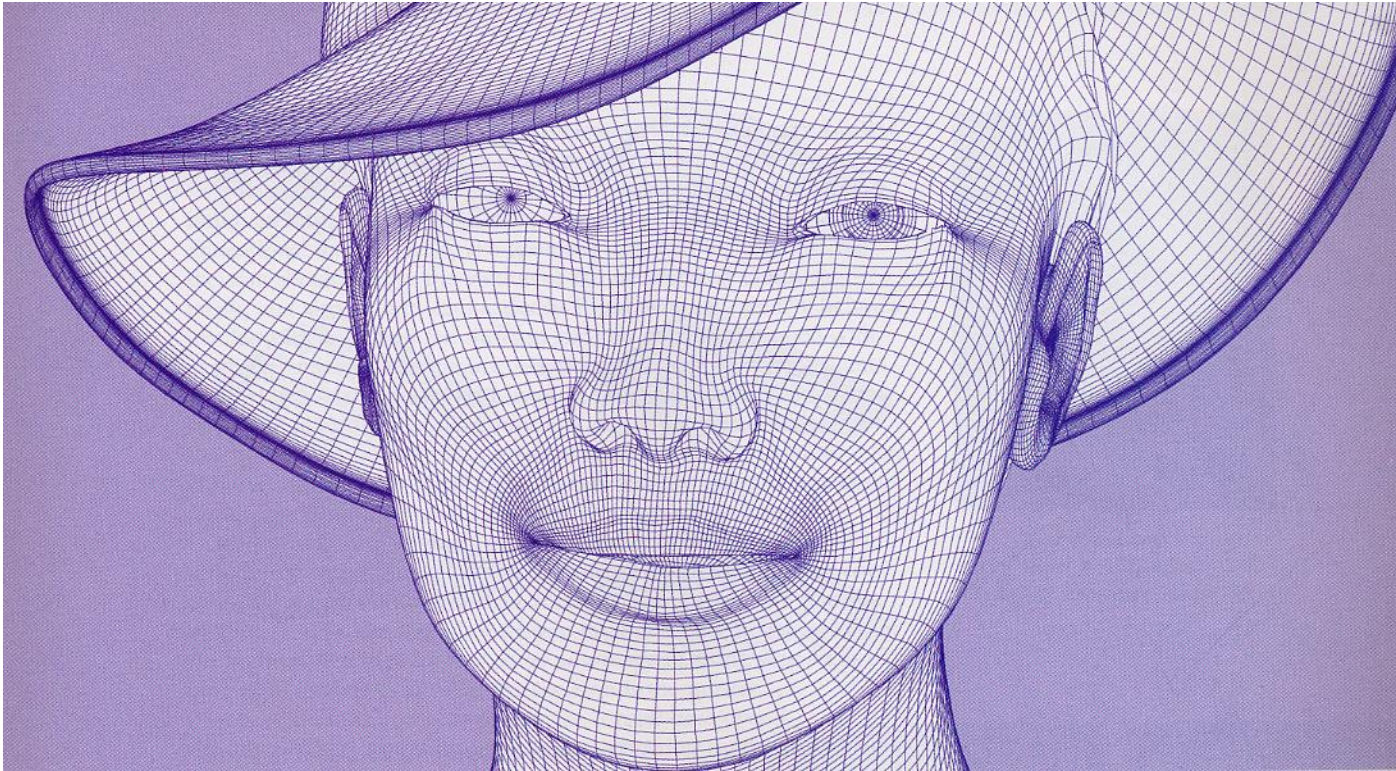
Reflectance Map

- Lambertian case

$$I = \cos \theta_i = \mathbf{n} \cdot \mathbf{s} = \frac{(pp_s + qq_s + 1)}{\sqrt{p^2 + q^2 + 1} \sqrt{p_s^2 + q_s^2 + 1}} = R(p, q)$$



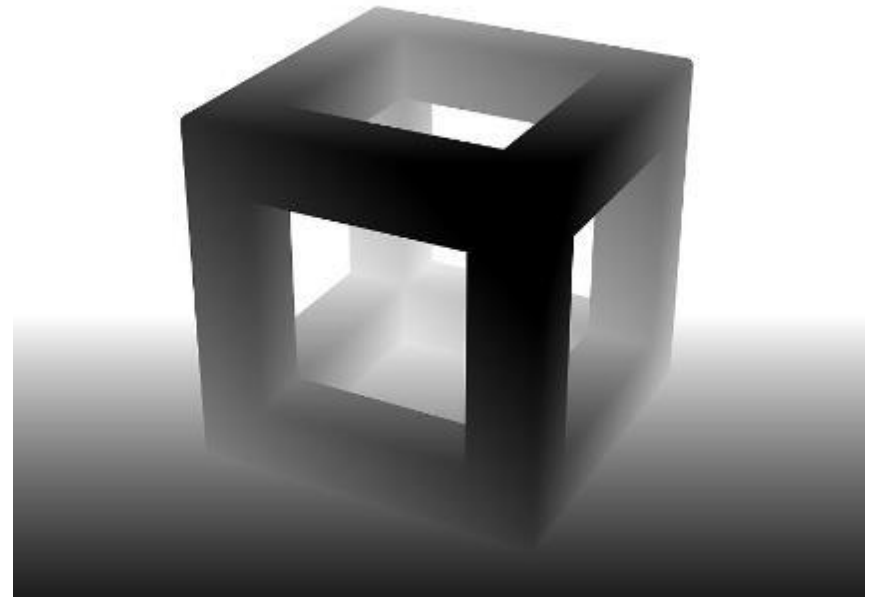
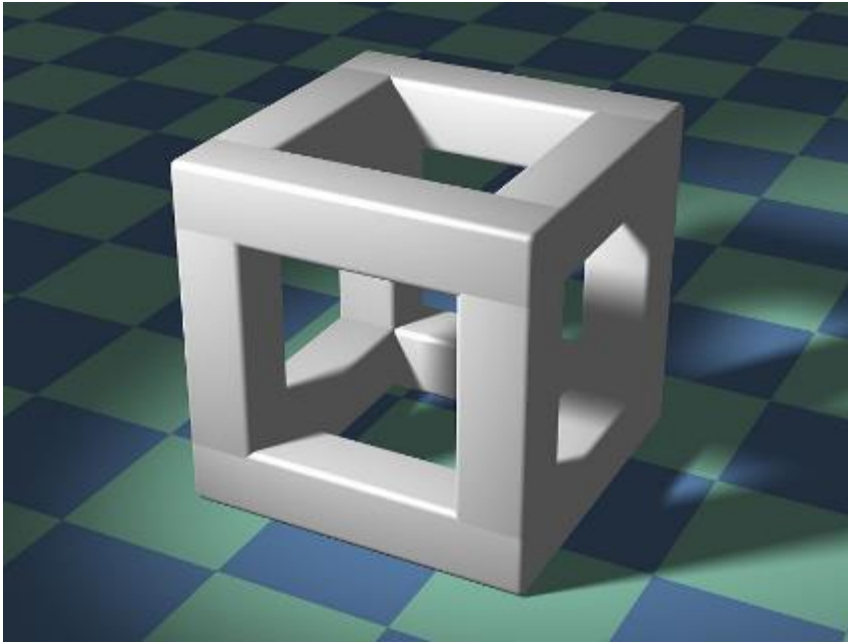
Wire frame surface presentation



