Deep Reinforcement Learning

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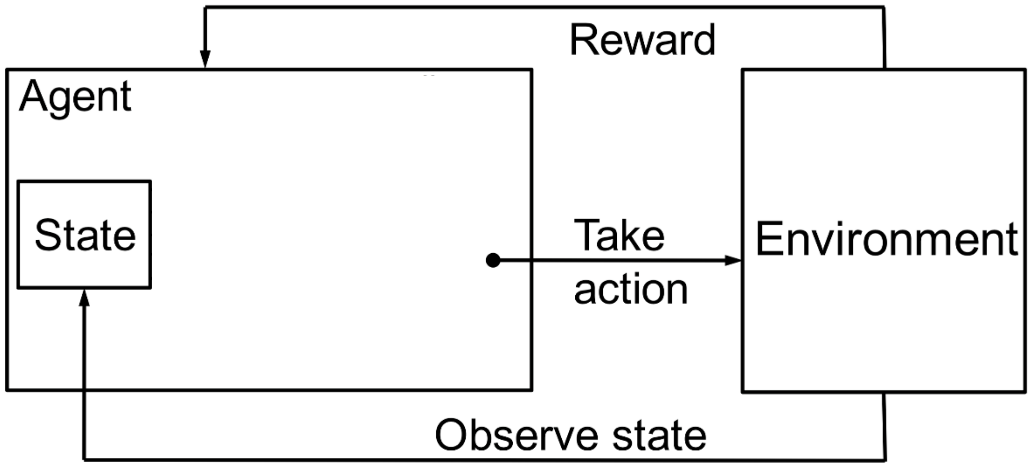
Basics (Intuition)

Deep Reinforcement Learning (DRL)

- ***Reinforcement Learning***



Q-Learning

- ***Q-Learning Algorithm***

Initialize $\hat{Q}(s,a)$ at random, put the agent in a random state s
Repeat:

1) Select the action $\text{argmax}_a \hat{Q}(s,a)$ with probability $(1-\varepsilon)$
   otherwise, select a at random
2) The agent is now in state s' and has received the reward r
3) Update $\hat{Q}(s,a)$ by

$$\Delta\hat{Q}(s,a) = \alpha[r + \gamma \max_{a'} \hat{Q}(s',a') - \hat{Q}(s,a)]$$

Critical aspects*:*

Tabular representation of $\hat{Q}(s,a)$

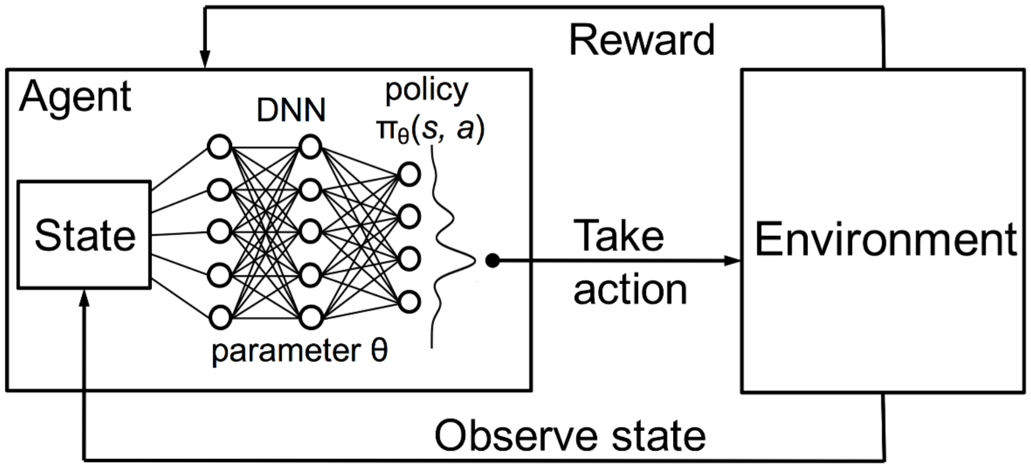
Suitable for discrete action spaces only

Very slow learner: each pair (s,a) *is a distinct entry, to be learned directly or by using a linear function approximator*

Deep Reinforcement Learning (DRL)

