Deep Learning

A course about <u>theory</u> & <u>practice</u>

Deep Learning and Time Series

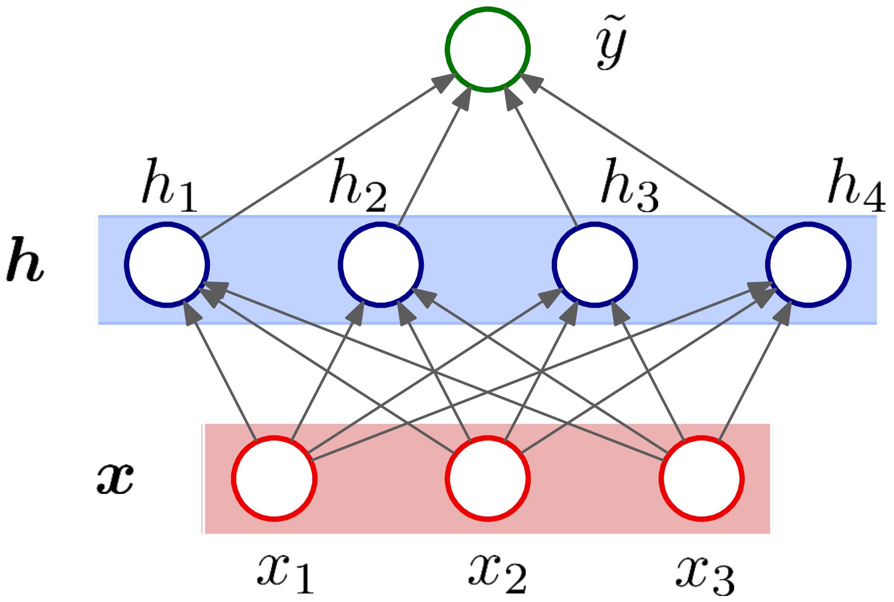
Marco Piastra

Recurrent Neural Networks

Recurrent Neural Networks

- **Feed-forward neural network**

$$\tilde{y} = \boldsymbol{w} \cdot \boldsymbol{h} + b \qquad \text{where} \qquad \boxed{\underbrace{\boldsymbol{h}}_{\text{hidden}} := g(\boldsymbol{W}\underbrace{\boldsymbol{x}}_{\text{input}} + \boldsymbol{b})}$$



Recurrent Neural Networks

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- **Recurrent Neural Network**

$$\tilde{y}^{(t)} = \boldsymbol{w} \cdot \boldsymbol{h}^{(t)} + b \qquad \text{where} \qquad \underbrace{\boldsymbol{h}^{(t)}}_{\text{hidden at } t} := g(\boldsymbol{W}\underbrace{\boldsymbol{x}^{(t)}}_{\text{input at } t} + \boldsymbol{U}\underbrace{\boldsymbol{h}^{(t-1)}}_{\text{hidden at } t\text{-}1} + \boldsymbol{b})$$

The basic idea is to make the network output depend on the past 'history'

Recurrent Neural Networks

- **Recurrent Neural Network**

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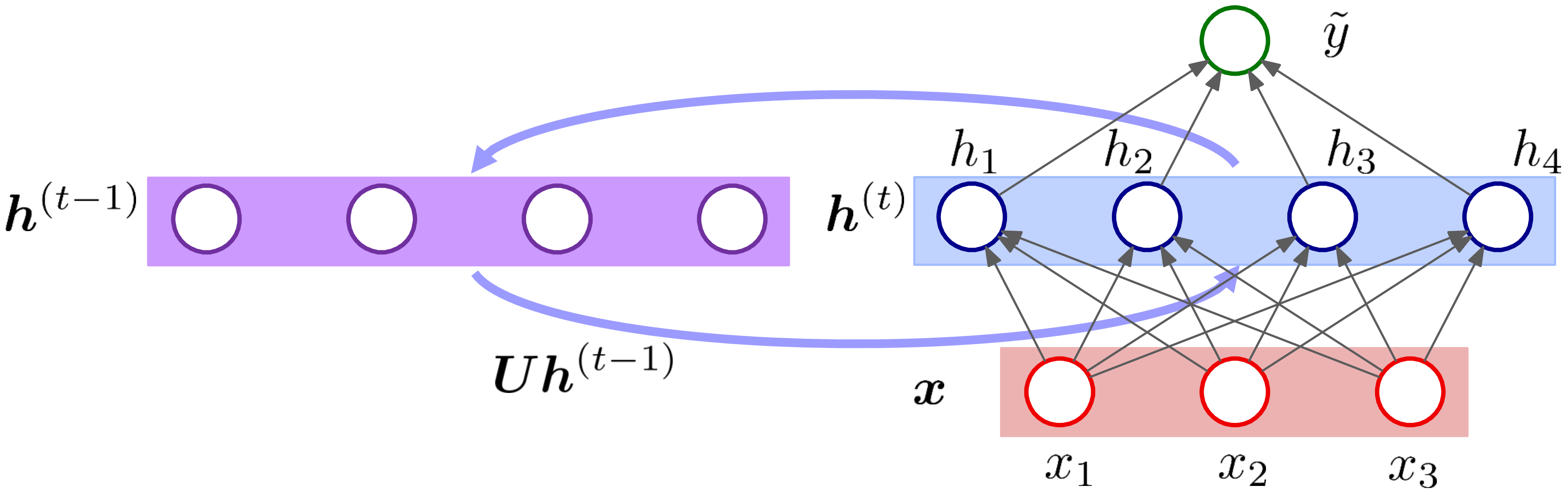
hidden at t input at t hidden at $t-1$

The basic idea is to make the network output depend on the past 'history'



Recurrent Neural Networks

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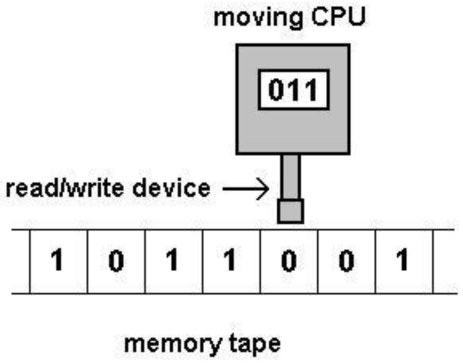
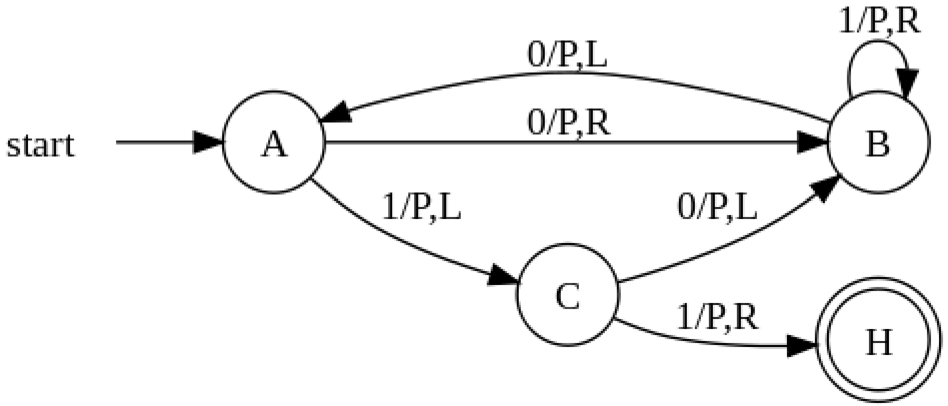
RNN are Turing Machines

- **Computational power of RNNs** (Siegelmann & Sontag, 1992)

  "RNNs can simulate any Turing machine"





This means that they can compute anything a Turing Machine could

[image from https://en.wikipedia.org/wiki/Turing_machine]

Recurrent Neural Networks

- ***Recurrent Neural Network***

$$\tilde{y}^{(t)} = \boldsymbol{w} \cdot \boldsymbol{h}^{(t)} + b \qquad \textit{where} \qquad \underbrace{\boldsymbol{h}^{(t)}}_{\text{hidden at } t} := g(\boldsymbol{W}\underbrace{\boldsymbol{x}^{(t)}}_{\text{input at } t} + \boldsymbol{U}\underbrace{\boldsymbol{h}^{(t-1)}}_{\text{hidden at } t-1} + \boldsymbol{b})$$

- ***General Properties***

  A recurrent neural network (RNN) is even more powerful than a FF neural network

  It can approximate any Turing machine (i.e. a general theoretical model of computation)

  It is much harder to train than a FF neural network

  Although, with *temporal unfolding* (see above), *gradient descent* methods can be applied

Recurrent Neural Networks

- **Recurrent Neural Network**

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where

$$\underbrace{\boldsymbol{h}^{(t)}}_{\text{hidden at } t} := g(\boldsymbol{W}\underbrace{\boldsymbol{x}^{(t)}}_{\text{input at } t} + \boldsymbol{U}\underbrace{\boldsymbol{h}^{(t-1)}}_{\text{hidden at } t-1} + \boldsymbol{b})$$

