Università degli
Studi di Pavia

Deep Learning

11 – Deep Reinforcement Learning

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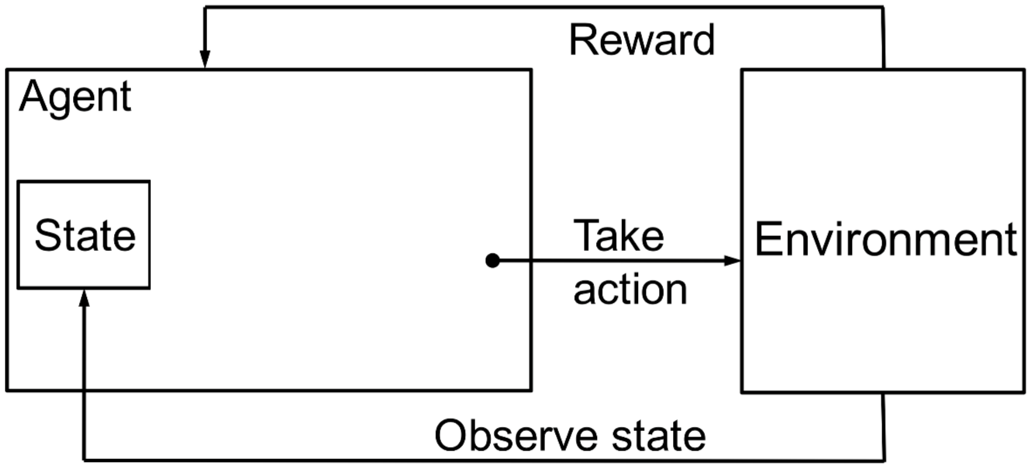
This presentation can be downloaded at:
http://vision.unipv.it/DL

Basics (Intuition)

Deep Reinforcement Learning (DRL)

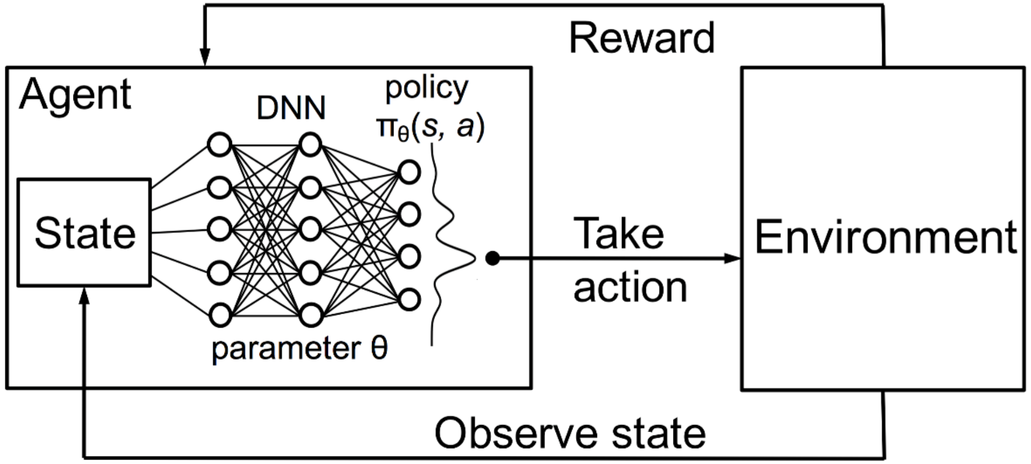
- ***Reinforcement Learning***



Deep Reinforcement Learning (DRL)

- ***Deep Reinforcement Learning***

  Using a deep neural network as the approximator $\hat{Q}(s, a)$



The optimal policy is learnt incrementally by using a deep neural network

Q-Learning

- ***Q-Learning Algorithm***

Initialize $\hat{Q}(s,a)$ at random, put the agent is in a random state s

Repeat:

1) Select the action $\text{argmax}_a \hat{Q}(s,a)$ with probability $(1-\varepsilon)$ otherwise, select a at random
2) The agent is now in state s' and has received the reward r
3) Update $\hat{Q}(s,a)$ by

$$\Delta\hat{Q}(s,a) = \alpha[r + \gamma \max_{a'} \hat{Q}(s',a') - \hat{Q}(s,a)]$$

Deep Reinforcement Learning

- ***Q-Learning Algorithm***

Initialize $\hat{Q}(s,a)$ at random, put the agent in a random state s
Repeat:

1) Select the action $\mathrm{argmax}_a \hat{Q}(s,a)$ with probability $(1-\varepsilon)$
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***Fundamental Idea**:*

Use a deep neural network to learn the approximator $\hat{Q}(s,a)$
from the examples collected while ***exploring*** *–* ***exploring***

Also replacing the update step with DNN training

Deep Reinforcement Learning

- ***Q-Learning Algorithm***

Initialize $\hat{Q}(s,a)$ at random, put the agent in a random state s

Repeat:

1) Select the action $\text{argmax}_a \hat{Q}(s,a)$ with probability $(1-\varepsilon)$
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$$\Delta\hat{Q}(s,a) = \alpha[r + \gamma \max_{a'} \hat{Q}(s',a') - \hat{Q}(s,a)]$$

CAREFUL

maximizing $\hat{Q}(s,a)$ *when this is a deep neural network may be non-trivial...*

Reinforcement Learning Reformulation

Reinforcement Learning Reformulation

Trajectory

$$\tau := \langle (s_t, a_t) \rangle_{t=0}^{T}$$

i.e., a sequence of states and actions.
It can be either finite or infinite, depending on T

Reward

Reward function:

$$r_t := r(s_t, a_t, s_{t+1})$$

Depending on the application, it can be simplified:

$$r_t := r(s_t, a_t), \quad r_t := r(s_t)$$

we will use these forms from now on, for brevity

Return

$$R(\tau) := \sum_{t=0}^{T} \gamma^t r_t$$

It is discounted when $\gamma < 1$ or undiscounted, when $\gamma = 1$ *(when trajectories are finite)*

Reinforcement Learning Reformulation

Probability of a trajectory

$$P(\tau|\pi) := P(s_0) \prod_{t=0}^{T-1} P(s_{t+1}|s_t, a_t)\pi(a_t|s_t)$$

probability of initial states

transition probability (i.e. the 'model')

Expected return of a policy

$$J(\pi) := \int_{\tau\sim\pi} P(\tau|\pi)R(\tau) \;=\; \mathop{\mathbb{E}}_{\tau\sim\pi}[R(\tau)]$$

where $\tau \sim \pi$ is the space of all the trajectories distributed as $\pi(a_t|s_t)$

Central RL Problem

$$\pi^* := \operatorname*{argmax}_{\pi} J(\pi)$$

i.e. finding the policy with the highest expected return

Reinforcement Learning Reformulation

Value Function (of a policy)

$$V^\pi(s) := \mathbb{E}[r(S_t) + \gamma r(S_{t+1}) + \gamma^2 r(S_{t+2}) + \ldots \mid \pi, S_t = s]$$

Action-Value function (of a policy)

$$Q^\pi(s,a) := \textstyle\sum_{S_{t+1}} P(S_{t+1} \mid s, a) \cdot V^\pi(S_{t+1})$$

$$= \mathbb{E}[r(S_t) + \gamma r(S_{t+1}) + \gamma^2 r(S_{t+2}) + \ldots \mid \pi, S_t = s, a_t = a]$$

Reinforcement Learning Reformulation

Value Function (of a policy)

$$V^{\pi}(s) := \underset{\tau \sim \pi}{\mathbb{E}} [R(\tau) \mid s_0 = s]$$

Action-Value function (of a policy)

$$Q^{\pi}(s,a) := \underset{\tau \sim \pi}{\mathbb{E}} [R(\tau) \mid s_0 = s, a_0 = a]$$

Optimal Value Function

$$V^{*}(s) := \max_{\pi} \underset{\tau \sim \pi}{\mathbb{E}} [R(\tau) \mid s_0 = s]$$

Optimal Action-Value Function

$$Q^{*}(s,a) := \max_{\pi} \underset{\tau \sim \pi}{\mathbb{E}} [R(\tau) \mid s_0 = s, a_0 = a]$$