- ***Deep Reinforcement Learning***

  Using a deep neural network for the $\hat{Q}(s,a)$ approximator



The optimal policy is learnt incrementally by using a deep neural network

Deep Reinforcement Learning

- ***Deep Q-Learning Algorithm*** *(intuitive)*

Initialize $\hat{Q}(s,a;\theta)$, put the agent in a random state s
Repeat:

1) Select the action $\text{argmax}_a \hat{Q}(s,a;\theta)$ with probability $(1-\varepsilon)$
   otherwise, select a at random
2) The agent is now in state s' and has received the reward r
3) Define:

$$y = \begin{cases} r & \text{if } s' \text{ is terminal} \\ r + \text{argmax}_{a'} \hat{Q}(s',a';\theta) & \text{otherwise} \end{cases}$$

4) Perform gradient descent over $(y - \hat{Q}(s,a;\theta))^2$

Fundamental Idea:
Use a deep neural network to learn the approximator $\hat{Q}(s,a;\theta)$
*from the examples collected while **exploring** – **exploiting***

Replace the update step with DNN training

Deep Reinforcement Learning

- ***Deep Q-Learning Algorithm*** *(intuitive)*

Initialize $\hat{Q}(s, a; \theta)$, put the agent in a random state s

Repeat:

1) Select the action $\text{argmax}_a \hat{Q}(s, a; \theta)$ with probability $(1 - \varepsilon)$
otherwise, select a at random
2) The agent is now in state s' and has received the reward r
3) Define:

$$y = \begin{cases} r & \text{if } s' \text{ is terminal} \\ r + \text{argmax}_{a'} \hat{Q}(s', a'; \theta) & \text{otherwise} \end{cases}$$

4) Perform gradient descent over $(y - \hat{Q}(s, a; \theta))^2$

CAREFUL

Training $\hat{Q}(s, a; \theta)$ *may be non-trivial...*

DQN Algorithm

Deep Q-Learning

- Playing Atari with Deep Reinforcement Learning
  [2013, V. Mnih, K. Kavukcuoglu, D. Silver, A. Graves, I. Antonoglou, D. Wierstra, M. Riedmiller, http://arxiv.org/abs/1312.5602, see also http://www.nature.com/nature/journal/v518/n7540/full/nature14236.html]

A software system only

Runs on virtually any Linux-based system, it contains optional provisions for GPU

It's open source

https://github.com/kuz/DeepMind-Atari-Deep-Q-Learner

Sophisticated machine-learning techniques

Uses deep reinforcement learning

in combination with convolutional neural networks (CNN)

Same configuration, multiple games

Same configuration applied to arcade games

It learned to play 7 (2013) or 49 (2015) different games

It is *autonomous*

It learns by itself, it receives no human expertise as input

In many cases, it outperforms human players

(from GitHub)

Deep Q-Learning

- ***DQN Algorithm*** [https://www.cs.toronto.edu/~vmnih/docs/dqn.pdf]

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory $\mathcal{D}$ to capacity N
Initialize action-value function Q with random weights
for episode $= 1, M$ **do**
  Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
  for $t = 1, T$ **do**
    With probability ϵ select a random action a_t
    otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
    Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in $\mathcal{D}$
    Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
    Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$
    Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
  end for
end for

---

states are images, which require some preprocessing

(see next slide)

Deep Q-Learning

Loss function

$$L(\theta^{(t)}) = \mathbb{E}_{s,a}\left[\left(y^{(t)} - Q(s,a;\theta^{(t)})\right)^2\right]$$

$$= \mathbb{E}_{s,a,s'}\left[\left(r + \gamma \max_{a'} Q(s',a';\theta^{(t-1)}) - Q(s,a;\theta^{(t)})\right)^2\right]$$

these parameters are kept constant when computing the gradient

Gradient

$$\nabla_\theta L(\theta^{(t)}) = \mathbb{E}_{s,a,s'}\left[\left(r + \gamma \max_{a'} Q(s',a';\theta^{(t-1)}) - Q(s,a;\theta^{(t)})\right)\nabla_\theta Q(s,a;\theta^{(t)})\right]$$

- *It is computed at each iteration (see algorithm)*
- *It compares the last (actual) step (variable y in the algorithm) ... with the value given by Q*
- *The expectation is approximated by the average, on the minibatch*