- **Temporal Unfolding**

output at 3

$$\tilde{y}^{(3)} = \boldsymbol{w} \cdot \boldsymbol{h}^{(3)} + b$$

$$\boldsymbol{h}^{(3)} := g(\boldsymbol{W}\boldsymbol{x}^{(3)} + \boldsymbol{U}\boldsymbol{h}^{(2)} + \boldsymbol{b})$$

$$\boldsymbol{h}^{(2)} := g(\boldsymbol{W}\boldsymbol{x}^{(2)} + \boldsymbol{U}\boldsymbol{h}^{(1)} + \boldsymbol{b})$$

$$\boldsymbol{h}^{(1)} := g(\boldsymbol{W}\boldsymbol{x}^{(1)} + \boldsymbol{U}\boldsymbol{h}^{(0)} + \boldsymbol{b})$$

$$\boldsymbol{h}^{(0)} := g(\boldsymbol{W}\boldsymbol{x}^{(0)} + \boldsymbol{U}\boldsymbol{h}^{(-1)} + \boldsymbol{b})$$

default hidden state $\boldsymbol{h}^{(-1)} := \boldsymbol{0}$

input sequence $\boldsymbol{x}^{(0)}, \boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \boldsymbol{x}^{(3)}$

Recurrent Neural Networks

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- **Temporal Unfolding**

This looks very similar to a deep feed-forward neural network …

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Recurrent Neural Networks

- ***Recurrent Neural Network***

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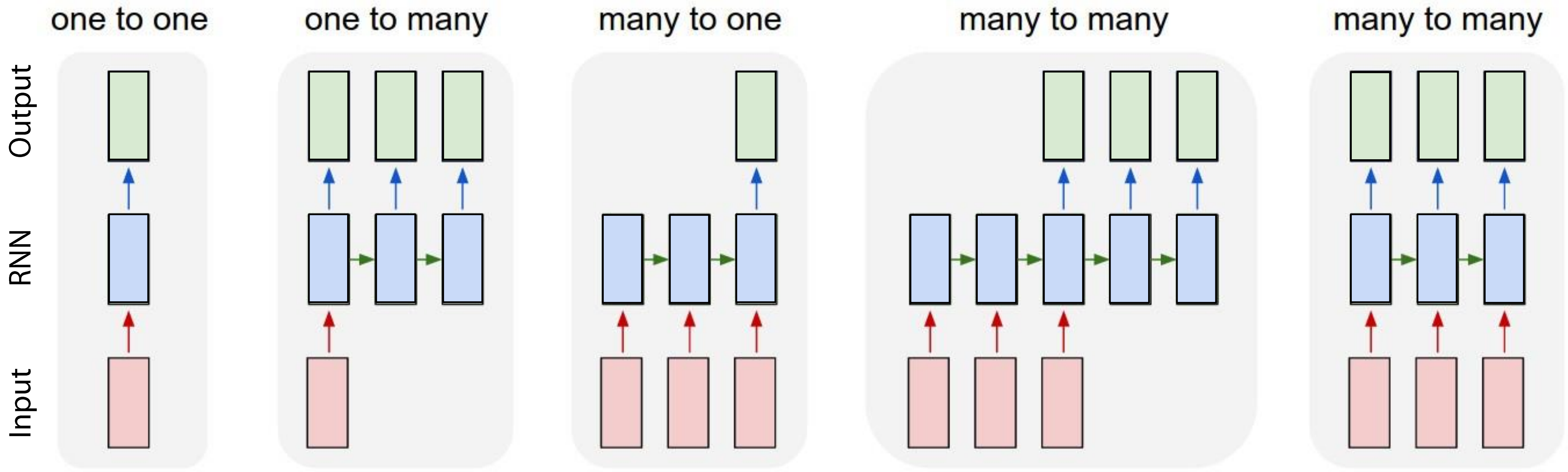
- ***Input-Output Modes***



Recurrent Neural Networks

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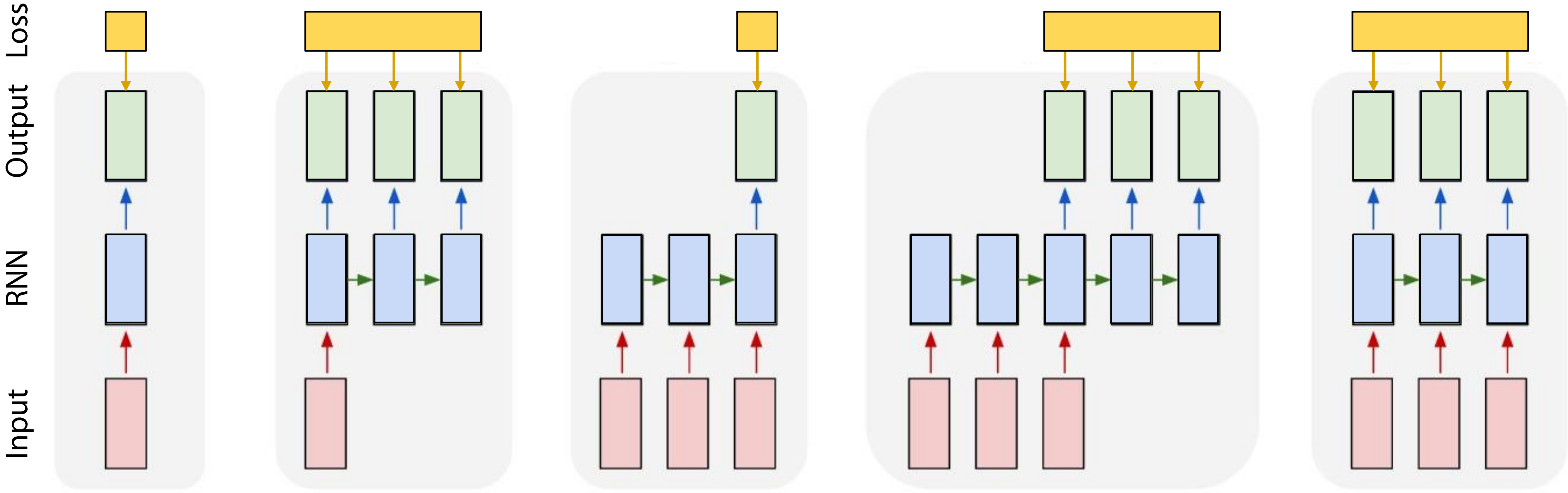
- ***Input-Output Modes***



Long-Short Term Memory (LSTM)

LSTM

- ***Long-Short Term Memory*** (Hochreiter & Schmidhuber, 1995)

$$\tilde{y}^{(t)} = \boldsymbol{w} \cdot \boldsymbol{h}^{(t)} + b$$

elementwise product

$$\boldsymbol{h}^{(t)} := \boldsymbol{o}^{(t)} \odot \tanh(\boldsymbol{c}^{(t)})$$

$$\boldsymbol{c}^{(t)} := \boldsymbol{f}^{(t)} \odot \boldsymbol{c}^{(t-1)} + \boldsymbol{i}^{(t)} \odot \boldsymbol{c}_{in}^{(t)}$$

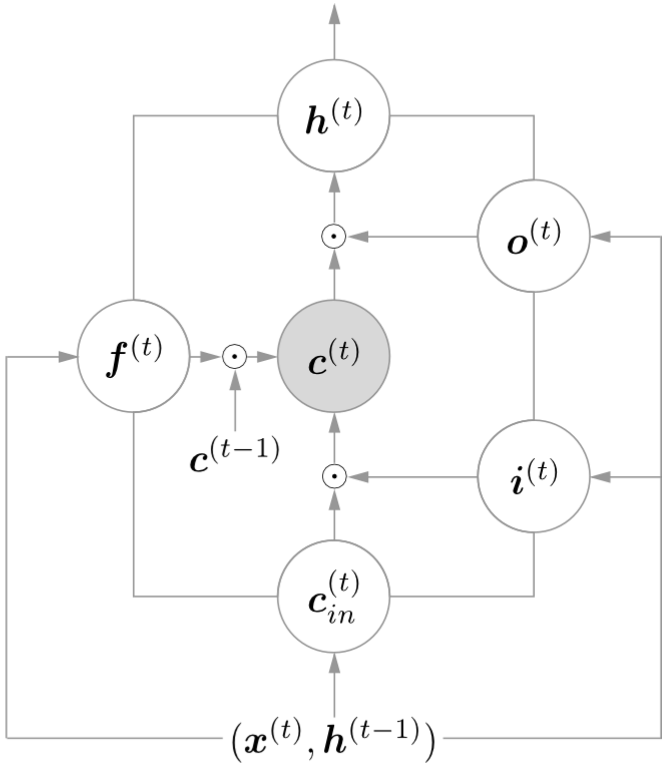
sigmoid function

$$\boldsymbol{o}^{(t)} := \sigma(\boldsymbol{W}_o \boldsymbol{x}^{(t)} + \boldsymbol{U}_o \boldsymbol{h}^{(t-1)} + \boldsymbol{b}_o)$$

$$\boldsymbol{f}^{(t)} := \sigma(\boldsymbol{W}_f \boldsymbol{x}^{(t)} + \boldsymbol{U}_f \boldsymbol{h}^{(t-1)} + \boldsymbol{b}_f)$$

$$\boldsymbol{i}^{(t)} := \sigma(\boldsymbol{W}_i \boldsymbol{x}^{(t)} + \boldsymbol{U}_i \boldsymbol{h}^{(t-1)} + \boldsymbol{b}_i)$$

$$\boldsymbol{c}_{in}^{(t)} := \tanh(\boldsymbol{W}_c \boldsymbol{x}^{(t)} + \boldsymbol{U}_c \boldsymbol{h}^{(t-1)} + \boldsymbol{b}_c)$$