Local rendering



Range finder



Reflectance maps

- Let us take a **reference system** where the optical axis of the acquisition system (the receiver) coincides with the z axis.
- The surface described by the function $z = f(x, y)$ has the normal vector: $(\partial z / \partial x, \partial z / \partial y, -1)^t$.
- Calling $p = \partial z / \partial x$ and $q = \partial z / \partial y$ there is a one-to-one correspondence between the plane p, q (called *gradient plane*) and the normal directions to the surface.
- The three angles \hat{i} , \hat{u} and \hat{e} may be computed with the following formulas:

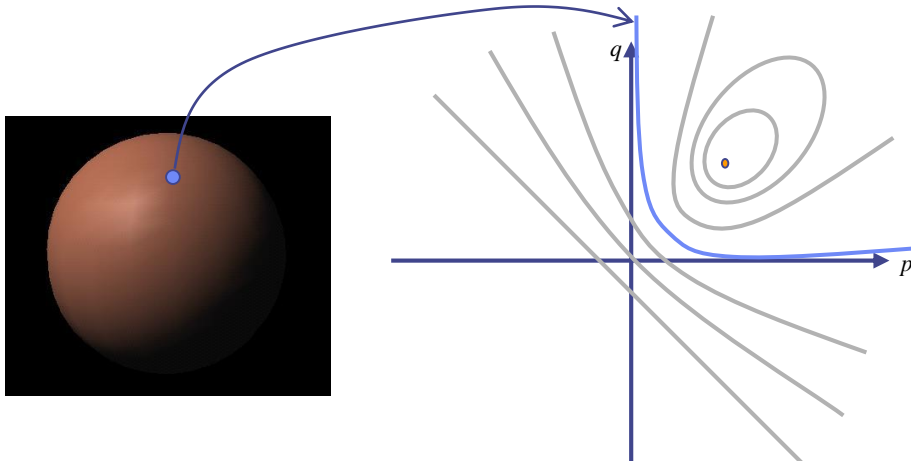
$$\cos \hat{e} = \frac{1}{\sqrt{1 + p^2 + q^2}}$$

$$\cos \hat{i} = \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2 + q^2} \sqrt{1 + p_s^2 + q_s^2}}$$

$$\cos \hat{u} = \frac{1}{\sqrt{1 + p_s^2 + q_s^2}}$$

Shape from a Single Image?

- Given a single image of an object with known surface reflectance taken under a known light source, can we recover the shape of the object?

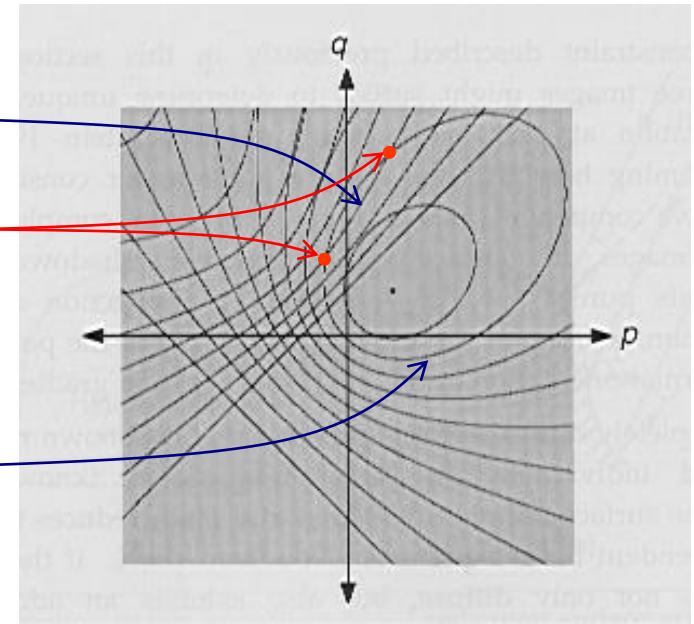


- Two reflectance maps?