Reinforcement Learning Reformulation

Reinforcement Learning Reformulation

Trajectory

$$\tau := \langle (s_t, a_t) \rangle_{t=0}^{T}$$

i.e., a sequence of *states* and *actions*.
It can be either finite or infinite, depending on T

Reward

Reward function:

$$r_t := r(s_t, a_t, s_{t+1})$$

Depending on the application, it could be *simplified*:

$$r_t := r(s_t, a_t), \quad r_t := r(s_t)$$

we will use these forms from now on, for brevity

Return

$$R(\tau) := \sum_{t=0}^{T} \gamma^t r_t$$

It is discounted when $\gamma < 1$ or undiscounted, when $\gamma = 1$ *(when trajectories are finite)*

Reinforcement Learning Reformulation

Value Function (of a policy)

$$V^\pi(s) := \mathbb{E}[r(S_t) + \gamma r(S_{t+1}) + \gamma^2 r(S_{t+2}) + \ldots \mid \pi, S_t = s]$$

Action-Value function (of a policy)

$$Q^\pi(s,a) := \sum_{S_{t+1}} P(S_{t+1} \mid s, a) \cdot V^\pi(S_{t+1})$$

$$= \mathbb{E}[r(S_t) + \gamma r(S_{t+1}) + \gamma^2 r(S_{t+2}) + \ldots \mid \pi, S_t = s, a_t = a]$$

Reinforcement Learning Reformulation

Value Function (of a policy)

$$V^{\pi}(s) := \mathop{\mathbb{E}}_{\tau\sim\pi}[R(\tau) \mid s_0 = s]$$

Action-Value function (of a policy)

$$Q^{\pi}(s,a) := \mathop{\mathbb{E}}_{\tau\sim\pi}[R(\tau) \mid s_0 = s, a_0 = a]$$

Optimal Value Function

$$V^{*}(s) := \max_{\pi} \mathop{\mathbb{E}}_{\tau\sim\pi}[R(\tau) \mid s_0 = s]$$

Optimal Action-Value Function

$$Q^{*}(s,a) := \max_{\pi} \mathop{\mathbb{E}}_{\tau\sim\pi}[R(\tau) \mid s_0 = s, a_0 = a]$$

Reinforcement Learning Reformulation

- **Connecting Value and Action-Value Functions**

$$V^\pi(s) = \mathop{\mathbb{E}}_{a \sim \pi} [Q^\pi(s, a)]$$

$$V^*(s) = \max_a [Q^*(s, a)]$$

- **Optimal Policy**

$$a^*(s) = \operatorname*{argmax}_a [Q^*(s, a)]$$

- **Advantage Function**

$$A^\pi(s, a) = Q^\pi(s, a) - V^\pi(s)$$

It tells how advantageous (or disadvantageous) is a particular action w.r.t. what is prescribed by the policy

Policy Gradient

Reinforcement Learning Reformulation (2)

Probability of a trajectory

$$P(\tau|\pi) := P(s_0) \prod_{t=0}^{T-1} P(s_{t+1}|s_t, a_t)\pi(a_t|s_t)$$

probability of initial states — *transition probability (i.e. the 'model')*

Expected return of a policy

$$J(\pi) := \mathop{\mathbb{E}}_{\tau\sim\pi}[R(\tau)] = \int_{\tau\sim\pi} P(\tau|\pi)R(\tau)$$

where $\tau \sim \pi$ is the space of all the trajectories distributed as $\pi(a_t|s_t)$

Central Reinforcement Learning Problem

$$\pi^* := \operatorname*{argmax}_{\pi} J(\pi)$$

i.e. finding the policy with the highest expected return

Policy Gradient

Parametric Policy

A generic policy that depends on parameters θ

$$\pi_\theta$$

For instance, in the **DQN Algorithm**, the **Action-Value Function** is approximator is a Deep Neural Network

$$\hat{Q}(s, a; \theta)$$

Policy Gradient Ascent

At each iteration, improve parameters using *expected returns* as the loss function:

$$\theta^{(k+1)} = \theta^{(k)} + \eta \nabla_\theta J(\pi_\theta|_{\theta^{(k)}})$$

easier said than done ...