LSTM

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$$\tilde{y}^{(t)} = \boldsymbol{w} \cdot \boldsymbol{h}^{(t)} + b$$

$$\boldsymbol{h}^{(t)} := \boldsymbol{o}^{(t)} \odot \tanh(\boldsymbol{c}^{(t)})$$

$$\boldsymbol{c}^{(t)} := \boldsymbol{f}^{(t)} \odot \boldsymbol{c}^{(t-1)} + \boldsymbol{i}^{(t)} \odot \boldsymbol{c}_{in}^{(t)}$$

$$\boldsymbol{o}^{(t)} := \sigma(\boldsymbol{W}_o \boldsymbol{x}^{(t)} + \boldsymbol{U}_o \boldsymbol{h}^{(t-1)} + \boldsymbol{b}_o)$$

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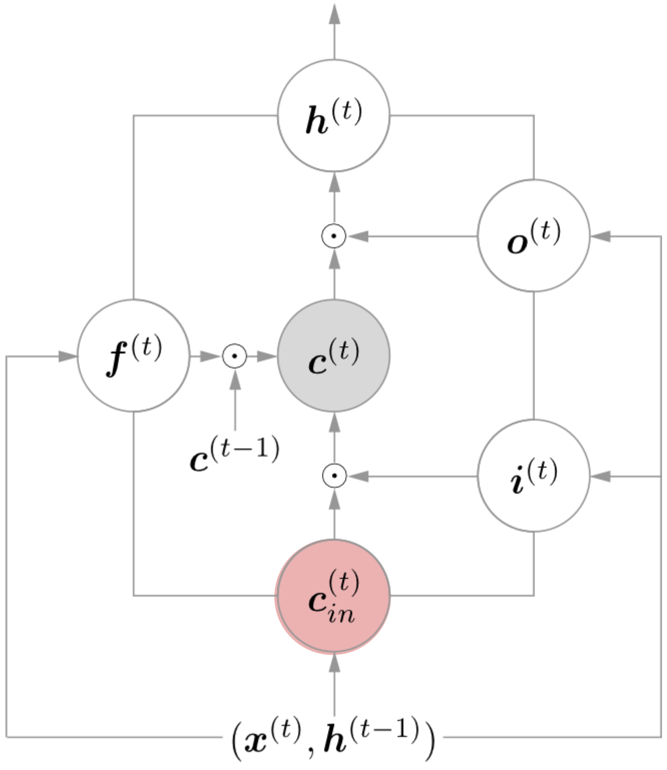
$$\boldsymbol{i}^{(t)} := \sigma(\boldsymbol{W}_i \boldsymbol{x}^{(t)} + \boldsymbol{U}_i \boldsymbol{h}^{(t-1)} + \boldsymbol{b}_i)$$

Combined input

$$\boldsymbol{c}_{in}^{(t)} := \tanh(\boldsymbol{W}_c \boldsymbol{x}^{(t)} + \boldsymbol{U}_c \boldsymbol{h}^{(t-1)} + \boldsymbol{b}_c)$$



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$$\tilde{y}^{(t)} = \boldsymbol{w} \cdot \boldsymbol{h}^{(t)} + b$$

$$\boldsymbol{h}^{(t)} := \boldsymbol{o}^{(t)} \odot \tanh(\boldsymbol{c}^{(t)})$$

$$\boldsymbol{c}^{(t)} := \boldsymbol{f}^{(t)} \odot \boldsymbol{c}^{(t-1)} + \boldsymbol{i}^{(t)} \odot \boldsymbol{c}_{in}^{(t)}$$

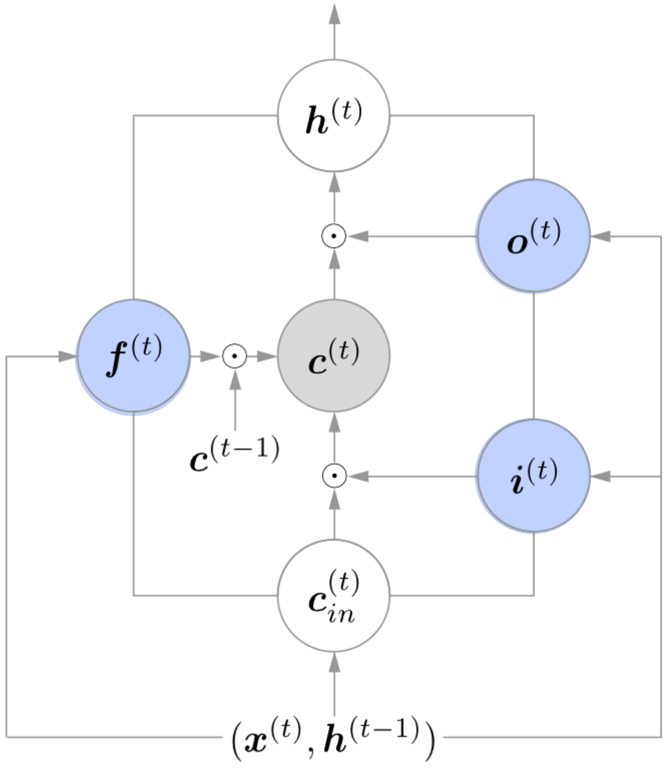
Gating values

$$\boldsymbol{o}^{(t)} := \sigma(\boldsymbol{W}_o \boldsymbol{x}^{(t)} + \boldsymbol{U}_o \boldsymbol{h}^{(t-1)} + \boldsymbol{b}_o) \quad \textit{output}$$

$$\boldsymbol{f}^{(t)} := \sigma(\boldsymbol{W}_f \boldsymbol{x}^{(t)} + \boldsymbol{U}_f \boldsymbol{h}^{(t-1)} + \boldsymbol{b}_f) \quad \textit{forget}$$

$$\boldsymbol{i}^{(t)} := \sigma(\boldsymbol{W}_i \boldsymbol{x}^{(t)} + \boldsymbol{U}_i \boldsymbol{h}^{(t-1)} + \boldsymbol{b}_i) \quad \textit{input}$$

$$\boldsymbol{c}_{in}^{(t)} := \tanh(\boldsymbol{W}_c \boldsymbol{x}^{(t)} + \boldsymbol{U}_c \boldsymbol{h}^{(t-1)} + \boldsymbol{b}_c)$$



LSTM

- ***Long-Short Term Memory*** (Hochreiter & Schmidhuber, 1995)

$$\tilde{y}^{(t)} = \boldsymbol{w} \cdot \boldsymbol{h}^{(t)} + b$$

Applying gates

$$\boldsymbol{h}^{(t)} := \boldsymbol{o}^{(t)} \odot \tanh(\boldsymbol{c}^{(t)})$$

hidden

$$\boldsymbol{c}^{(t)} := \boldsymbol{f}^{(t)} \odot \boldsymbol{c}^{(t-1)} + \boldsymbol{i}^{(t)} \odot \boldsymbol{c}_{in}^{(t)}$$

memory

$$\boldsymbol{o}^{(t)} := \sigma(\boldsymbol{W}_o \boldsymbol{x}^{(t)} + \boldsymbol{U}_o \boldsymbol{h}^{(t-1)} + \boldsymbol{b}_o)$$

$$\boldsymbol{f}^{(t)} := \sigma(\boldsymbol{W}_f \boldsymbol{x}^{(t)} + \boldsymbol{U}_f \boldsymbol{h}^{(t-1)} + \boldsymbol{b}_f)$$

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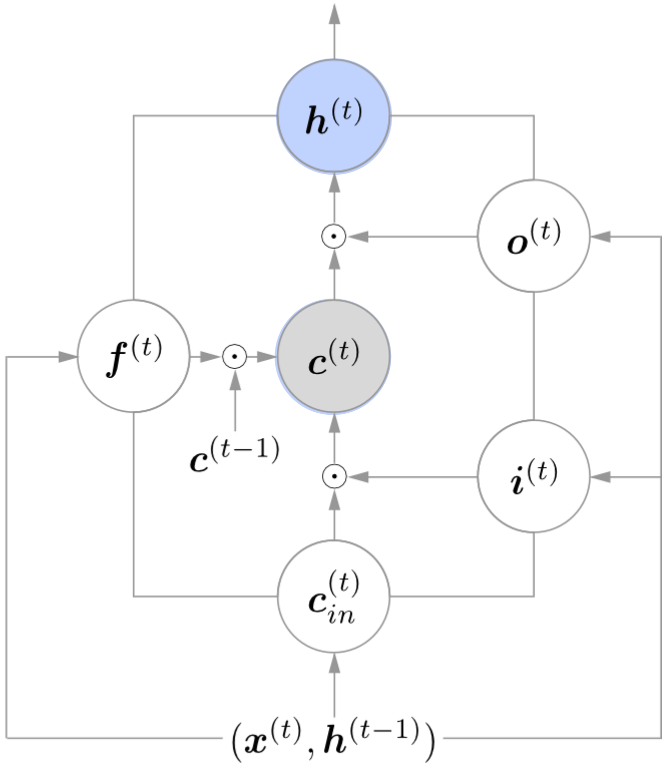