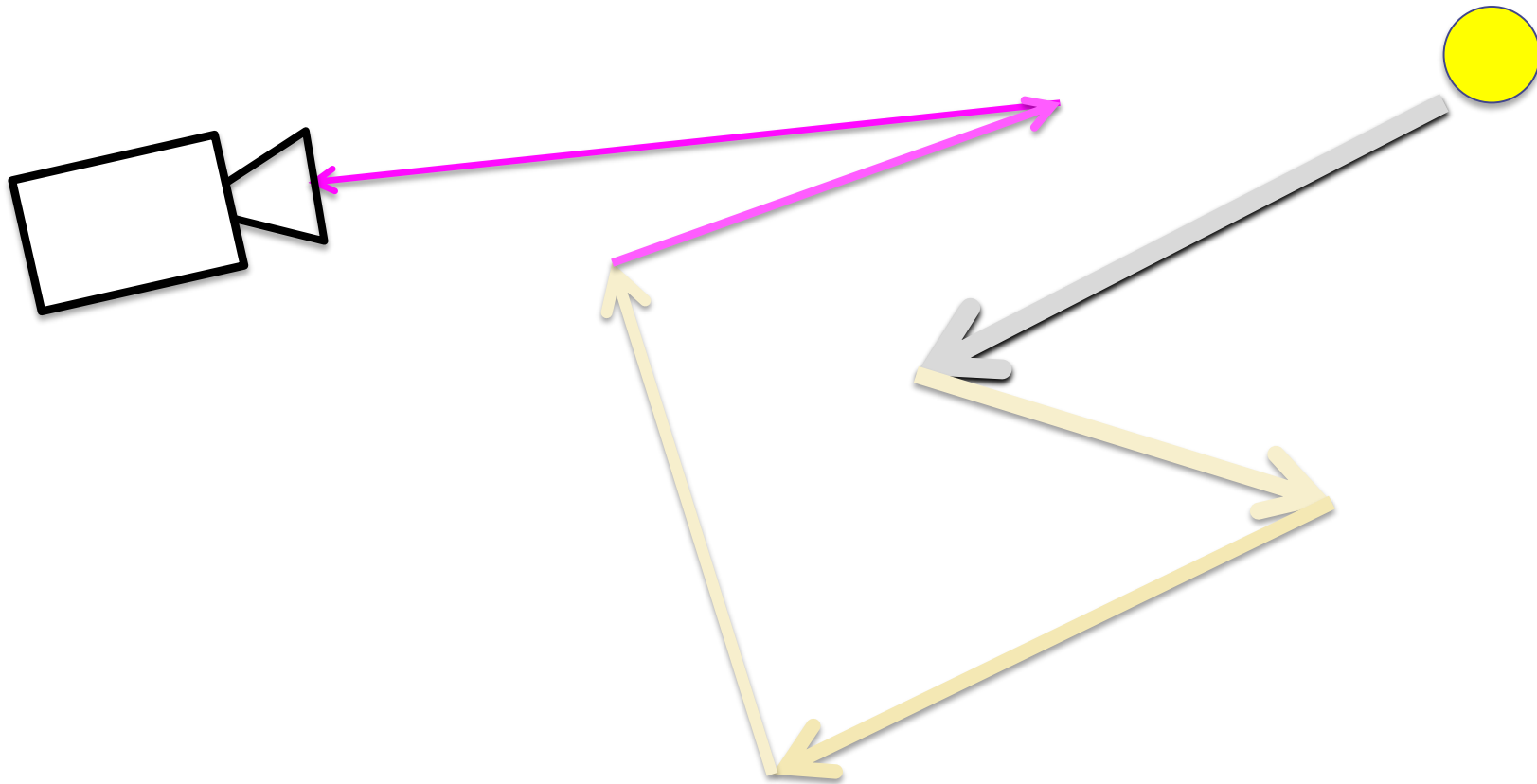
$$R_2(p, q) = a$$

Intersections:
2 solutions for
 p and q .

$$R_1(p, q) = b$$

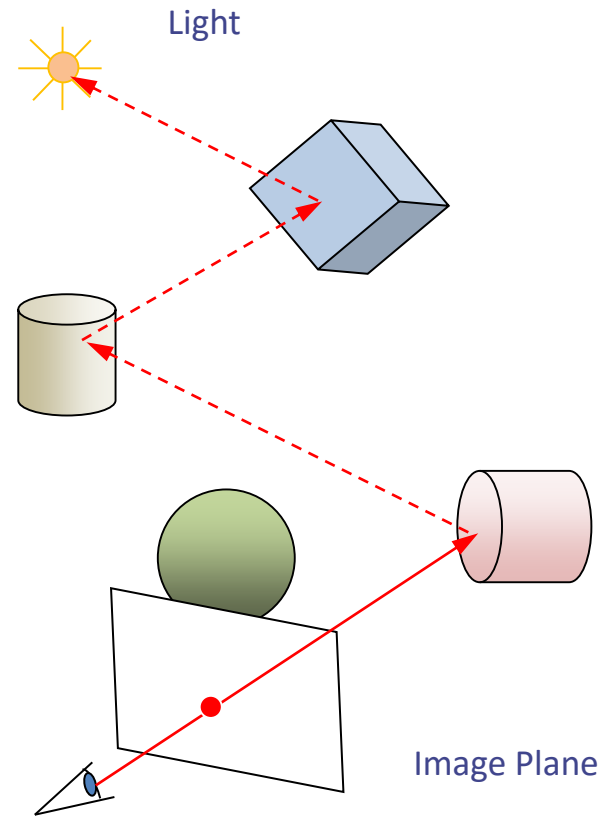
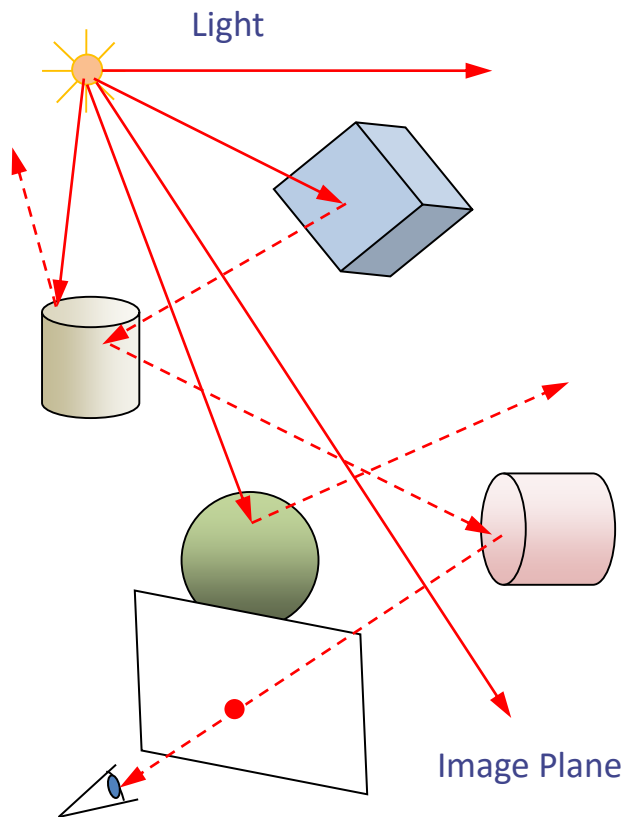