Reinforcement Learning Reformulation

- **Connecting Value and Action-Value Functions**

$$V^{\pi}(s) = \mathop{\mathbb{E}}_{a \sim \pi} \left[Q^{\pi}(s,a) \right]$$

$$V^{*}(s) = \max_{a} \left[Q^{*}(s,a) \right]$$

- **Optimal Policy**

$$a^{*}(s) = \operatorname*{argmax}_{a} \left[Q^{*}(s,a) \right]$$

- **Advantage Function**

$$A^{\pi}(s,a) = Q^{\pi}(s,a) - V^{\pi}(s)$$

It tells how advantageous (or disadvantageous) is a particular action w.r.t. what is prescribed by the policy

DQN Algorithm

Deep Q-Learning

- Playing Atari with Deep Reinforcement Learning
  [2013, V. Mnih, K. Kavukcuoglu, D. Silver, A. Graves, I. Antonoglou, D. Wierstra, M. Riedmiller, http://arxiv.org/abs/1312.5602, see also http://www.nature.com/nature/journal/v518/n7540/full/nature14236.html]

  A software system only
    Runs on virtually any Linux-based system, it contains optional provisions for GPU

  It's open source
    https://github.com/kuz/DeepMind-Atari-Deep-Q-Learner

  Sophisticated machine-learning techniques
    Uses deep reinforcement learning
    in combination with convolutional neural networks (CNN)

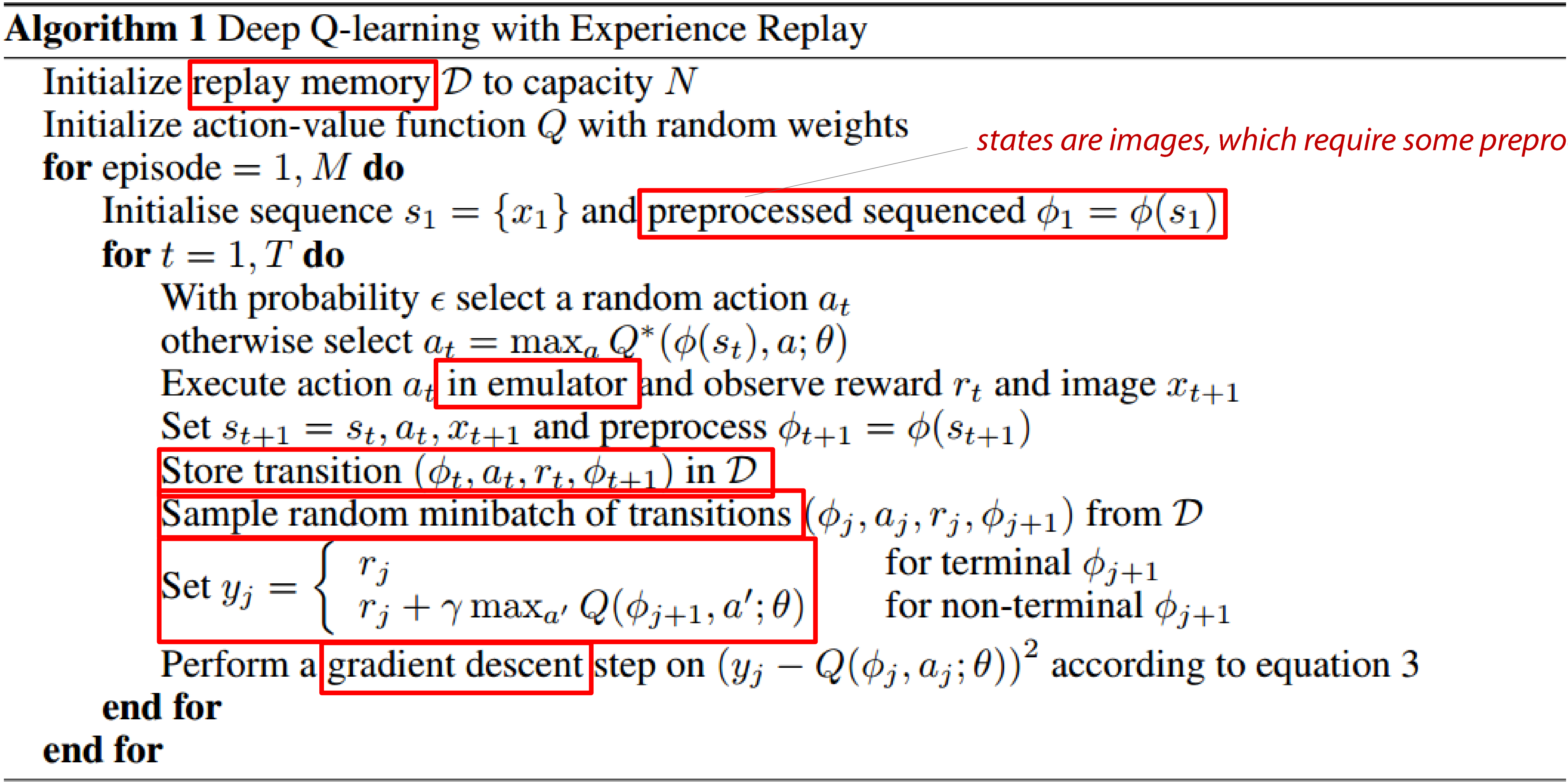
  Same configuration, multiple games
    Same configuration applied to arcade games
    It learned to play 7 (2013) or 49 (2015) different games

  It is *autonomous*
    It learns by itself, it receives no human expertise as input
    In many cases, it outperforms human players

(from GitHub)

Deep Q-Learning

- ***DQN Algorithm*** [https://www.cs.toronto.edu/~vmnih/docs/dqn.pdf]



Algorithm 1 Deep Q-learning with Experience Replay

```
Initialize replay memory $\mathcal{D}$ to capacity $N$
Initialize action-value function $Q$ with random weights
for episode $= 1, M$ do
    Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
    for $t = 1, T$ do
        With probability $\epsilon$ select a random action $a_t$
        otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
        Execute action $a_t$ in emulator and observe reward $r_t$ and image $x_{t+1}$
        Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
        Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in $\mathcal{D}$
        Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
        Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$
        Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
    end for
end for
```

states are images, which require some preprocessing

Policy Gradient

Policy Gradient

Parametric Policy

A generic policy that depends on parameters θ

$$\pi_\theta$$

For instance, in the **DQN Algorithm**, the **Action-Value Function** is approximator is a Deep Neural Network

$$\hat{Q}(s, a; \theta)$$

Policy Gradient Ascent

At each iteration, improve parameters using *expected returns* as the loss function:

$$\theta^{(k+1)} = \theta^{(k)} + \eta \nabla_\theta J(\pi_\theta)\,\big|_{\theta^{(k)}}$$

easier said than done ...

Policy Gradient

1) Probability of a trajectory, given a parametric policy

$$P(\tau|\pi_\theta) := P(s_0) \prod_{t=0}^{T-1} P(s_{t+1}|s_t, a_t)\pi_\theta(a_t|s_t)$$

2) Log-Derivative

By applying the chain rule:

$$\nabla_\theta \log P(\tau|\pi_\theta) = \frac{1}{P(\tau|\pi_\theta)} \nabla_\theta P(\tau|\pi_\theta)$$

It follows:

$$\nabla_\theta P(\tau|\pi_\theta) = P(\tau|\pi_\theta) \nabla_\theta \log P(\tau|\pi_\theta)$$

Policy Gradient

3) Log-Probability

$$\log P(\tau|\pi_\theta) := \log P(s_0) + \sum_{t=0}^{T-1} \left[\log P(s_{t+1}|s_t, a_t) + \log \pi_\theta(a_t|s_t)\right]$$

these terms do NOT depend on θ

4) Gradient of the Log-Probability

$$\nabla_\theta \log P(\tau|\pi_\theta) := \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t)$$

5) Expected return

$$J(\pi_\theta) := \int_{\tau\sim\pi_\theta} P(\tau|\pi_\theta) R(\tau) \;=\; \mathop{\mathbb{E}}_{\tau\sim\pi_\theta} [R(\tau)]$$

Policy Gradient

- **Basic Policy Gradient**

$$\nabla_\theta J(\pi_\theta) = \int_{\tau \sim \pi_\theta} \nabla_\theta P(\tau|\pi_\theta) R(\tau)$$

this term does NOT depend on θ

$$= \int_{\tau \sim \pi_\theta} P(\tau|\pi_\theta) \nabla_\theta \log P(\tau|\pi_\theta) R(\tau)$$

$$= \mathop{\mathbb{E}}_{\tau \sim \pi_\theta} \left[\nabla_\theta \log P(\tau|\pi_\theta) R(\tau)\right]$$

$$= \mathop{\mathbb{E}}_{\tau \sim \pi_\theta} \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) R(\tau)\right]$$

This last term is an <u>expectation</u>: it can be estimated from a sample mean

Policy Gradient

- **Basic Policy Gradient**

$$\nabla_\theta J(\pi_\theta) = \underset{\tau\sim\pi_\theta}{\mathbb{E}}\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) R(\tau)\right]$$

$$\hat{g} := \frac{1}{|\mathcal{D}|} \sum_{\tau\in\mathcal{D}} \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) R(\tau)$$

dataset: a sample of actual trajectories

estimated gradient (mean)

Policy Gradient

- **Basic Policy Gradient**

$$\nabla_\theta J(\pi_\theta) = \mathop{\mathbb{E}}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) R(\tau)\right]$$

an entire trajectory? even in the past?