LSTM

- ***Long-Short Term Memory*** (Hochreiter & Schmidhuber, 1995)

$$\boxed{\tilde{y}^{(t)} = \boldsymbol{w} \cdot \boldsymbol{h}^{(t)} + b}$$

Cell output

$$\boldsymbol{h}^{(t)} := \boldsymbol{o}^{(t)} \odot \tanh(\boldsymbol{c}^{(t)})$$

$$\boldsymbol{c}^{(t)} := \boldsymbol{f}^{(t)} \odot \boldsymbol{c}^{(t-1)} + \boldsymbol{i}^{(t)} \odot \boldsymbol{c}_{in}^{(t)}$$

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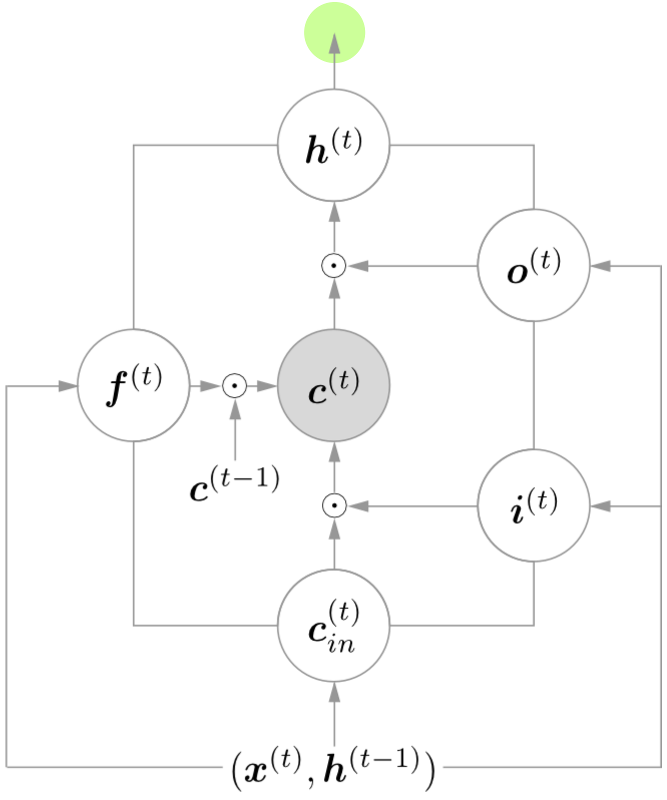
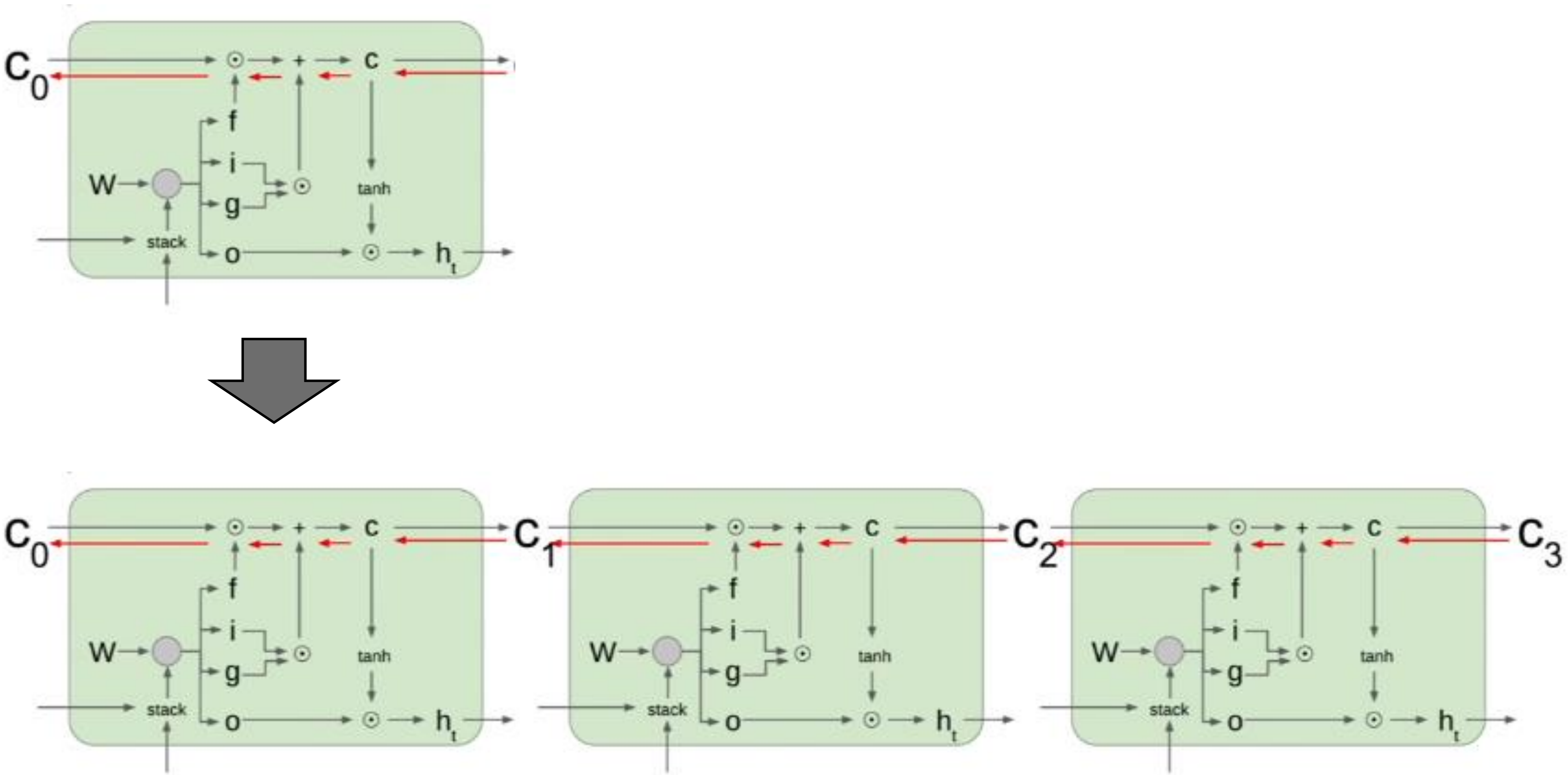


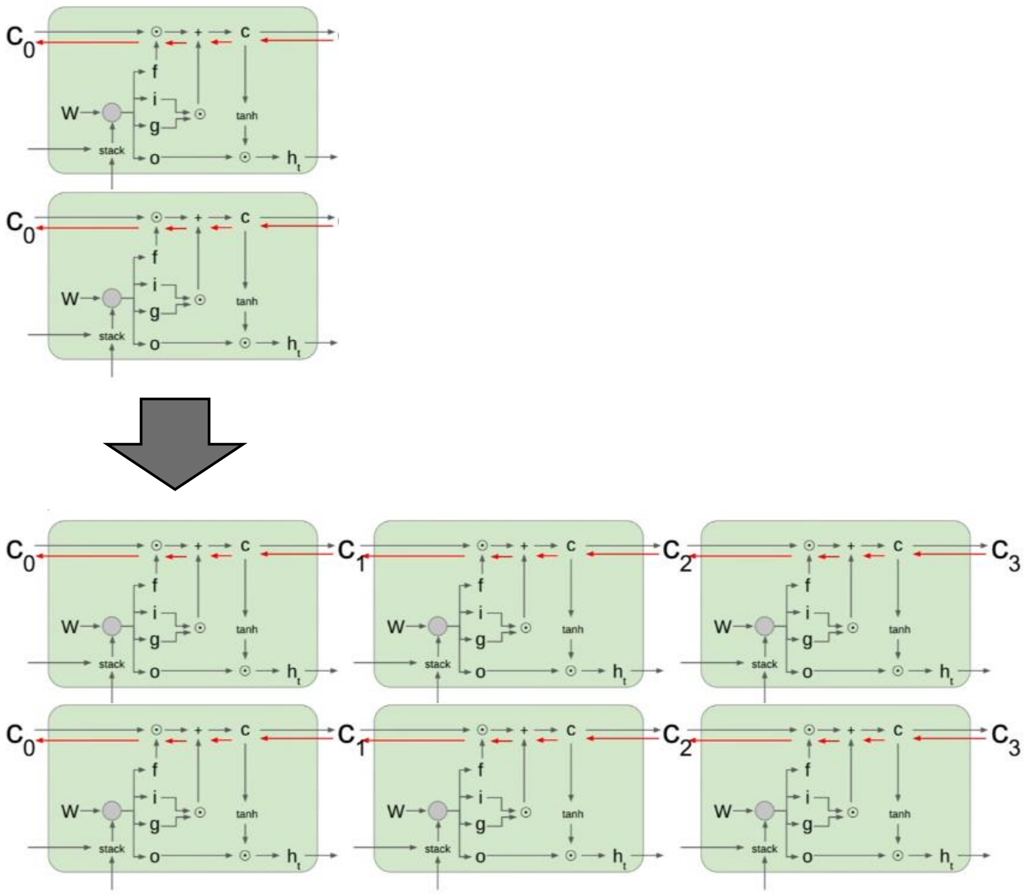
LSTM

- ***Temporal Unfolding***



LSTM

- ***Stacking and Temporal Unfolding***



GRU

- ***Gated Recurrent Unit*** (Kyunghyun Cho et al., 1995)