The life of a photon



Global rendering: ray tracing

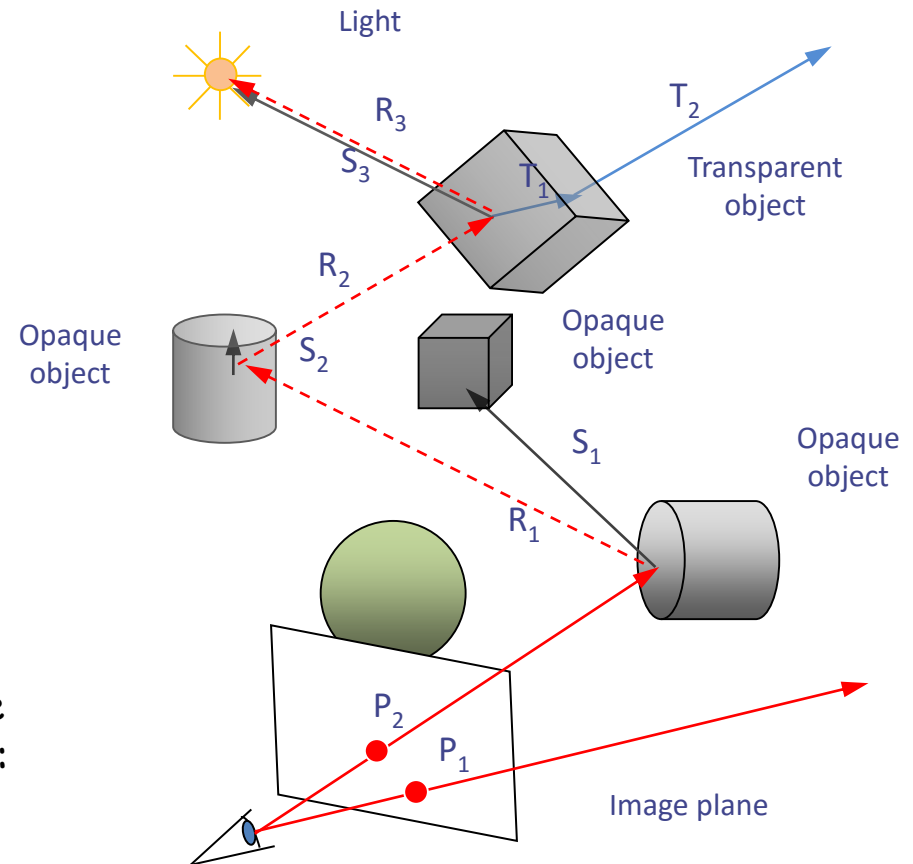
- The two basic schemes of **forward (source to eye)** and **backward (eye to source)** ray tracing, this last is computationally efficient but cannot properly model diffuse reflections and all other changes in light intensity due to non-specular reflections



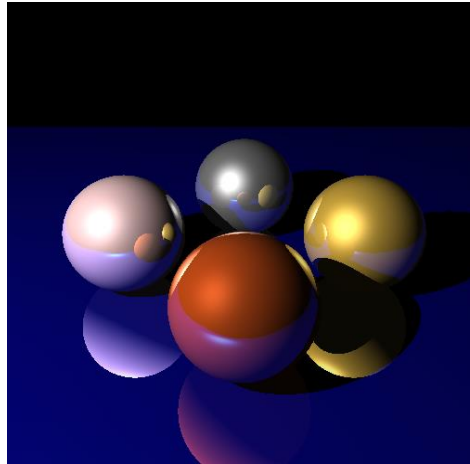
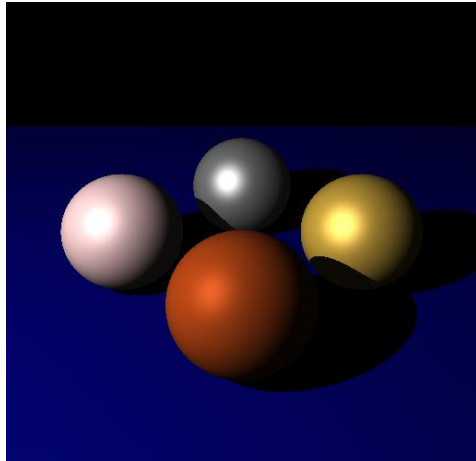
Global rendering: ray tracing

- A ray sent from the eye to the scene through the image plane may intersect an object; in this case secondary rays are sent out and three different lighting models can be considered:
 - **transmission**. The secondary ray is sent in the direction of refraction, following the Descartes' law;
 - **reflection**. The secondary ray is sent in the direction of reflection, and the Phong model is applied;
 - **shadowing**. The secondary ray is sent toward a light source, if intercepted by an object the point is shadowed.