More precisely:

$$\nabla_\theta J(\pi_\theta) = \mathop{\mathbb{E}}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) \sum_{t'=t}^{T-1} r(s_{t'}, a_{t'})\right]$$

reward from t onward
('reward-to-go')

Simple Policy Gradient

- ***Pseudo-Algorithm***

```
Initialize the weights $\theta$ of a DNN $\hat{Q}(s,a;\theta)$ at random
Repeat:
1)  For $M$ episodes
        Start in initial state $s_0$
        For $t$ from $0$ to $T$
            play by $\boxed{a_t \sim \pi_\theta(a|s_t)}$
        Collect the episode trajectory $\tau = \langle(s_t, a_t)\rangle_{t=0}^{T}$ and store it in $\mathcal{D}$
2)  Sample a random minibatch $\mathcal{B} = \{(s_i, a_i)\}$ from $\mathcal{D}$
```

How can we 'sample a policy' in practice?

$$\Delta\theta = \eta \frac{1}{|\mathcal{B}|} \sum_{\tau \in \mathcal{B}} \nabla_\theta \log \pi_\theta(a_t|s_t) R(\tau)$$

Sampling a Policy

Problem

Sampling actions from a stochastic policy

$$a_t \sim \pi_\theta(a|s_t)$$

Intended meaning:

$$\pi_\theta(a_t|s_t) \propto \hat{Q}(a_t, s_t; \theta)$$

the probability of each action should be proportional to the expected return

Discrete Action Space

Consider $\hat{Q}(a_t, s_t; \theta)$ as the **logit** of a softmax

$$\pi_\theta(a_t|s_t) := \frac{\exp(\hat{Q}(a_t, s_t; \theta))}{\sum_{a \in \mathcal{A}(s_t)} \exp(\hat{Q}(a, s_t; \theta))}$$

and sample accordingly

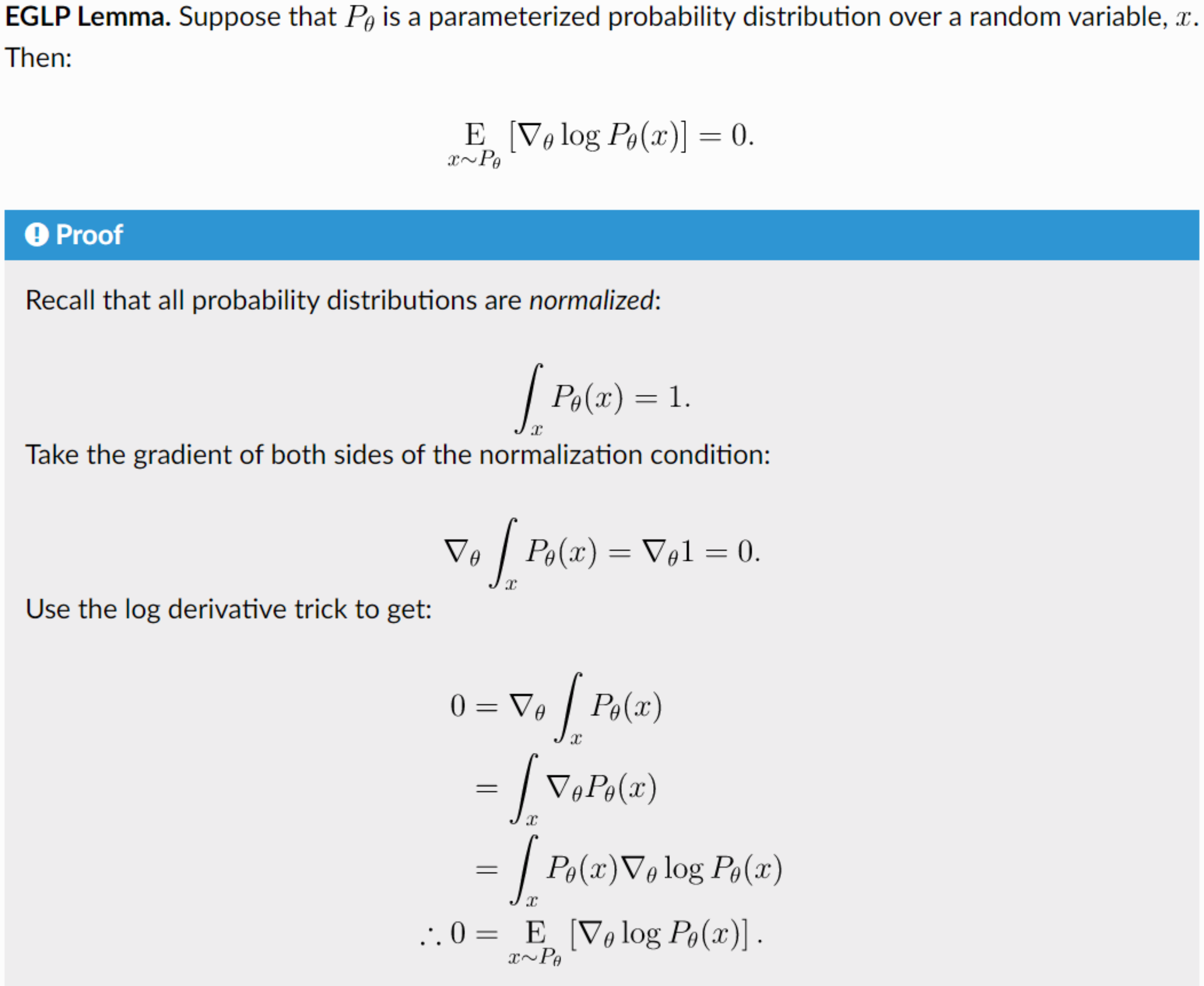
all possible actions in state s_t

The Continuous Case is a bit more complex ...

Actor-Critic

Actor-Critic

An Aside:
Expected Grad-Log Probability
(EGLP lemma)



EGLP Lemma. Suppose that P_θ is a parameterized probability distribution over a random variable, x. Then:

$$\underset{x \sim P_\theta}{\mathrm{E}} \left[\nabla_\theta \log P_\theta(x) \right] = 0.$$

Proof

Recall that all probability distributions are *normalized*:

$$\int_x P_\theta(x) = 1.$$

Take the gradient of both sides of the normalization condition:

$$\nabla_\theta \int_x P_\theta(x) = \nabla_\theta 1 = 0.$$

Use the log derivative trick to get:

$$\begin{aligned} 0 &= \nabla_\theta \int_x P_\theta(x) \\ &= \int_x \nabla_\theta P_\theta(x) \\ &= \int_x P_\theta(x) \nabla_\theta \log P_\theta(x) \\ \therefore 0 &= \underset{x \sim P_\theta}{\mathrm{E}} \left[\nabla_\theta \log P_\theta(x) \right]. \end{aligned}$$

[image from: https://spinningup.openai.com/en/latest/spinningup/rl_intro3.html]

Actor-Critic

Policy Gradient

$$\nabla_\theta J(\pi_\theta) = \underset{\tau \sim \pi_\theta}{\mathbb{E}} \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) \sum_{t'=t}^{T-1} r(s_{t'}, a_{t'}) \right]$$