Simpler structure, no internal memory

$$\tilde{y}^{(t)} = \boldsymbol{w} \cdot \boldsymbol{h}^{(t)} + b$$

(exponential moving average)

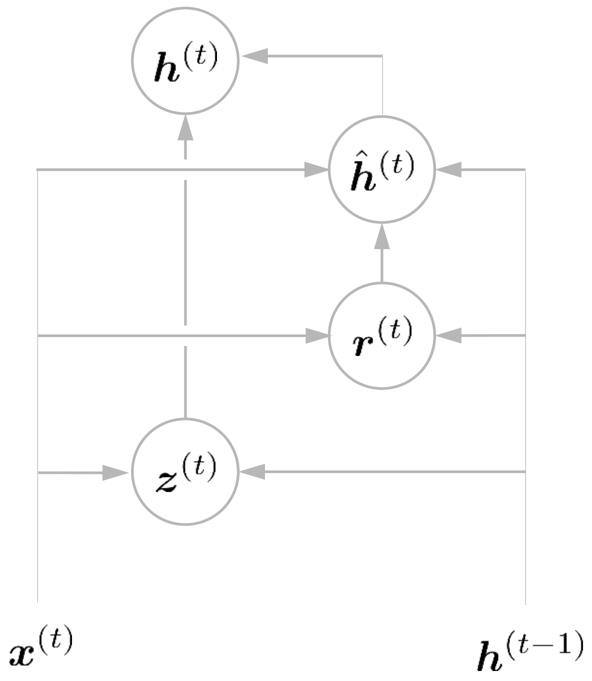
$$\boldsymbol{h}^{(t)} := (1 - \boldsymbol{z}^{(t)}) \odot \boldsymbol{h}^{(t-1)} + \boldsymbol{z}^{(t)} \odot \hat{\boldsymbol{h}}^{(t)}$$

$$\hat{\boldsymbol{h}}^{(t)} := \tanh(\boldsymbol{W}_h \boldsymbol{x}^{(t)} + \boldsymbol{U}_h (\boldsymbol{r}^{(t)} \odot \boldsymbol{h}^{(t-1)}) + \boldsymbol{b}_h)$$

sigmoid function

$$\boldsymbol{r}^{(t)} := \sigma(\boldsymbol{W}_r \boldsymbol{x}^{(t)} + \boldsymbol{U}_r \boldsymbol{h}^{(t-1)} + \boldsymbol{b}_r)$$

$$\boldsymbol{z}^{(t)} := \sigma(\boldsymbol{W}_z \boldsymbol{x}^{(t)} + \boldsymbol{U}_z \boldsymbol{h}^{(t-1)} + \boldsymbol{b}_z)$$



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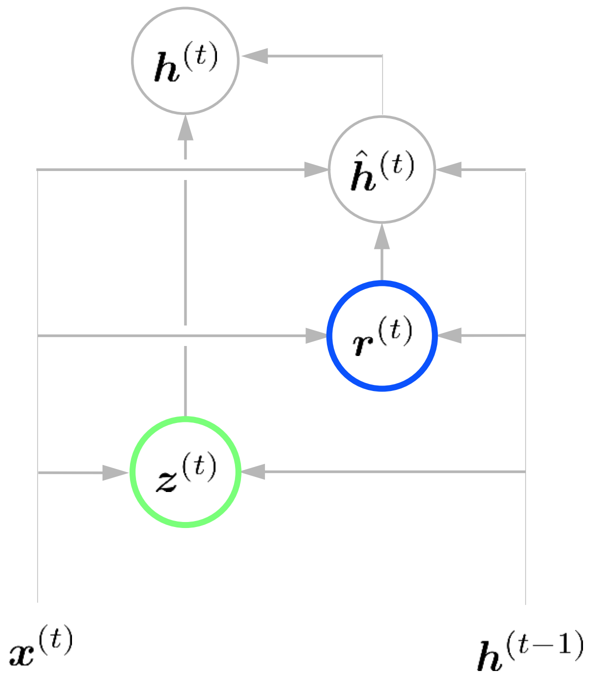
sigmoid function

Gating values

$$\boldsymbol{r}^{(t)} := \sigma(\boldsymbol{W}_r \boldsymbol{x}^{(t)} + \boldsymbol{U}_r \boldsymbol{h}^{(t-1)} + \boldsymbol{b}_r)$$ *reset*

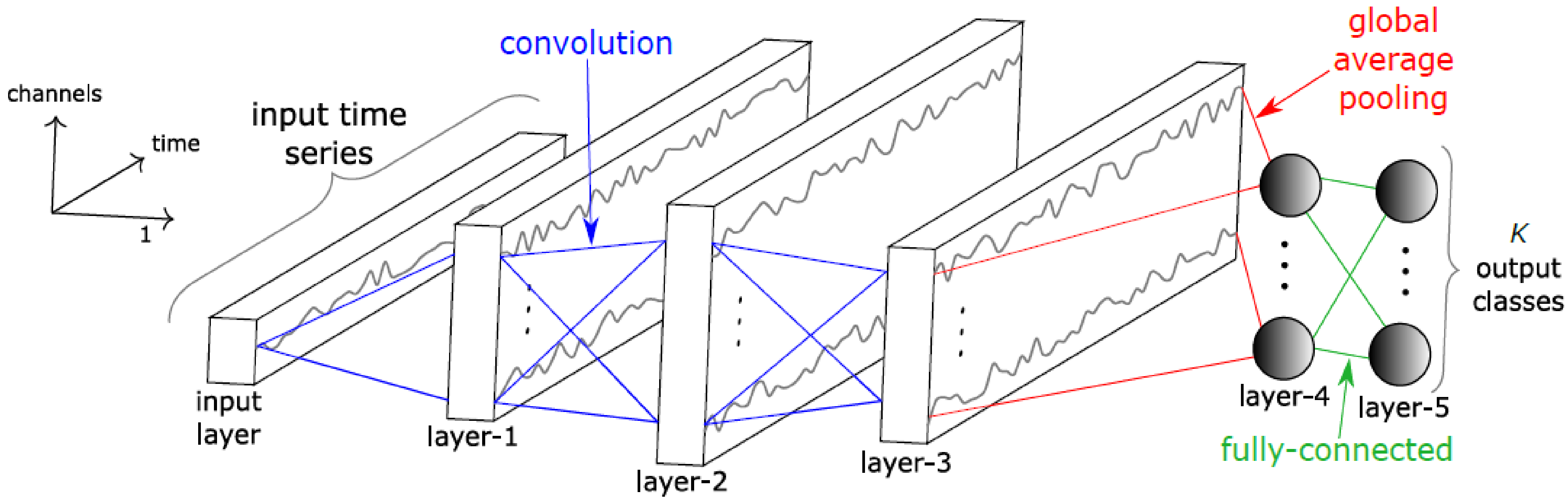
$$\boldsymbol{z}^{(t)} := \sigma(\boldsymbol{W}_z \boldsymbol{x}^{(t)} + \boldsymbol{U}_z \boldsymbol{h}^{(t-1)} + \boldsymbol{b}_z)$$ *update*



Deep Convolutional Networks for Time Series Analysis

1D Convolution Over Time

Time windows are treated as 1D 'images'

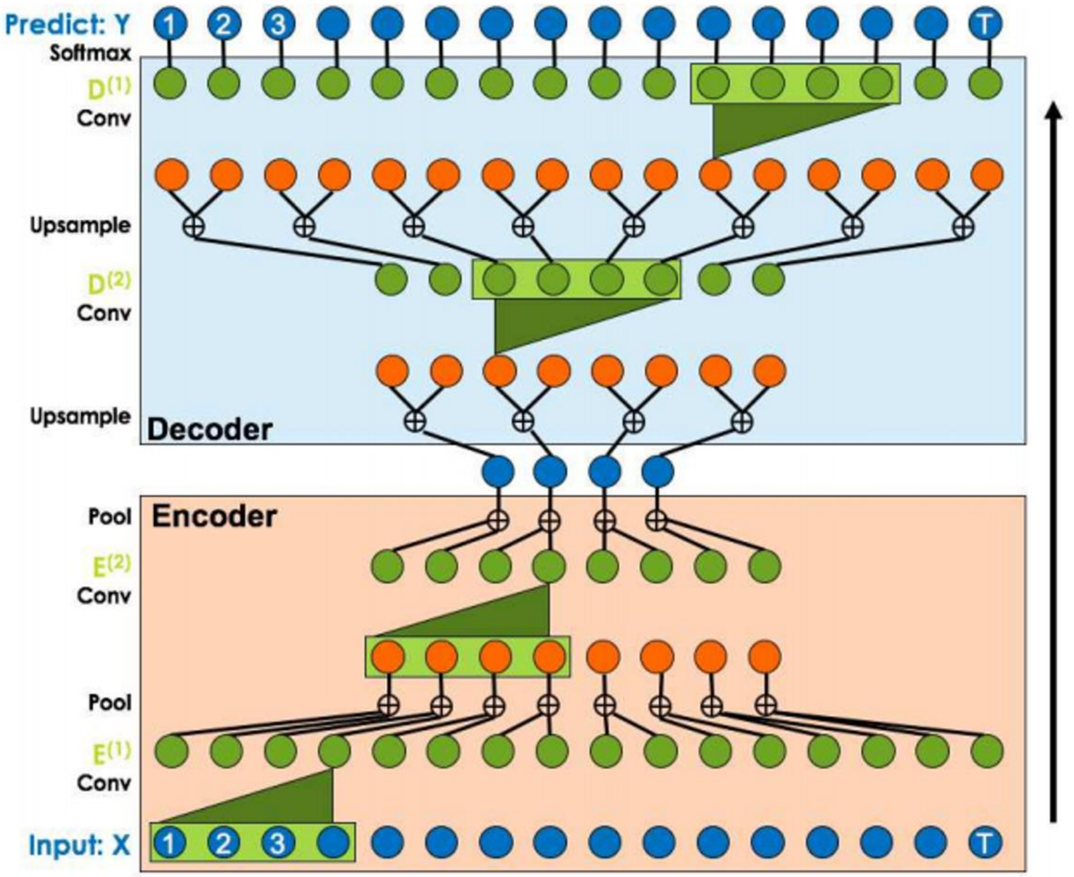


[image from https://link.springer.com/article/10.1007%2Fs10618-019-00619-1]