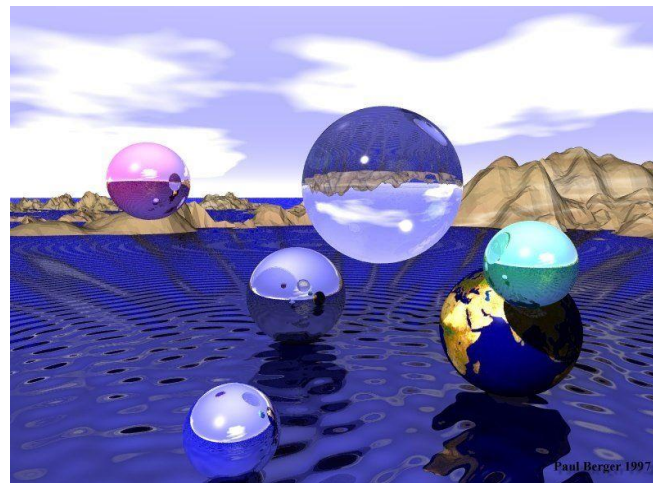
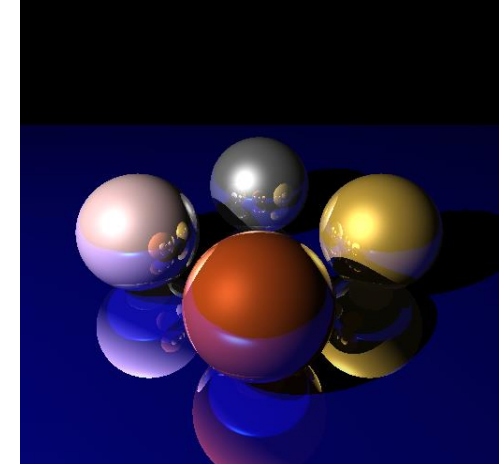
Figure. A background pixel corresponding to the ray P_1 and a pixel representing an object (ray P_2). The secondary rays triggered by the primary ray P_2 according to the three models: transmission (T_1 and T_2), reflection (R_1 , R_2 and R_3), and tentative shadowing (S_1 , S_2 : true shadows, and S_3).



Reflections and transparencies



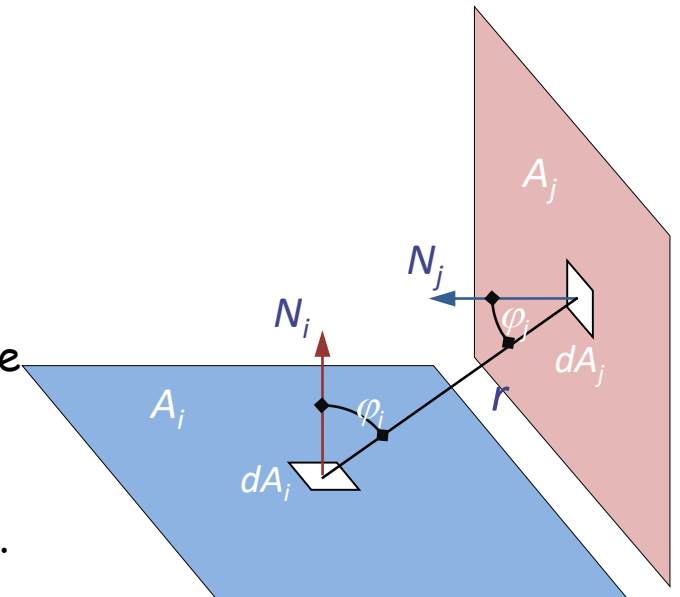
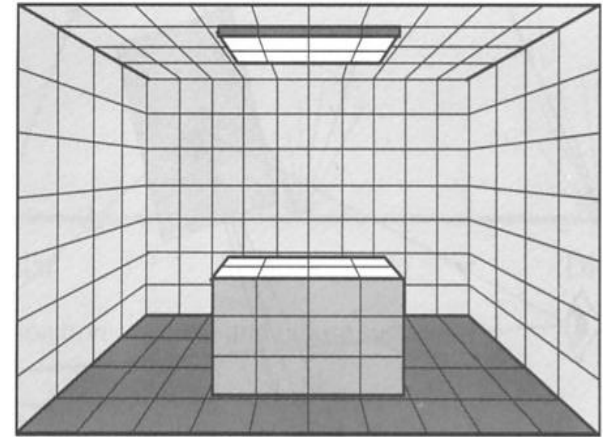
Created by David Derman – CISC 440



Global rendering: radiosity

- The **radiosity** method has been developed to model the diffuse-diffuse interactions so gaining a more realistic visualization of surfaces.
- The diffused surfaces scatter light in all directions (i.e. in Lambertian way). Thus a scene is divided into **patches** - small flat polygons. For each patch the goal is to measure energies emitted from and reflected to respectively.
The radiosity of the patch i is given by:

$$B_i = E_i + \rho_i \sum_{j=1}^n B_j F_{ij}$$
- Where E_i represents the energy emitted by patch i , ρ_i the reflectivity parameter of patch i , and $\sum_{j=1}^n B_j F_{ij}$ the energy reflected to patch i from the n patches j around it, depending on the **form factors** F_{ij} .
- The form factor represents the fraction of light that reaches patch i from patch j . It depends on the distance and orientation of the two patches.
- A scene with n patches, follows a system of n equations for which the solution yields the radiosity of each patch.

