

Deep Learning

A course about theory & practice



Exponential Moving Average (EMA)

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An aside: *moving averages*

Following non-stationary phenomena

■ Average

Definition:
$$\bar{v}_T := \frac{1}{T} \sum_{k=1}^T v_k$$

Running implementation:

$$\begin{aligned}\bar{v}_T &= \frac{1}{T} \left(v_T + \sum_{k=1}^{T-1} v_k \right) = \frac{1}{T} \left(v_T + (T-1)\bar{v}_{T-1} \right) \\ &= \bar{v}_{T-1} + \frac{1}{T} (v_T - \bar{v}_{T-1}) = \frac{1}{T} v_T + \left(1 - \frac{1}{T} \right) \bar{v}_{T-1}\end{aligned}$$

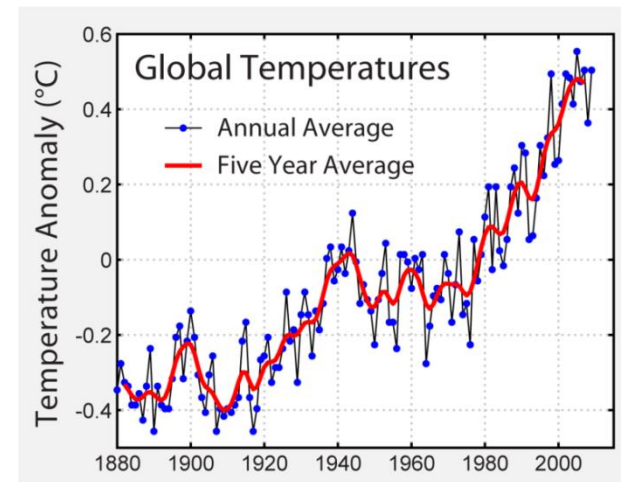
■ Simple Moving Average (SMA)

$$\bar{v}_{T,n} := \frac{1}{n} \sum_{k=T-n}^T v_k$$

■ Exponential Moving Average (EMA)

$$\bar{v}_{T,\alpha} := \alpha v_T + (1 - \alpha) \bar{v}_{T-1,\alpha}, \quad \alpha \in [0, 1]$$

“the weight of newer observations remains constant”



[image from wikipedia]

“the weight of newer observations diminishes with time”

An aside: *moving averages*

■ Exponential Moving Average (EMA)

$$\bar{v}_{T,\alpha} := \alpha v_T + (1 - \alpha) \bar{v}_{T-1,\alpha}, \quad \alpha \in [0, 1]$$

Expanding:

$$\begin{aligned} \bar{v}_{t,\alpha} &= \alpha v_t + (1 - \alpha) \bar{v}_{t-1,\alpha} \\ &= \alpha v_t + (1 - \alpha)(\alpha v_{t-1} + (1 - \alpha) \bar{v}_{t-2,\alpha}) \\ &= \alpha v_t + (1 - \alpha)(\alpha v_{t-1} + (1 - \alpha)(\alpha v_{t-2} + (1 - \alpha) \bar{v}_{t-3,\alpha})) \\ &= \alpha (v_t + (1 - \alpha) v_{t-1} + (1 - \alpha)^2 v_{t-2}) + (1 - \alpha)^3 \bar{v}_{t-3,\alpha} \end{aligned}$$

The weight of past contributions decays as

$$(1 - \alpha)^{\Delta t}$$

A SMA with n previous values
is approximately equal to an EMA with

$$\alpha = \frac{2}{n + 1}$$