Policy Gradient

1) **Probability of a trajectory, given a parametric policy**

$$P(\tau|\pi_\theta) := P(s_0) \prod_{t=0}^{T-1} P(s_{t+1}|s_t, a_t)\pi_\theta(a_t|s_t)$$

2) **Log-Derivative**

(Let's take another direction)

By the chain rule, this derivative is:

$$\nabla_\theta \log P(\tau|\pi_\theta) = \frac{1}{P(\tau|\pi_\theta)} \nabla_\theta P(\tau|\pi_\theta)$$

It follows:

$$\nabla_\theta P(\tau|\pi_\theta) = P(\tau|\pi_\theta) \nabla_\theta \log P(\tau|\pi_\theta)$$

Remember this 'mathematical trick':
it will be played again...

Policy Gradient

3) Log-Probability of a trajectory

probability of a trajectory

$$P(\tau|\pi_\theta) := P(s_0) \prod_{t=0}^{T-1} P(s_{t+1}|s_t, a_t)\pi_\theta(a_t|s_t)$$

$$\log P(\tau|\pi_\theta) := \log P(s_0) + \sum_{t=0}^{T-1} [\log P(s_{t+1}|s_t, a_t) + \log \pi_\theta(a_t|s_t)]$$

these terms do NOT depend on θ

4) Gradient of the Log-Probability

$$\nabla_\theta \log P(\tau|\pi_\theta) := \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t)$$

This is the purpose of the 'mathematical trick': discounting the probability model P

Policy Gradient

- **Basic Policy Gradient**

expected return

$$J(\pi) = \int_{\tau\sim\pi} P(\tau|\pi)R(\tau)$$

$$\nabla_\theta J(\pi_\theta) = \int_{\tau\sim\pi_\theta} \nabla_\theta P(\tau|\pi_\theta)R(\tau)$$

this term does NOT depend on θ

$$= \int_{\tau\sim\pi_\theta} P(\tau|\pi_\theta)\nabla_\theta \log P(\tau|\pi_\theta)R(\tau)$$

$$= \mathop{\mathbb{E}}_{\tau\sim\pi_\theta}\left[\nabla_\theta \log P(\tau|\pi_\theta)R(\tau)\right]$$

$$= \mathop{\mathbb{E}}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t)R(\tau)\right]$$

This last term is an <u>expectation</u>: it can be estimated from a <u>sample mean</u>

Policy Gradient

- **Basic Policy Gradient**

$$\nabla_\theta J(\pi_\theta) = \mathop{\mathbb{E}}_{\tau \sim \pi_\theta} \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) R(\tau) \right]$$

$$\hat{g} := \frac{1}{|\mathcal{D}|} \sum_{\tau \in \mathcal{D}} \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) R(\tau)$$

Dataset: a sample of actual trajectories

Estimated gradient (mean)

Policy Gradient

- **Basic Policy Gradient**

$$\nabla_\theta J(\pi_\theta) = \mathop{\mathbb{E}}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) R(\tau)\right]$$

An entire trajectory? Even in the past?

Better switch to this:

$$\nabla_\theta J(\pi_\theta) = \mathop{\mathbb{E}}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) \sum_{t'=t}^{T-1} r(s_{t'}, a_{t'})\right]$$

Reward from t onward
('reward-to-go')

Simple Policy Gradient

- ***Pseudo-Algorithm***

Initialize the weights θ of a DNN $\hat{Q}(s, a; \theta)$ at random

Repeat:

1) For M episodes

   Start in initial state s_0

   For t from 0 to T

   play by $a_t \sim \pi_\theta(a|s_t)$

   Collect the episode trajectory $\tau = \langle(s_t, a_t)\rangle_{t=0}^{T}$ and store it in $\mathcal{D}$

2) Sample a random minibatch $\mathcal{B} = \{\tau_i\}$ from $\mathcal{D}$

$$\Delta\theta = \eta \, \frac{1}{|\mathcal{B}|} \sum_{\tau \in \mathcal{B}} \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) R(\tau)$$

How can we 'sample a policy' in practice?