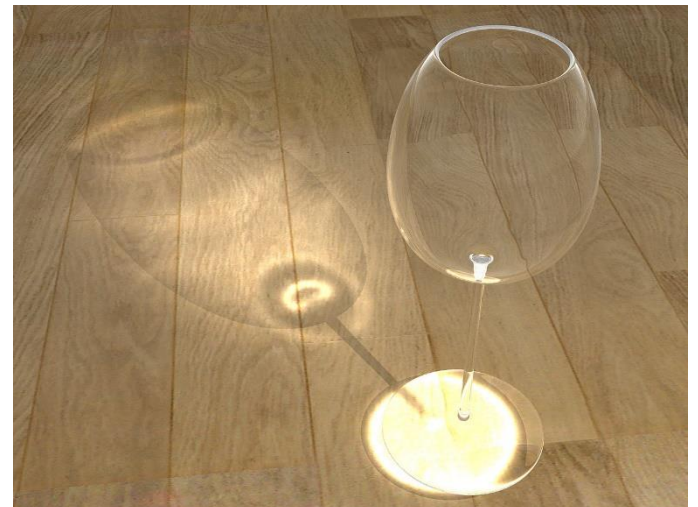
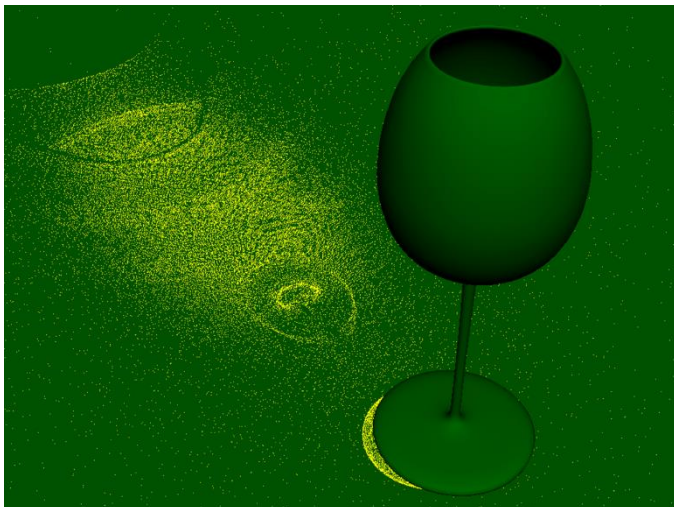