Due to the EGLP lemma:

$$\underset{a_t \sim \pi_\theta}{\mathbb{E}} \left[\nabla_\theta \log \pi_\theta(a_t|s_t)\, b(s_t) \right] = 0$$

for any function $b(s_t)$ that depends on s_t only *(i.e., $b(s_t)$ is constant w.r.t. to a_t)*

Policy Gradient with Baseline

$$\nabla_\theta J(\pi_\theta) = \underset{\tau \sim \pi_\theta}{\mathbb{E}} \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) \left(\left(\sum_{t'=t}^{T-1} r(s_{t'}, a_{t'}) \right) - \overset{\text{baseline}}{b(s_t)} \right) \right]$$

We can subtract term-wise any function $b(s_t)$ without altering the expectation

Actor-Critic

Actor-Critic
(typical formulation)

$$\nabla_\theta J(\pi_\theta) = \mathop{\mathbb{E}}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T-1}\nabla_\theta \log \pi_\theta(a_t|s_t)\left(\left(\sum_{t'=t}^{T-1} r(s_{t'},a_{t'})\right) - V^\pi(s_t)\right)\right]$$

Note that:

$$\begin{aligned}\left(\sum_{t'=t}^{T-1} r(s_{t'},a_{t'})\right) - V^\pi(s_t) &= (r(s_t,a_t) + V^\pi(s_{t+1})) - V^\pi(s_t)\\ &= Q^\pi(s_t,a_t) - V^\pi(s_t)\\ &= A^\pi(s_t,a_t)\end{aligned}$$

it’s the ***advantage function***

Actor-Critic

Actor-Critic
(typical formulation)

$$\nabla_\theta J(\pi_\theta) = \mathop{\mathbb{E}}_{\tau \sim \pi_\theta} \left[\sum_{t=0}^{T-1} \nabla_\theta \boxed{\log \pi_\theta(a_t | s_t)} \boxed{A^\pi(s_t, a_t)} \right]$$

'Actor' *'Critic'*

In practice, $V^\pi(s_t)$ is estimated via $\hat{V}(s; \phi)$
namely, <u>another</u> DNN with specific parameters ϕ

$$\hat{A}(s_t, a_t) := \Big(r(s_t, a_t) + \hat{V}(s_{t+1}; \phi) \Big) - \hat{V}(s_t; \phi)$$

What are the advantages? "It reduces variance"

Intuitively $\hat{Q}(s, a; \theta)$ depends also on how the action space is explored
whereas $\hat{V}(s_t; \phi)$ depends only on <u>actual rewards</u> $r(s_t, a_t)$

Actor -Critic

- ***Pseudo-Algorithm***

```
Initialize the weights $\theta, \phi$ of two DNNs $\pi_\theta(a|s),\ \hat{V}(s;\phi)$ at random
Repeat:
1) For $M$ episodes
        Start in initial state $s_0$
        For $t$ from 0 to $T$
            play by $a_t \sim \pi_\theta(a|s_t)$
        Collect all episode transitions $\tau_r := \langle (s_t, a_t, r_t, s_{t+1}) \rangle_{t=0}^{T}$ and store them in $\mathcal{D}$

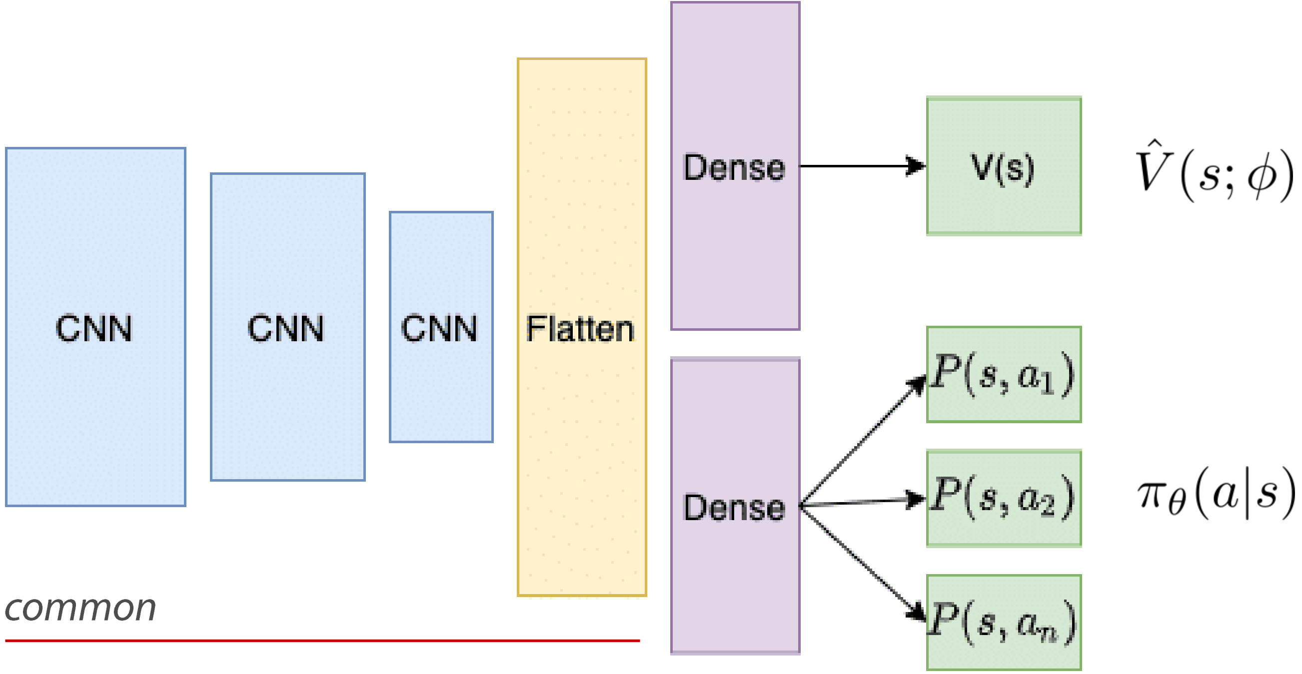
2) For a random minibatch $\mathcal{B} = \{(s_i, a_i, r_i, s_{i+1})\}$ from $\mathcal{D}$
        Evaluate
            $\hat{A}(s_i, a_i) = \Big(r_i + \hat{V}(s_{i+1}, \phi)\Big) - \hat{V}(s_i, \phi)$
        Update weights
            $\Delta\phi = -\eta_\phi \nabla_\phi \left(\hat{A}(s_i, a_i)\right)^2$
            $\Delta\theta = \eta_\theta \nabla_\theta J(\pi_\theta) = \eta_\theta \nabla_\theta \log \pi_\theta(a_i|s_i)\ \hat{A}(s_i, a_i)$
```

Actor -Critic

- ***Network Architecture***

  A bifurcated structure which includes:
  - A common part
  - A V-head
  - A π-head