replay buffer

Sampling a Policy

Problem

Sampling actions from a stochastic policy

$$a_t \sim \pi_\theta(a|s_t)$$

Intended meaning:

$$\pi_\theta(a_t|s_t) \propto \hat{Q}(a_t, s_t; \theta)$$

the probability of each action should be proportional to the expected return

Discrete Action Space

Consider $\hat{Q}(a_t, s_t; \theta)$ as the **logit** of a softmax

$$\pi_\theta(a_t|s_t) := \frac{\exp(\hat{Q}(a_t, s_t; \theta))}{\sum_{a \in \mathcal{A}(s_t)} \exp(\hat{Q}(a, s_t; \theta))}$$

and sample accordingly

All possible actions in state s_t

Actor-Critic

Actor-Critic

An Aside:
Expected Grad-Log Probability
(EGLP lemma)

EGLP Lemma. Suppose that P_θ is a parameterized probability distribution over a random variable, x. Then:

$$\underset{x \sim P_\theta}{\mathrm{E}} \left[\nabla_\theta \log P_\theta(x) \right] = 0.$$

Proof

Recall that all probability distributions are *normalized*:

$$\int_x P_\theta(x) = 1.$$

Take the gradient of both sides of the normalization condition:

$$\nabla_\theta \int_x P_\theta(x) = \nabla_\theta 1 = 0.$$

Use the log derivative trick to get:

$$\begin{aligned} 0 &= \nabla_\theta \int_x P_\theta(x) \\ &= \int_x \nabla_\theta P_\theta(x) \\ &= \int_x P_\theta(x) \nabla_\theta \log P_\theta(x) \\ \therefore 0 &= \underset{x \sim P_\theta}{\mathrm{E}} \left[\nabla_\theta \log P_\theta(x) \right]. \end{aligned}$$

[image from: https://spinningup.openai.com/en/latest/spinningup/rl_intro3.html]

Actor-Critic

Policy Gradient

$$\nabla_\theta J(\pi_\theta) = \mathop{\mathbb{E}}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) \sum_{t'=t}^{T-1} r(s_{t'}, a_{t'})\right]$$

Due to the EGLP lemma:

$$\mathop{\mathbb{E}}_{a_t\sim\pi_\theta}\left[\nabla_\theta \log \pi_\theta(a_t|s_t)\, b(s_t)\right] = 0$$

for any function $b(s_t)$ that depends on s_t only *(i.e., $b(s_t)$ is constant w.r.t. to a_t)*

Policy Gradient with Baseline

baseline

$$\nabla_\theta J(\pi_\theta) = \mathop{\mathbb{E}}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) \left(\left(\sum_{t'=t}^{T-1} r(s_{t'}, a_{t'})\right) - b(s_t)\right)\right]$$

We can subtract term-wise any function $b(s_t)$ without altering the expectation

Actor-Critic

Actor-Critic
(typical formulation)

state-value function *as baseline*

$$\nabla_\theta J(\pi_\theta) = \mathop{\mathbb{E}}_{\tau \sim \pi_\theta} \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) \left(\left(\sum_{t'=t}^{T-1} r(s_{t'}, a_{t'}) \right) - V^\pi(s_t) \right) \right]$$

Note that:

$$\begin{aligned} \left(\sum_{t'=t}^{T-1} r(s_{t'}, a_{t'}) \right) - V^\pi(s_t) &= (r(s_t, a_t) + V^\pi(s_{t+1})) - V^\pi(s_t) \\ &= Q^\pi(s_t, a_t) - V^\pi(s_t) \\ &= A^\pi(s_t, a_t) \end{aligned}$$

it's the ***advantage function***

Actor-Critic

Actor-Critic
(typical formulation)

$$\nabla_\theta J(\pi_\theta) = \mathop{\mathbb{E}}_{\tau \sim \pi_\theta}\left[\sum_{t=0}^{T-1} \nabla_\theta \boxed{\log \pi_\theta(a_t|s_t)}\ \boxed{A^\pi(s_t, a_t)}\right]$$

'Actor' *'Critic'*

In practice, $V^\pi(s_t)$ is estimated via $\hat{V}(s;\phi)$
namely, <u>another</u> DNN with specific parameters ϕ

$$\hat{A}(s_t, a_t) := \left(r(s_t, a_t) + \hat{V}(s_{t+1};\phi)\right) - \hat{V}(s_t;\phi)$$

What are the advantages? "It reduces variance"

Intuitively $\hat{Q}(s, a;\theta)$ depends also on how the action space is explored
whereas $\hat{V}(s_t;\phi)$ depends only on <u>actual rewards</u> $r(s_t, a_t)$