Temporal Convolution Networks

1D Convolution-Deconvolution in an autoencoder architecture (Lea et al., 2016)

Effective in segmenting actions and <u>predicting</u> time series



[image from https://arxiv.org/abs/1611.05267]

RNN applications

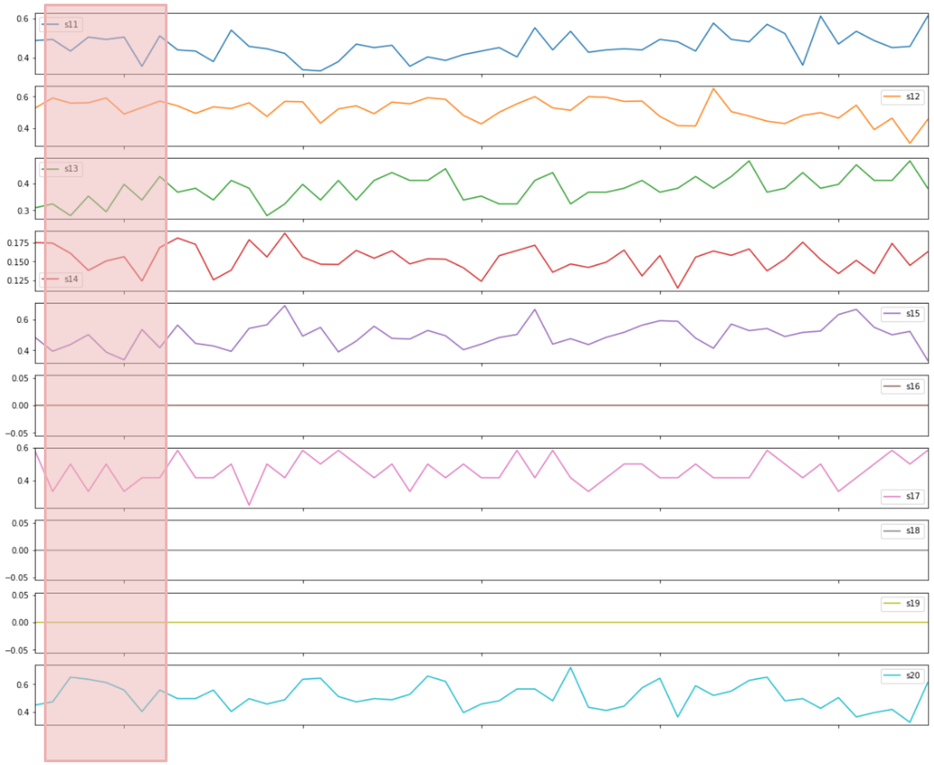
Predictive Maintenance

- ***Detecting failure conditions from sensor readings***

Nominal Condition

Input

Before Failure

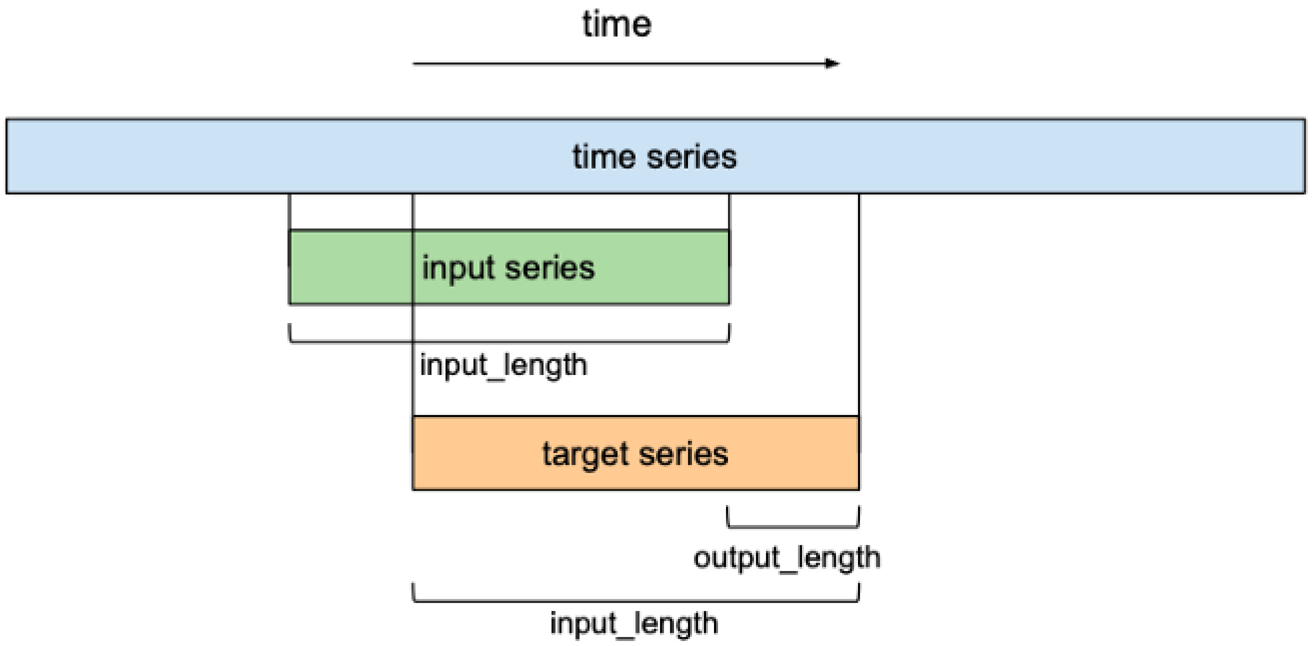


Input

Training and Prediction occurs by using a sliding window of sensor readings as input

Time Series Forecasting

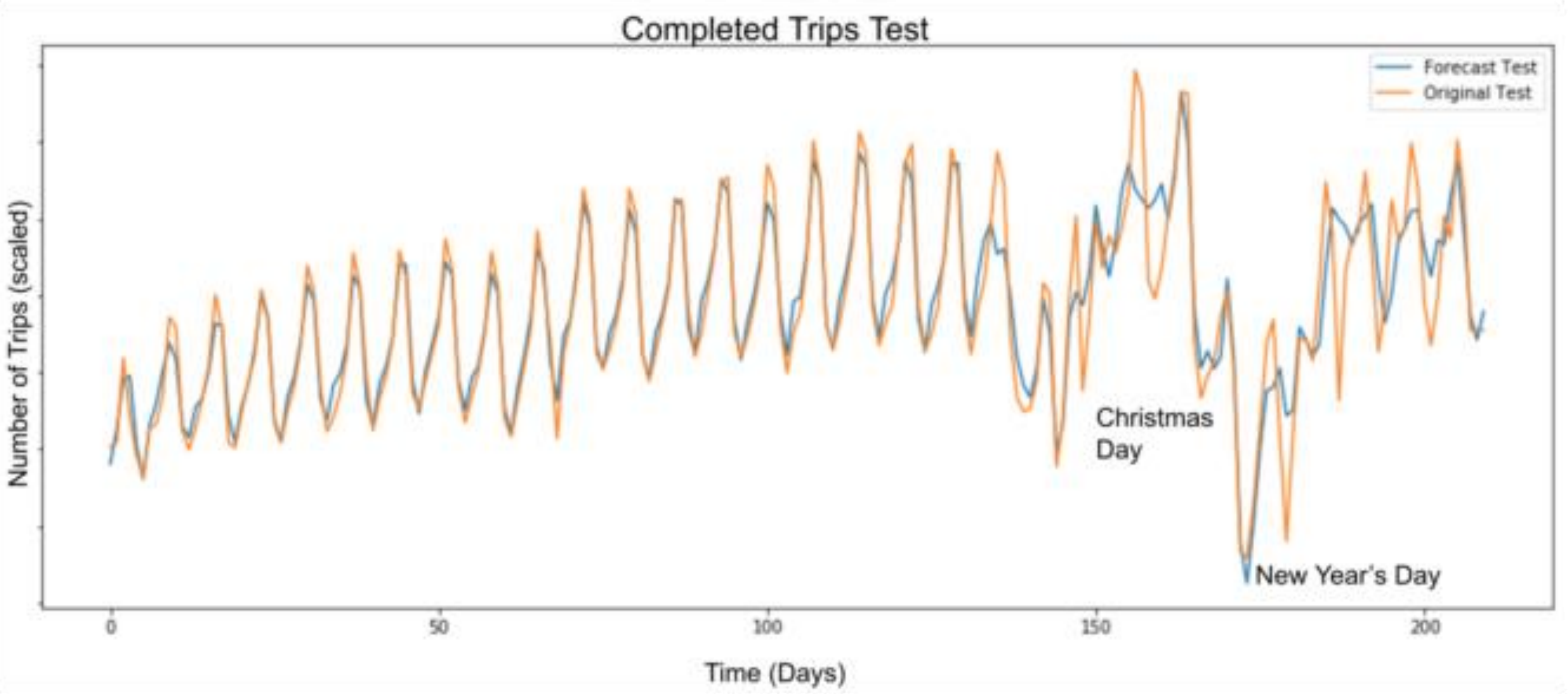
Forecasting time series ahead of time



[image from https://medium.com/unit8-machine-learning-publication/temporal-convolutional-networks-and-forecasting-5ce1b6e97ce4]