  It follows that part of the weights are *shared*

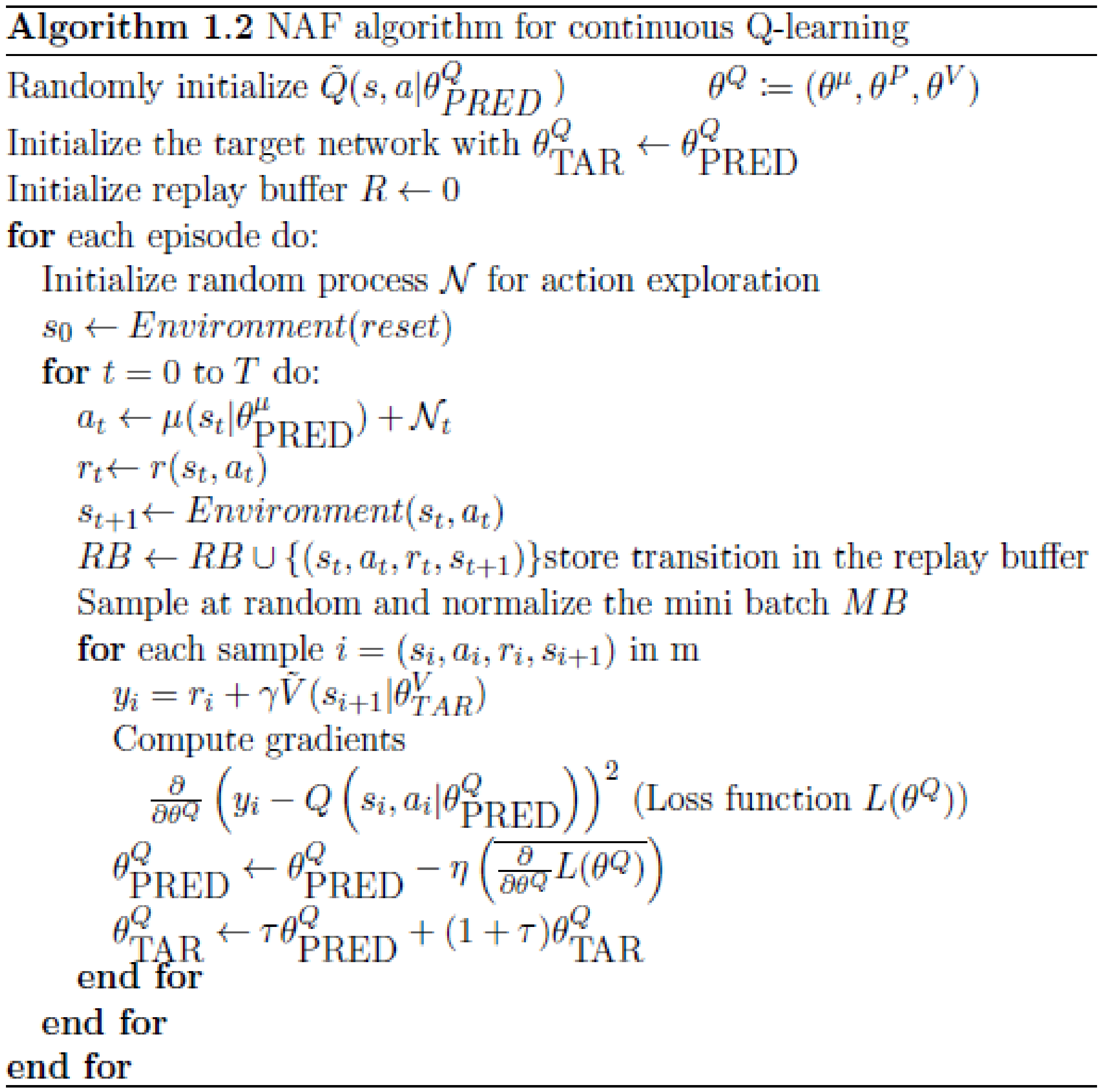


[image from: https://adventuresinmachinelearning.com/a2c-advantage-actor-critic-tensorflow-2/]

Normalized Advantage Function (NAF)

Normalized Advantage Function (NAF) algorithm

S. Gu, T. P. Lillicrap, I. Sutskever, S. Levine.
Continuous deep Q-learning
with model-based acceleration*, 2016*



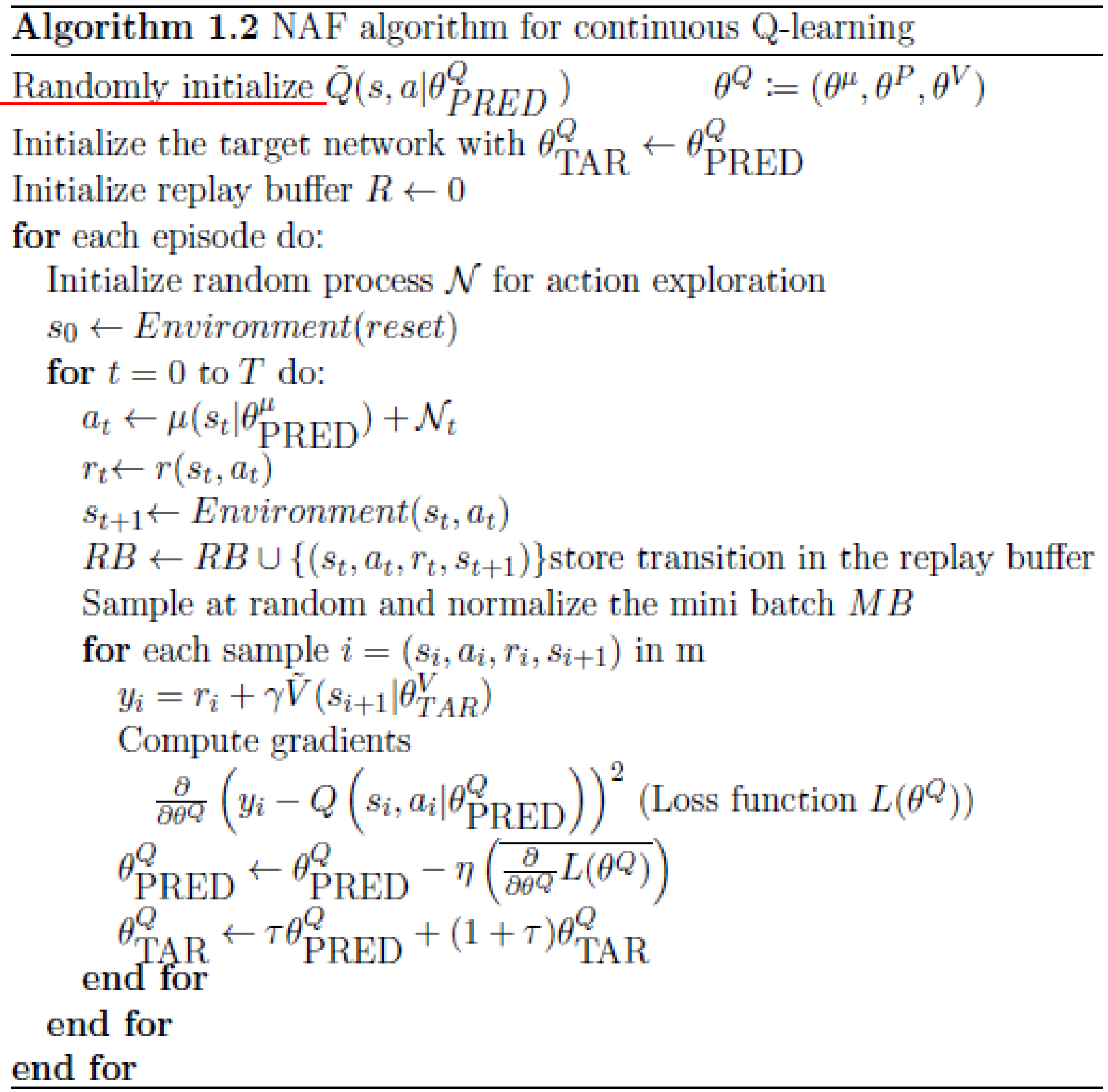
Algorithm 1.2 NAF algorithm for continuous Q-learning

```
Randomly initialize $\tilde{Q}(s,a|\theta^Q_{PRED})$        $\theta^Q := (\theta^\mu, \theta^P, \theta^V)$
Initialize the target network with $\theta^Q_{\text{TAR}} \leftarrow \theta^Q_{\text{PRED}}$
Initialize replay buffer $R \leftarrow 0$
for each episode do:
  Initialize random process $\mathcal{N}$ for action exploration
  $s_0 \leftarrow Environment(reset)$
  for $t = 0$ to $T$ do:
    $a_t \leftarrow \mu(s_t|\theta^\mu_{\text{PRED}}) + \mathcal{N}_t$
    $r_t \leftarrow r(s_t, a_t)$
    $s_{t+1} \leftarrow Environment(s_t, a_t)$
    $RB \leftarrow RB \cup \{(s_t, a_t, r_t, s_{t+1})\}$store transition in the replay buffer
    Sample at random and normalize the mini batch $MB$
    for each sample $i = (s_i, a_i, r_i, s_{i+1})$ in m
      $y_i = r_i + \gamma \tilde{V}(s_{i+1}|\theta^V_{TAR})$
      Compute gradients
        $\frac{\partial}{\partial \theta^Q}\left(y_i - Q\left(s_i, a_i|\theta^Q_{\text{PRED}}\right)\right)^2$ (Loss function $L(\theta^Q)$)
      $\theta^Q_{\text{PRED}} \leftarrow \theta^Q_{\text{PRED}} - \eta\left(\overline{\frac{\partial}{\partial \theta^Q} L(\theta^Q)}\right)$
      $\theta^Q_{\text{TAR}} \leftarrow \tau\theta^Q_{\text{PRED}} + (1+\tau)\theta^Q_{\text{TAR}}$
    end for
  end for
end for
```