Example

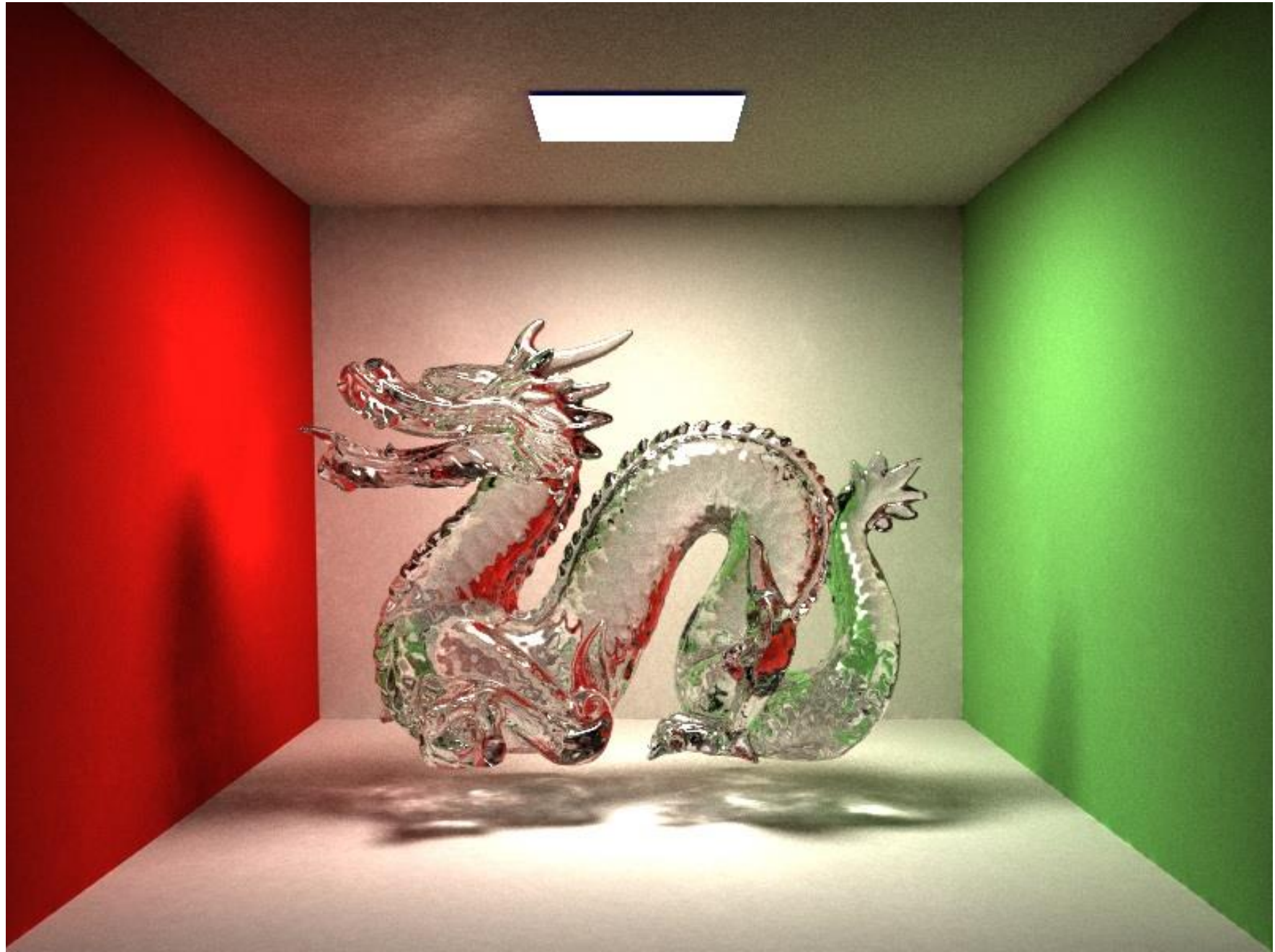


Global illumination photon mapping

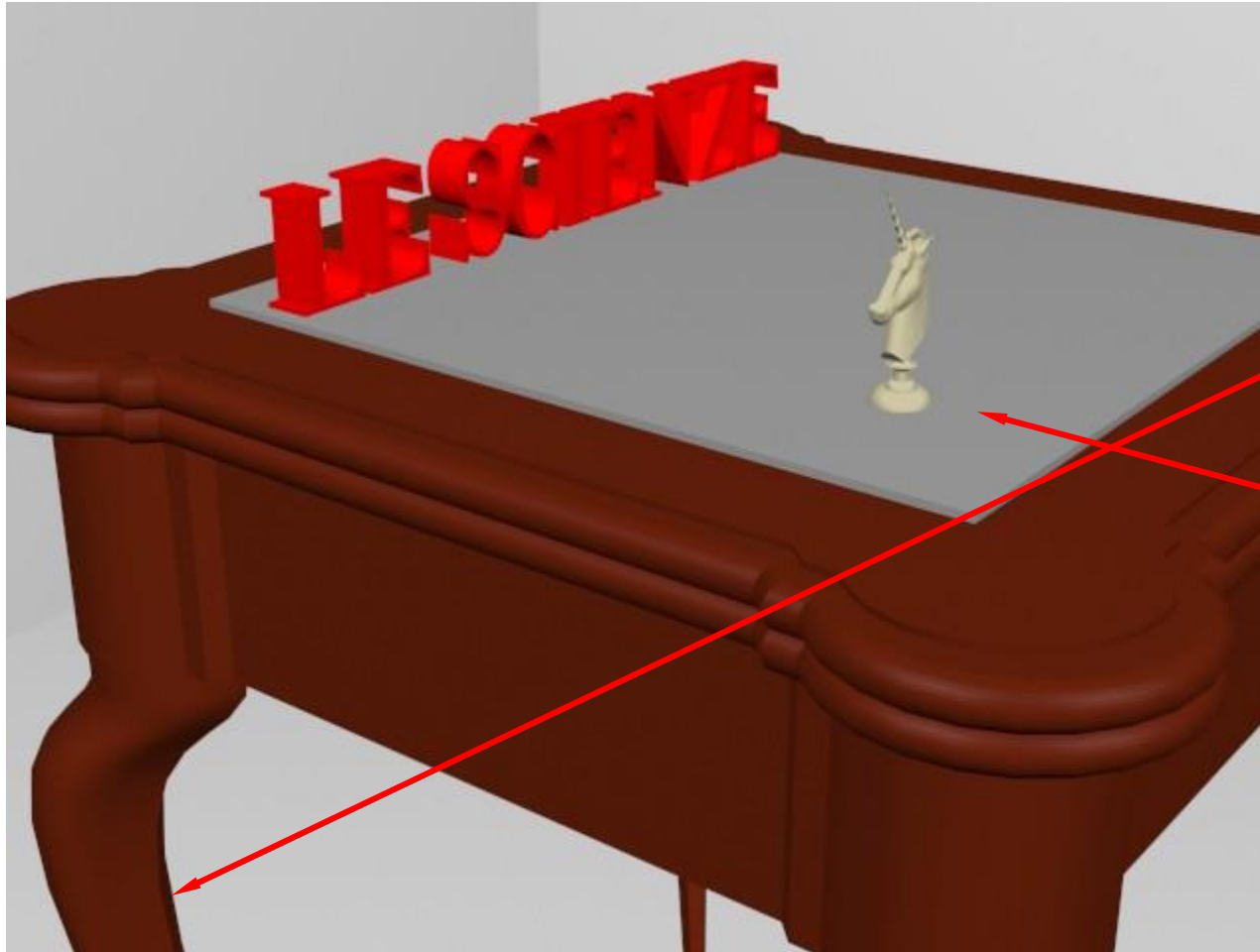
- **Photon mapping** is a two-pass algorithm developed by Henrik Wann Jensen that approximately solves the rendering equation.
- Rays from the light source and rays from the camera are traced independently until some termination criterion is met, then they are connected in a second step to produce a radiance value.
- It is used to realistically simulate the interaction of light with different objects. Specifically, it is capable of simulating the refraction of light through a transparent substance such as glass or water, diffuse interreflection between illuminated objects, the subsurface scattering of light in translucent materials, and some of the effects caused by particulate matter such as smoke or water vapor.