Actor-Critic

- ***Pseudo-Algorithm***

Initialize the weights θ, ϕ of <u>*two*</u> DNNs $\pi_\theta(a|s)$, $\hat{V}(s;\phi)$ at random

Repeat:

1) For M episodes

   Start in initial state s_0

   For t from 0 to T

   play by $a_t \sim \pi_\theta(a|s_t)$

   Collect all episode ***trajectories*** $\tau_r := \langle(s_t, a_t, r_t, s_{t+1})\rangle_{t=0}^{T}$ and store them in $\mathcal{D}$ — *replay buffer*

2) For a random minibatch $\mathcal{B} = \{(s_i, a_i, r_i, s_{i+1})\}$ from $\mathcal{D}$

   Evaluate

$$\hat{A}(s_i, a_i) = \left(r_i + \hat{V}(s_{i+1};\phi)\right) - \hat{V}(s_i;\phi)$$

   Update weights

$$\Delta\phi = -\eta_\phi \nabla_\phi \left(\hat{A}(s_i, a_i)\right)^2$$

gradient <u>descent</u>: $\hat{V}(s;\phi)$ *should converge to* $V(s)$

$$\Delta\theta = \eta_\theta \nabla_\theta J(\pi_\theta) = \eta_\theta \nabla_\theta \log \pi_\theta(a_i|s_i)\ \hat{A}(s_i, a_i)$$

gradient <u>ascent</u>: increase $J(\pi_\theta)$ *as much as possible*

Actor-Critic

- ***Pseudo-Algorithm*** *(update step in more detail)*

2) For a random minibatch $\mathcal{B} = \{(s_i, a_i, r_i, s_{i+1})\}$ from $\mathcal{D}$

Evaluate

$$\hat{h} = \frac{1}{|\mathcal{D}|} \sum_{\tau \in \mathcal{D}} \frac{1}{T} \sum_{t=0}^{T} \nabla_\phi \left(\hat{V}(s_t; \phi) - \hat{R}_t) \right)^2 \qquad \hat{R}_t = \left(r_t + \hat{V}(s_{t+1}; \phi) \right)$$

and:

$$\hat{g} = \frac{1}{|\mathcal{D}|} \sum_{\tau \in \mathcal{D}} \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) \hat{A}_t \qquad \hat{A}_t = \left(r_t + \hat{V}(s_{t+1}; \phi) \right) - \hat{V}(s_t; \phi)$$

Update

$$\phi^{(t+1)} = \phi^{(t)} - \eta_\phi \; \hat{h}$$

$$\theta^{(t+1)} = \theta^{(t)} - \eta_\theta \; \hat{g}$$

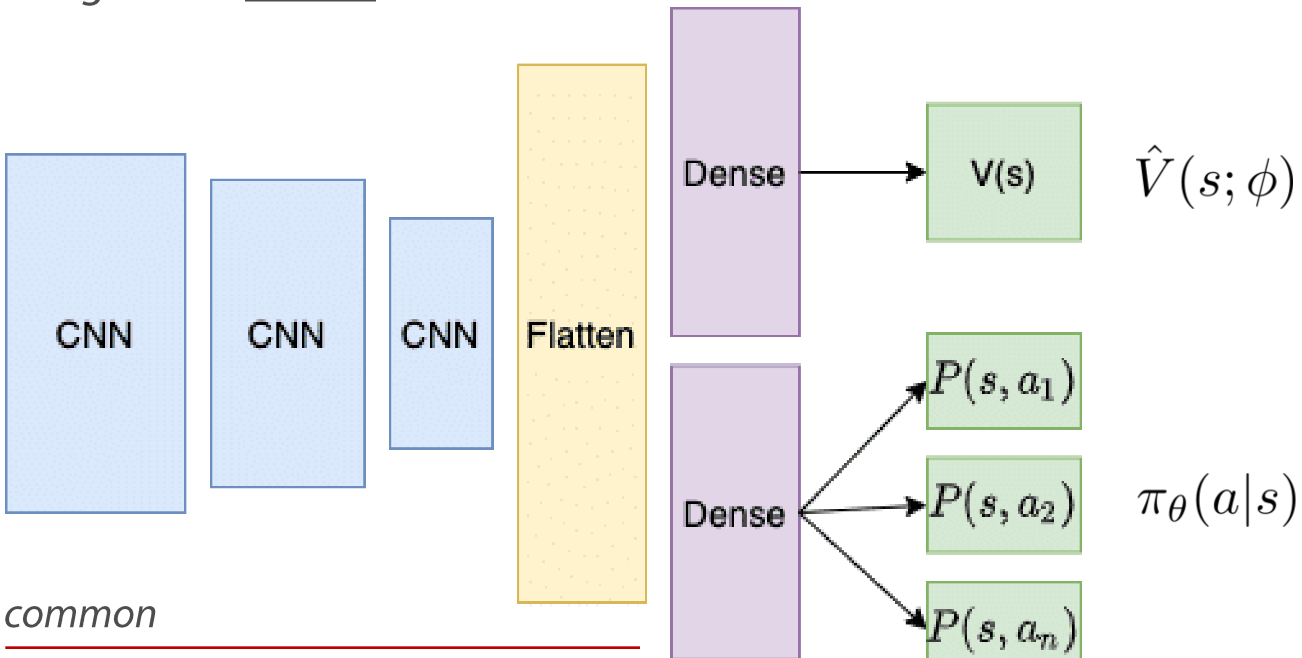
NOTE: standard mini-batch gradient optimization can be replaced by Adam or other optimizers