Normalized Advantage Function (NAF) algorithm

Algorithm Highlights

- *a deep neural network for* $\hat{Q}(s,a)$



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        for each sample $i = (s_i, a_i, r_i, s_{i+1})$ in m
            $y_i = r_i + \gamma\tilde{V}(s_{i+1}|\theta^V_{TAR})$
            Compute gradients
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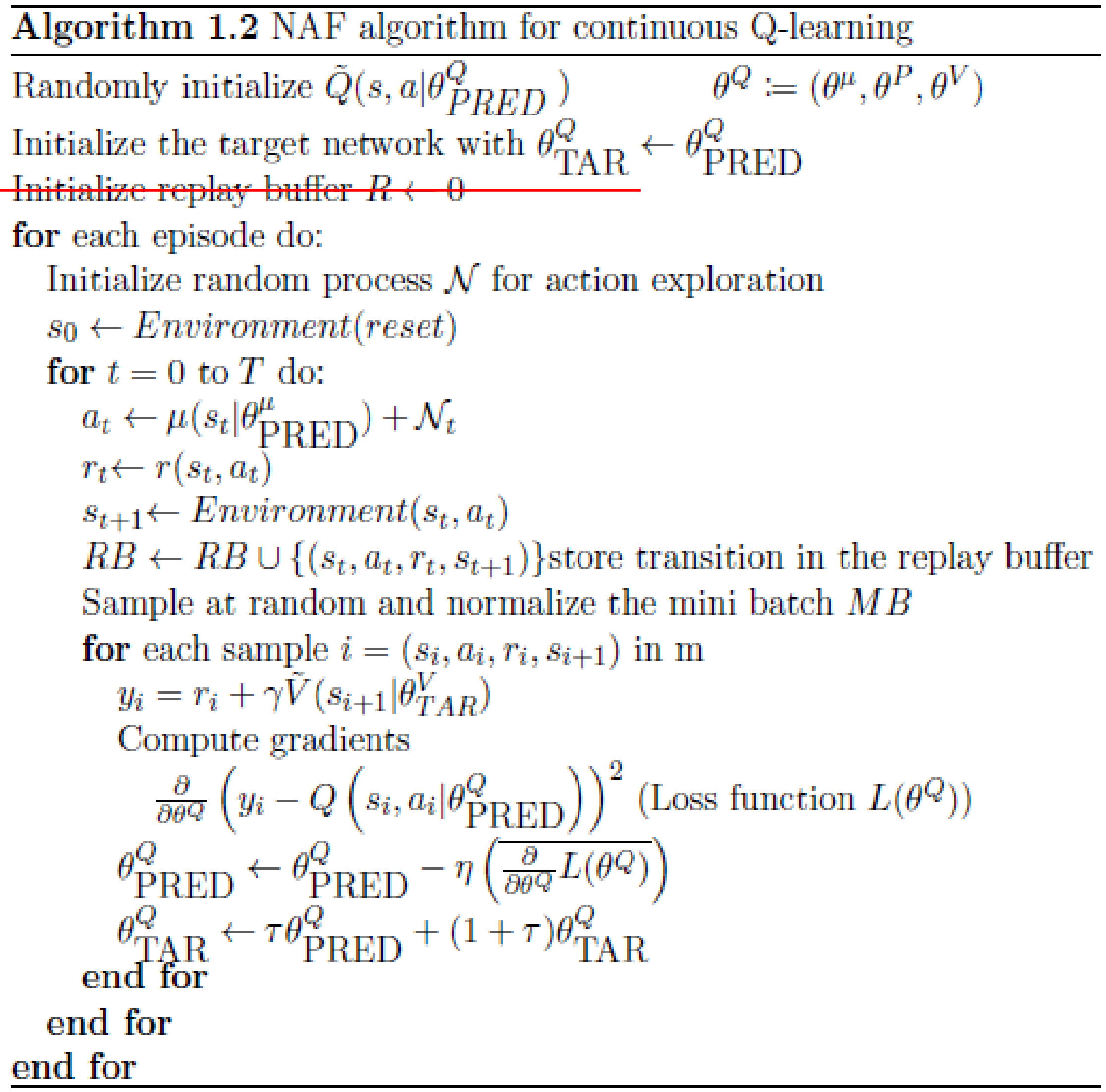
            $\theta^Q_{\text{PRED}} \leftarrow \theta^Q_{\text{PRED}} - \eta\left(\overline{\frac{\partial}{\partial\theta^Q}L(\theta^Q)}\right)$
            $\theta^Q_{\text{TAR}} \leftarrow \tau\theta^Q_{\text{PRED}} + (1+\tau)\theta^Q_{\text{TAR}}$
        end for
    end for
end for
```

Normalized Advantage Function (NAF) algorithm

Algorithm Highlights

- *a deep neural network for* $\hat{Q}(s, a)$
- *two deep networks:*
  one TARget, which is the objective and one PREDictor for transient approximations



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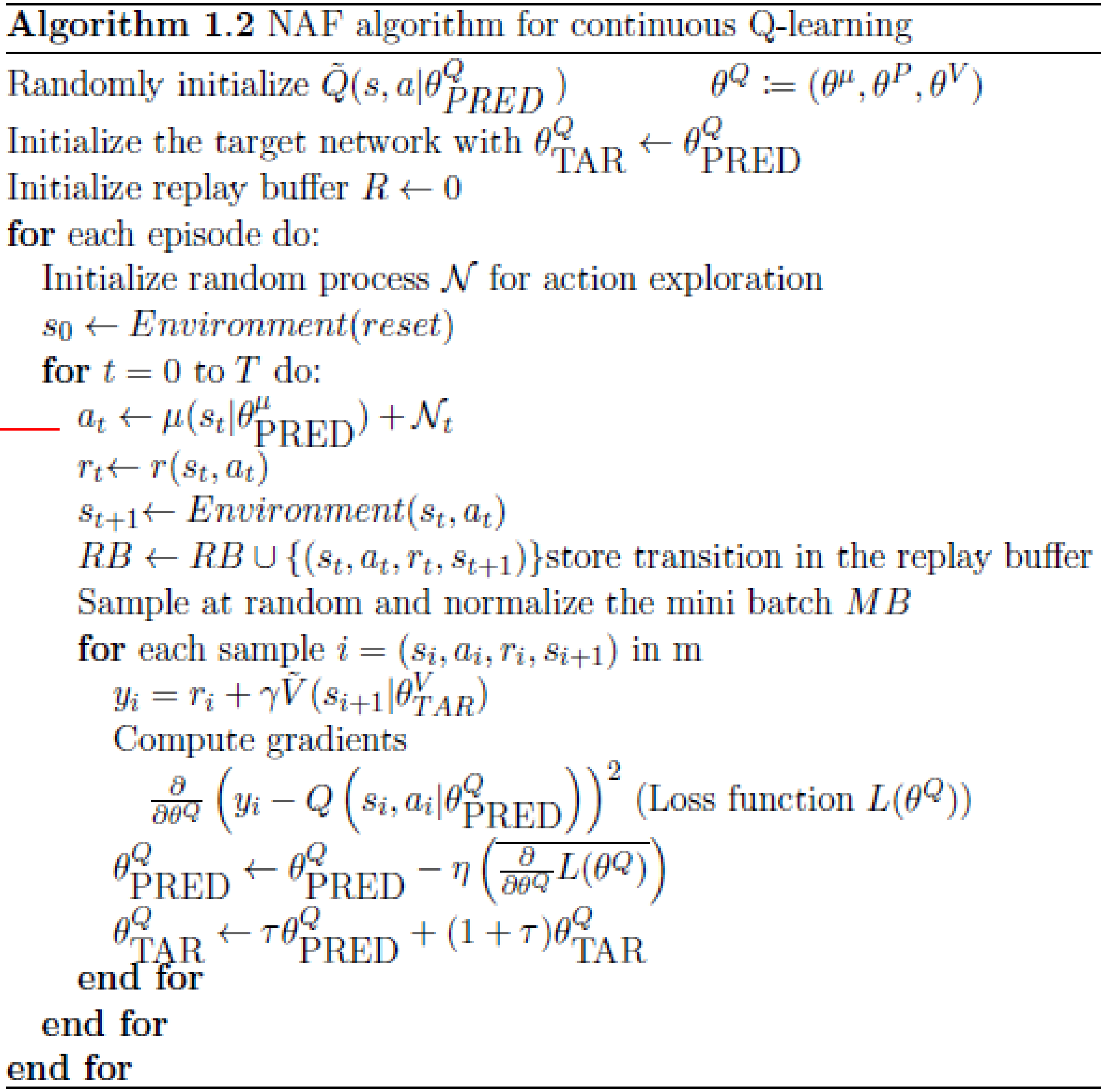
    Sample at random and normalize the mini batch $MB$
    for each sample $i = (s_i, a_i, r_i, s_{i+1})$ in m
      $y_i = r_i + \gamma\tilde{V}(s_{i+1}|\theta^V_{TAR})$
      Compute gradients
        $\frac{\partial}{\partial\theta^Q}\left(y_i - Q\left(s_i, a_i|\theta^Q_{\text{PRED}}\right)\right)^2$ (Loss function $L(\theta^Q)$)
      $\theta^Q_{\text{PRED}} \leftarrow \theta^Q_{\text{PRED}} - \eta\left(\overline{\frac{\partial}{\partial\theta^Q}L(\theta^Q)}\right)$
      $\theta^Q_{\text{TAR}} \leftarrow \tau\theta^Q_{\text{PRED}} + (1+\tau)\theta^Q_{\text{TAR}}$
    end for
  end for
end for
```