Example

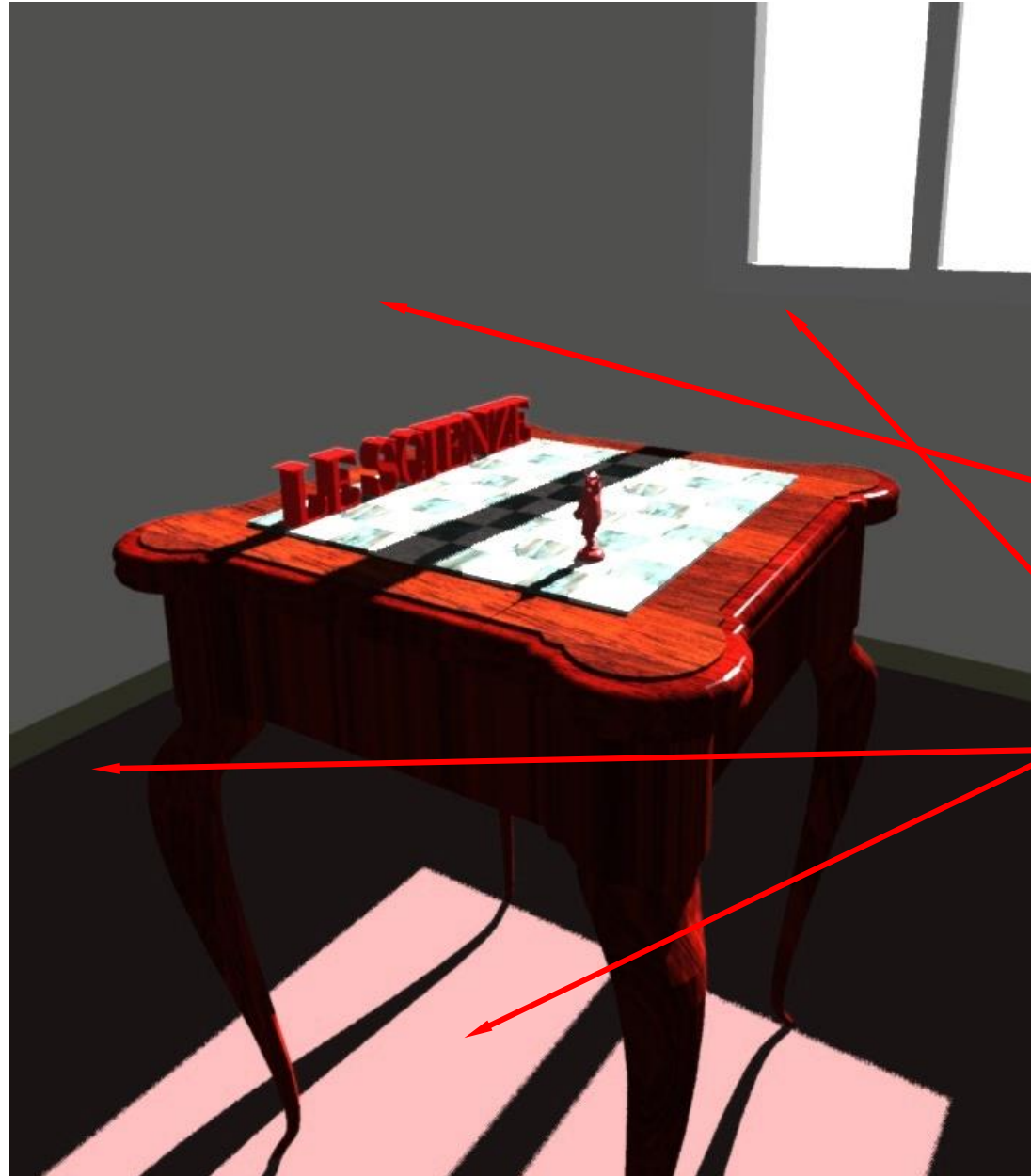


.... Phong model



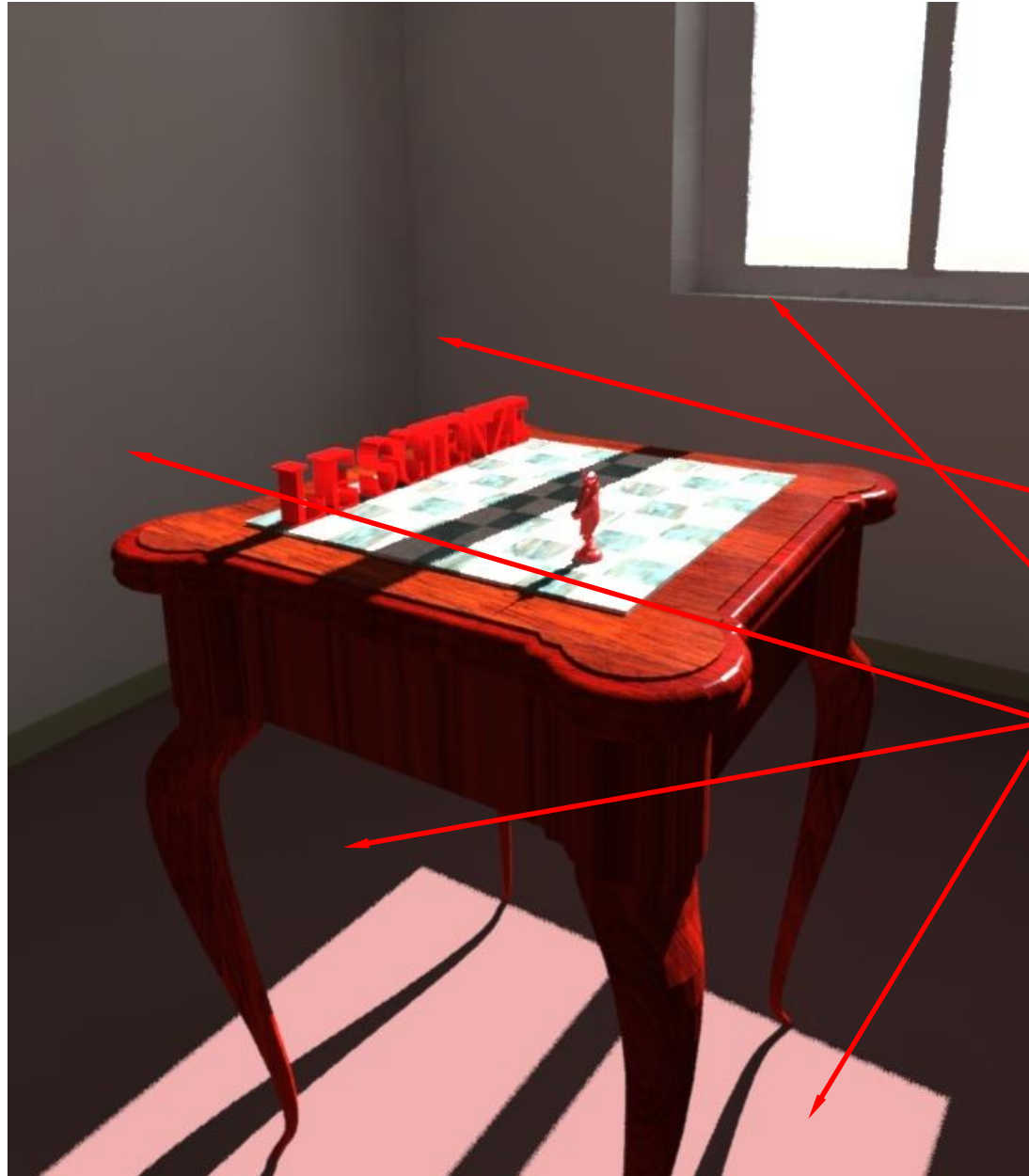
- No objects interactions
- No shadows

... ray tracing



- is there a corner?
- is the window in depth?
- is the floor properly rendered?

.. radiosity



- there is a corner
- the window is in depth
- floor and walls are clearer and pink