Time Series Forecasting

Detecting anomalies as differences from forecasted and actual



[image from https://eng.uber.com/neural-networks/]

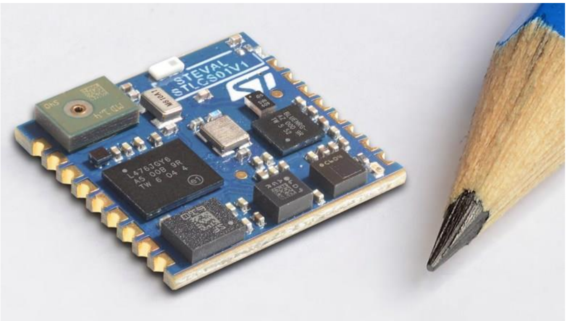
An Aside: Embedded AI
(at UniPV)

Credits are due to M. Musci and E. Torti

STM SensorTile for Wearable Devices

Microcontroller STM32 (ARM Cortex M4)

- 80MHz Clock Frequency
- 128KB SRAM
- 1MB Flash memory



Multiple Onboard Sensors

- Microphone
- 3D Accelerometer + 3D Gyroscope
- e-Compass, 3D Accelerometer, 3D Magnetic sensor
- Barometer

Peripherals

- 100 mAh Li-Ion battery
- Bluetooth Low Energy (BLE) radio module

Why Embedding?

Edge computing** is a method of optimizing **cloud computing systems
by taking some portion of an application, its data, or services
away from one or more central nodes (the "core") to the other logical extreme
(the "edge") of the Internet which makes contact with the physical world or end users
[Wikipedia]

- **A *critical* problem for (any) Intelligent *Wireless* Sensor**

  Wireless transmission of raw sensor data to the cloud requires
  a substantial amount of power

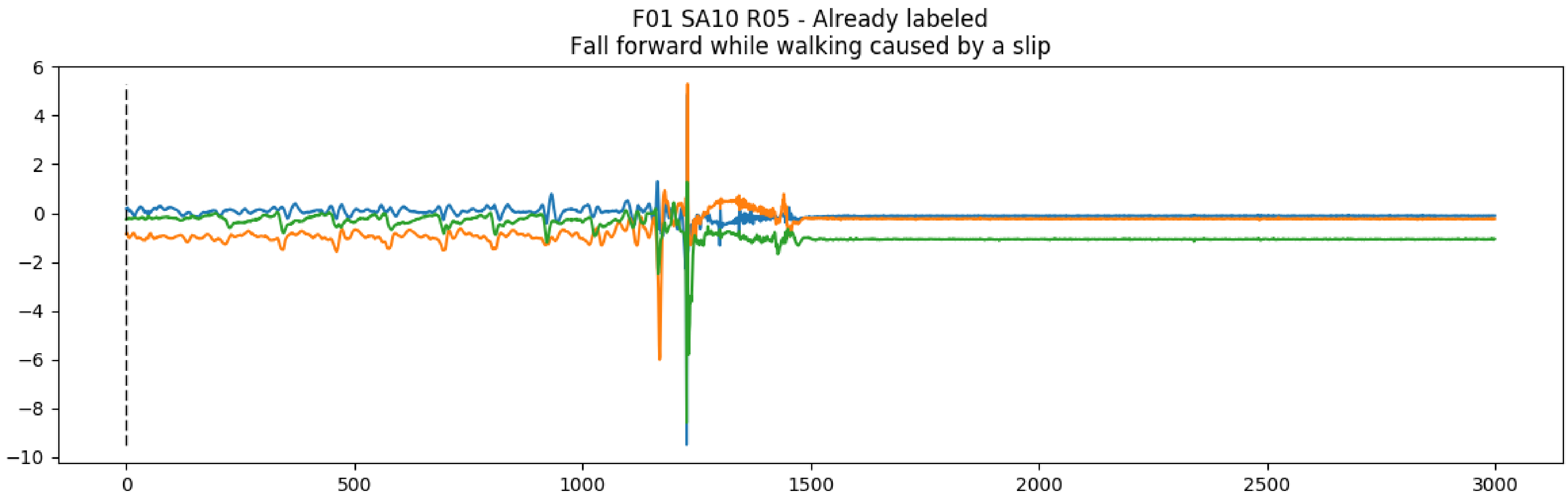
  All Internet-of-Things Low-Power Wide Area protocols (LPWA: Lora, Sigfox, NB-IoT)
  but also BLE, ZigBee, etc. are optimized for sparse and infrequent *short* messages

  *Ideally, in an Intelligent Sensor, data processing must be performed **onboard**:*
  short messages should be sent only when relevant events or state transitions occur

Human Fall (rehearsed)

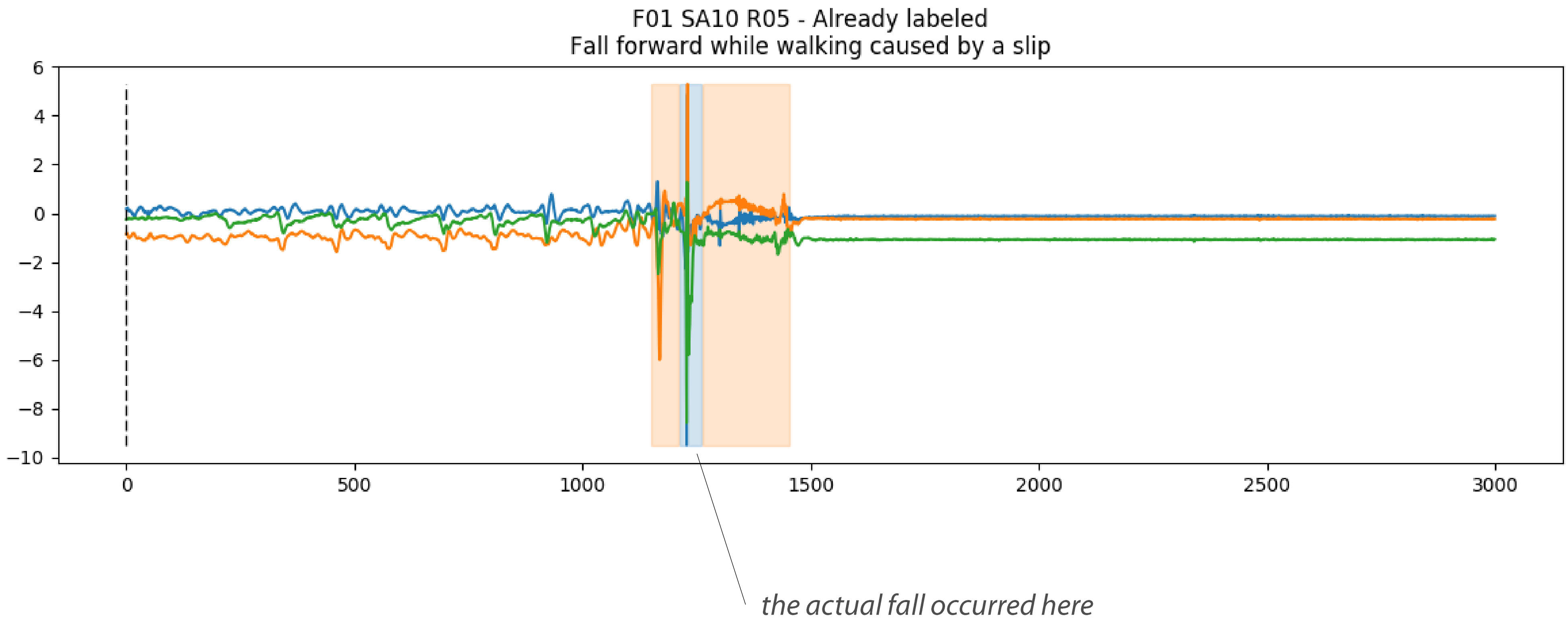
Human Fall (rehearsed)