Normalized Advantage Function (NAF) algorithm

Algorithm Highlights

- *a deep neural network for* $\hat{Q}(s, a)$
- *two deep networks:*
  one TARget, which is the objective and one PREDictor for transient approximations
- *careful tensorial formulation to avoid the argmax step (see after)*



Algorithm 1.2 NAF algorithm for continuous Q-learning

```
Randomly initialize $\tilde{Q}(s, a|\theta^Q_{PRED})$        $\theta^Q := (\theta^\mu, \theta^P, \theta^V)$
Initialize the target network with $\theta^Q_{\text{TAR}} \leftarrow \theta^Q_{\text{PRED}}$
Initialize replay buffer $R \leftarrow 0$
for each episode do:
  Initialize random process $\mathcal{N}$ for action exploration
  $s_0 \leftarrow Environment(reset)$
  for $t = 0$ to $T$ do:
    $a_t \leftarrow \mu(s_t|\theta^\mu_{\text{PRED}}) + \mathcal{N}_t$
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    $RB \leftarrow RB \cup \{(s_t, a_t, r_t, s_{t+1})\}$store transition in the replay buffer
    Sample at random and normalize the mini batch $MB$
    for each sample $i = (s_i, a_i, r_i, s_{i+1})$ in m
      $y_i = r_i + \gamma\tilde{V}(s_{i+1}|\theta^V_{TAR})$
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Normalized Advantage Function (NAF) algorithm

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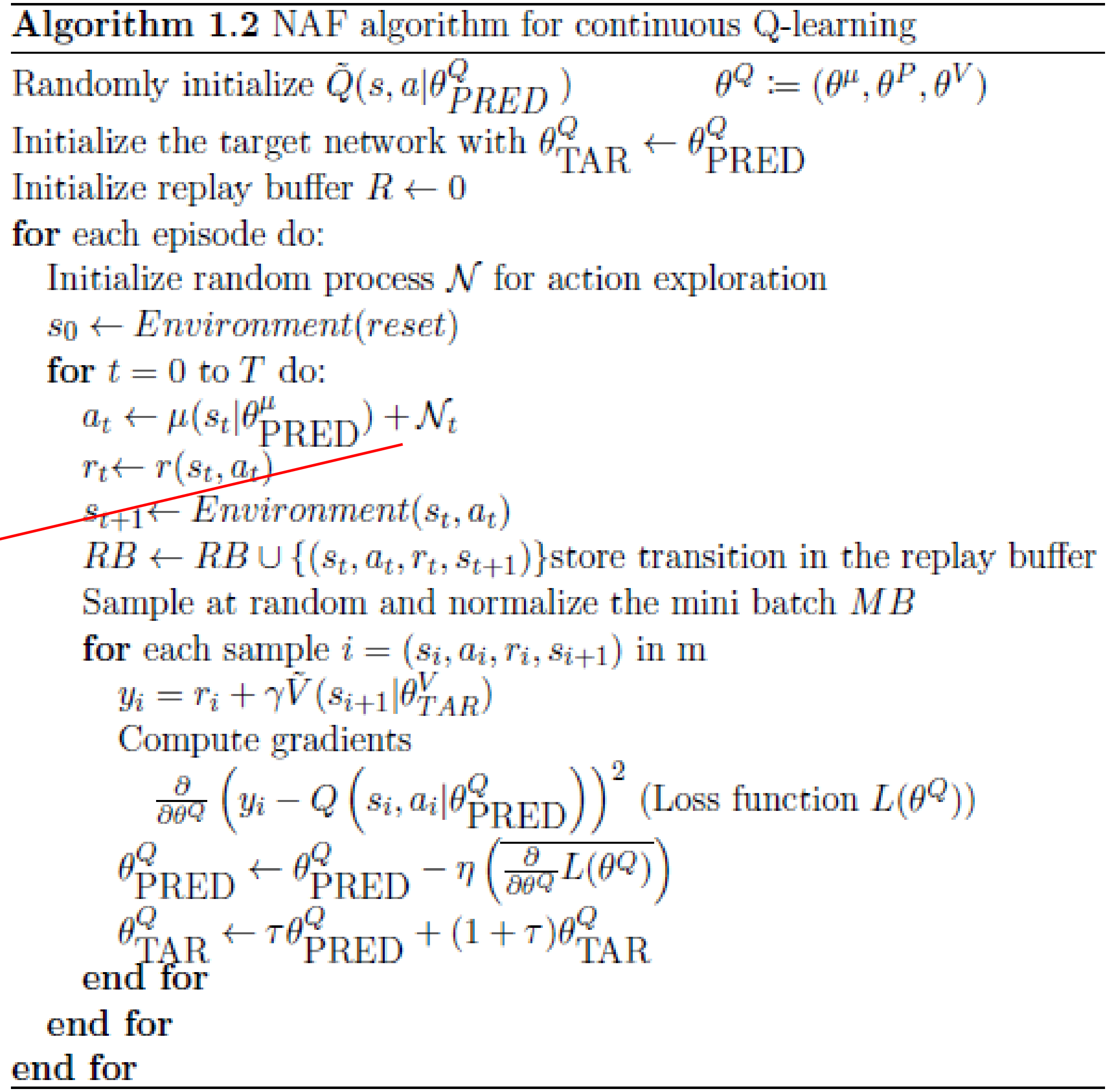
- *careful <u>tensorial</u> formulation*
  to avoid the argmax step (see after)
- *noise based on a stochastic process*