Actor-Critic

- ***Network Architecture (typical)***

  A bifurcated structure which includes:
  - A common part
  - A V-head
  - A π-head

  It follows that part of the weights are shared



[image from: https://adventuresinmachinelearning.com/a2c-advantage-actor-critic-tensorflow-2/]

Proximal Policy Optimization (PPO)

Improving Training Stability

In the Actor-Critic approach, the *policy update* step is critical:

$$\Delta\theta = \eta_\theta \nabla_\theta J(\pi_\theta) = \eta_\theta \nabla_\theta \log \pi_\theta(a_i|s_i)\ \hat{A}(s_i, a_i)$$

Unlike supervised learning, deep reinforcement learning is a very delicate process:
even small steps $\Delta\theta$ may induce large changes in policy, which may compromise the entire optimization

The objective of Proximal Policy Optimization (PPO) is ensuring that steps $\Delta\theta$
are kept within a viable range, determined via hyperparameters

Improving Training Stability

The *policy update* step rewritten:

$$\theta^{(t+1)} = \theta^{(t)} + \eta_\theta \nabla_\theta J(\pi_\theta)$$

where:

$$\nabla_\theta J(\pi_\theta) = \nabla_\theta \log \pi_\theta(a_i|s_i) \ \hat{A}(s_i, a_i)$$

Introduce the *surrogate advantage* term,
a measure of how a new policy θ performs relative to a current policy $\theta^{(t)}$:

$$\mathcal{L}(\theta, \theta^{(t)}) = \mathop{\mathbb{E}}_{s,a \sim \pi_{\theta^{(t)}}} \left[\frac{\pi_\theta(a|s)}{\pi_{\theta^{(t)}}(a|s)} \hat{A}^{\pi_{\theta^{(t)}}}(s, a) \right]$$

It can be proven that:

$$\nabla_\theta J(\pi_\theta) \Big|_{\theta^{(t)}} = \nabla_\theta \mathcal{L}(\theta, \theta^{(t)})$$

PPO-clip

In PPO clip, the term $J(\pi_\theta)$

is replaced by the clipped *surrogate advantage* term:

$$L(\theta, \theta^{(t)}) = \underset{s,a\sim\pi_{\theta^{(t)}}}{\mathbb{E}} \left[\min\left(\frac{\pi_\theta(a|s)}{\pi_{\theta^{(t)}}(a|s)} \hat{A}^{\pi_{\theta^{(t)}}}(s,a),\ \text{clip}\left(\epsilon, \hat{A}^{\pi_{\theta^{(t)}}}(s,a)\right)\right)\right]$$

where:

$$\text{clip}\left(\epsilon, \hat{A}\right) := \begin{cases} 1+\epsilon & \text{if } A \geq 0 \\ 1-\epsilon & \text{otherwise} \end{cases}$$

and ϵ is a suitable hyperparameter value