- Accelerometers



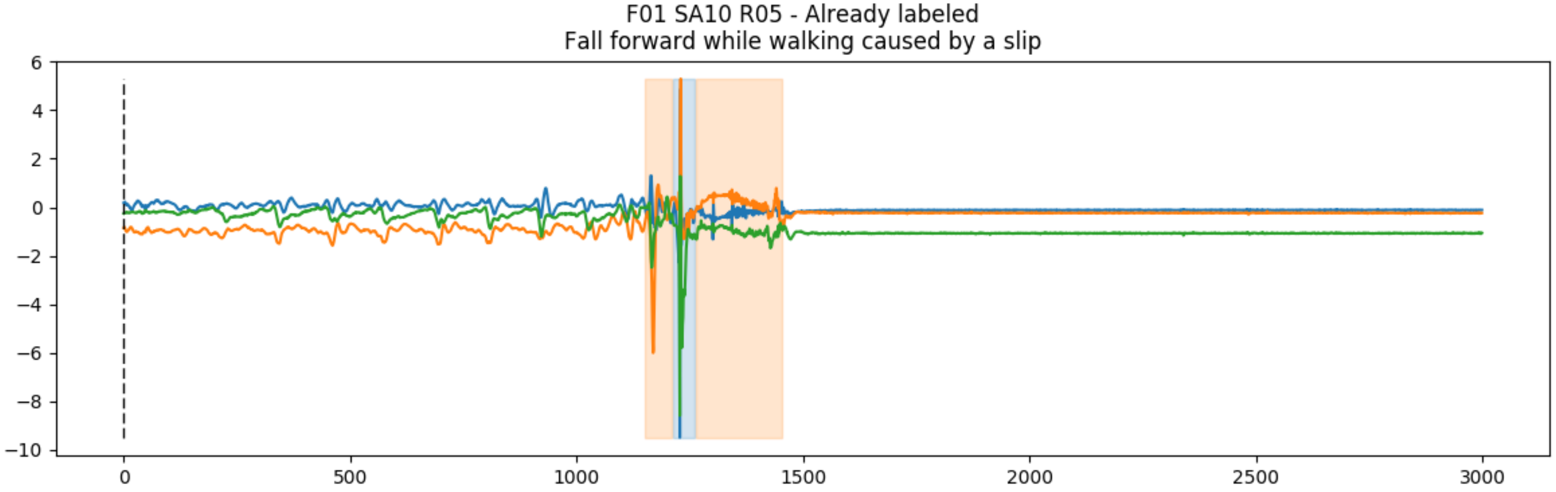
Human Fall (rehearsed)

- Accelerometers (annotated)



Fall Detection

- *Detecting human falls from accelerometer signals is difficult*
    - The input is time-variant (*unlike a still image*)
    - Information is limited (*just three scalar values at each time instant*)
    - It is the 'history' (*i.e. the shape*) of signals that describes the event
    - False negatives are to be avoided, but even false positives….



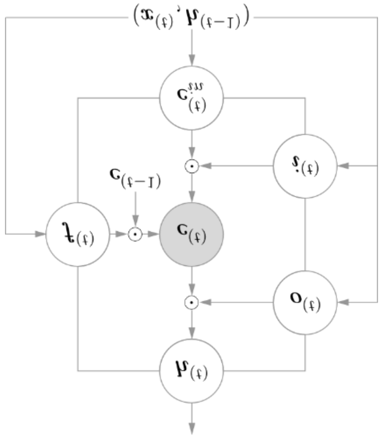
Smart Sensors: a Case Study

- ***Fall detection with wearable sensors (IPHSDM)***

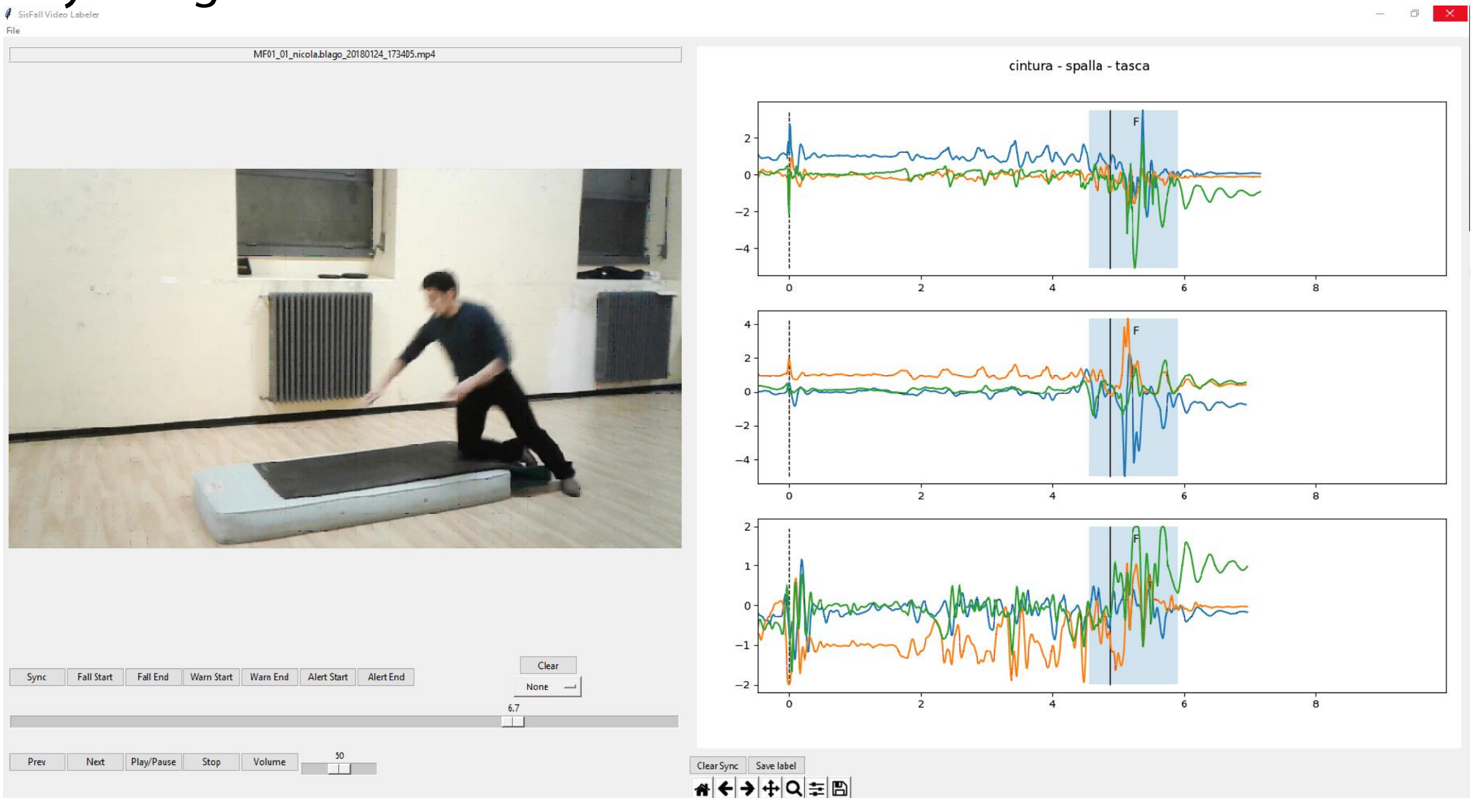
A project co-funded by Regione Lombardia

Event detection is performed by the smart sensors (SensorTile)
Messages are sent over BLE only to signal relevant events

Long-Short Term Memory (LSTM)

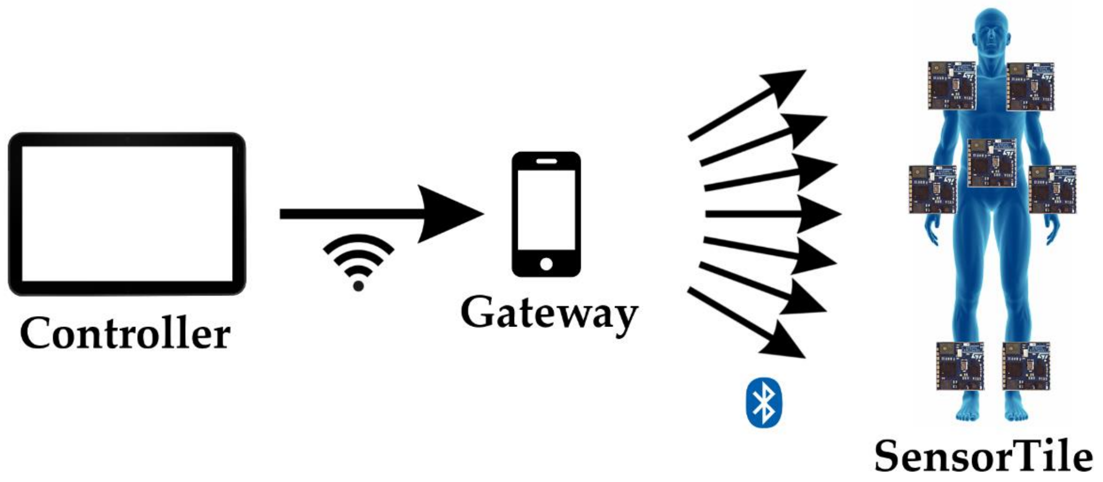


Embedded on Device



Creating a dataset
(for fall detection)

Body Network



Body Network

Simulated Falls

Dataset Annotation