  (i.e. a random walk, see later)
  *forcing **exploration***



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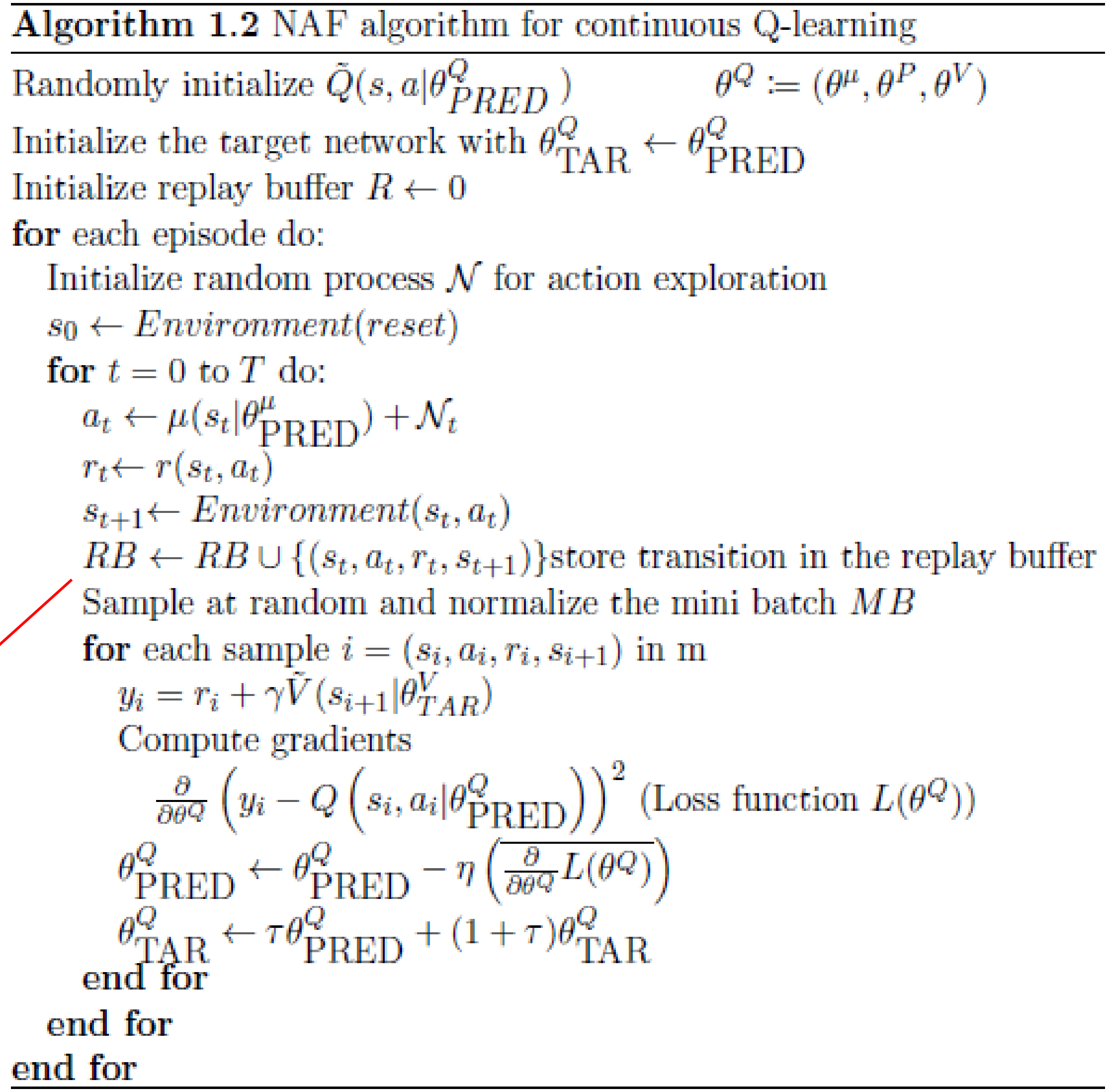
- ***replay buffer*** *with random extraction of* ***mini-batches*** *to avoid temporal correlation arising from sequential exploration*



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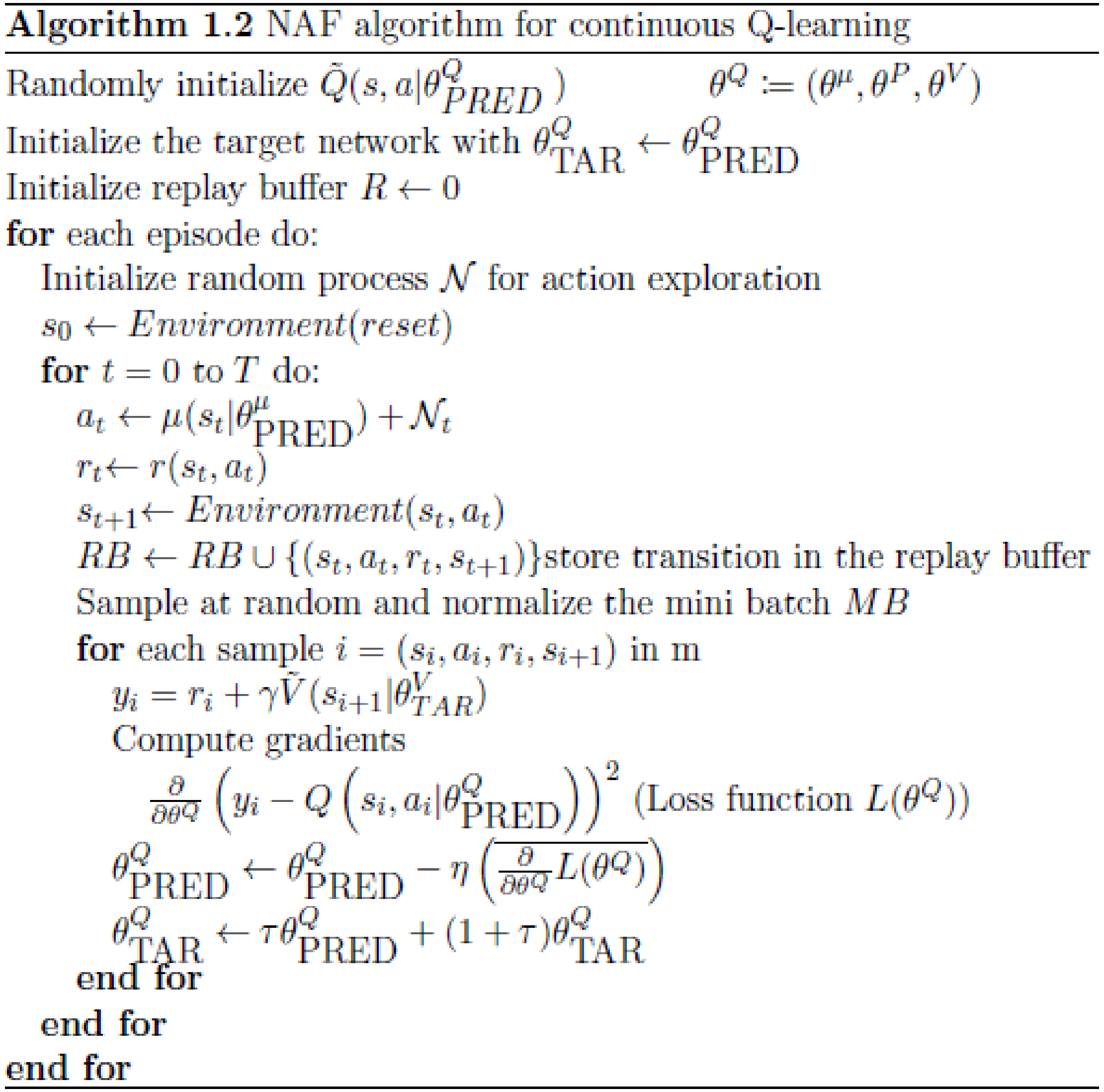
- ***replay buffer*** *with random extraction of* ***mini-batches*** *to avoid temporal correlation arising from sequential exploration*
- ***Can cope with continuous*** $\mathcal{A}$ ***and*** $\mathcal{S}$



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```
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    end for
  end for
end for
```

Normalized Advantage Function (NAF) algorithm

- ***A special approximator***

*NOTE: all functions here are **continuous** and of **vector** parameters*

From the definition of the **Advantage Function**

$$A^\pi(s,a) := Q^\pi(s,a) - V^\pi(s)$$

The NAF approximator becomes:

$$\hat{Q}(s,a) := \hat{A}(s,a;\theta) - \hat{V}(s;\phi)$$

Define:

$\mu,\ P$ *are 'Deep Neural Networks'*

$$\hat{A}(s,a;\theta) = \tfrac{1}{2}(a-\mu(s;\theta_\mu))^T P(s;\theta_P)(a-\mu(s;\theta_\mu))$$

this is a quadratic form

Then the solution to

$$\frac{\partial}{\partial a}\hat{Q}(s,a) = 0 \qquad \Longleftrightarrow \qquad \frac{\partial}{\partial a}\hat{A}(s,a;\theta) = 0$$

is

$$a^* = \mu(s;\theta_\mu)$$