COMPUTER VISION
Two-view Geometry

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Outline

- The 3D representation of points
- The pinhole camera model
- Applying a coordinate transformation
- Homogeneous representations and algebraic operations
- The fundamental matrix
- The essential matrix
- Rectification

The 3D representation of points

In the 3D space :

$$\underbrace{\mathbf{p} = (X, Y, Z)^T = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}}_{\text{initial point}} \quad \underbrace{\mathbf{p}' = (X', Y', Z')^T = \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix}}_{\text{same point in different coordinate system}}$$

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Euclidean transform $\mathbf{p}' = \mathbf{R}\mathbf{p} + \mathbf{t}$ becomes in homogeneous coordinates :

$$\begin{bmatrix} X' \\ Y' \\ Z' \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

or otherwise $\tilde{\mathbf{p}}' = \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix} \tilde{\mathbf{p}}$, avec $\mathbf{R}^T\mathbf{R} = \mathbf{I}$, $\det \mathbf{R} = 1$

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- the transform has six degrees of freedom (three elementary rotations, three elementary translations)
- we discard the ˜ for the sake of simplicity, but when it makes sense the variables are homogeneous

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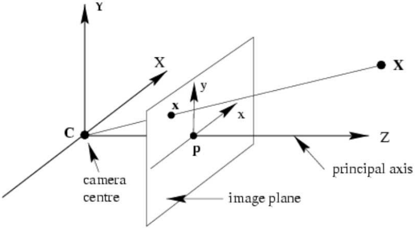

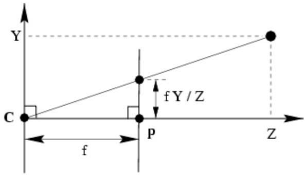

3D $\Rightarrow$ 2D projection

- In the 3D focal plance : $(X, Y, Z)^T \Rightarrow (fX/Z, fY/Z, f)^T$
- In the image 2D plane : $(X, Y, Z)^T \Rightarrow (fX/Z, fY/Z) = (x, y)$

The pinhole camera model

The image plane projection $(fX/Z, fY/Z)$ gives in homogeneous coordinates :

$$\begin{bmatrix} fX \\ fY \\ Z \end{bmatrix} = \begin{bmatrix} f & & \\ & f & \\ & & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & & & 0 \\ & 1 & & 0 \\ & & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathrm{diag}(f, f, 1)[\mathbf{I}|\mathbf{0}]\mathbf{X}$$

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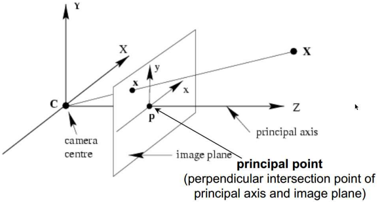
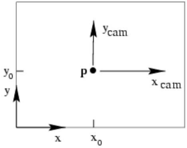
Problem : usually, the chosen reference in the image plane is not the projection of the optical axis :





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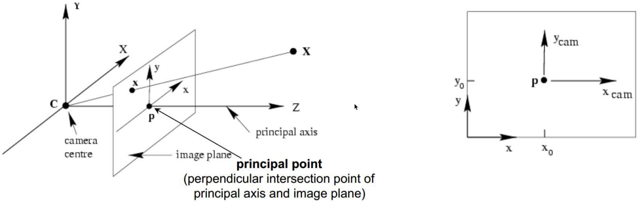

This gives in the reference system we use commonly :

$$(X, Y, Z) \Rightarrow (fX/Z + p_x, fY/Z + p_y)$$

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K - intrinsic calibration matrix

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- constant as long as the optical system is not physically adjusted
- usually determined using specific calibration objects (the most common ones being planar checkerboards)

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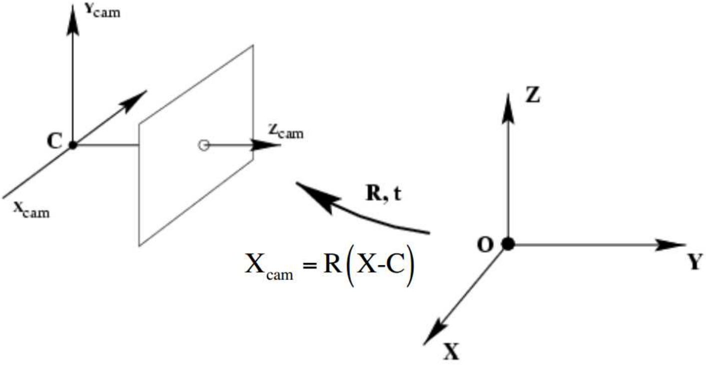
Transformation to an inertial (fixed) frame

Final step of the modelling : we express the 3D variables in a frame which is not attached to the camera and which is fixed (typical setting for mobile robotics) :



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Final step of the modelling : we express the 3D variables in a frame which is not attached to the camera and which is fixed (typical setting for mobile robotics) : By denoting as $\mathbf{C}$ the center of the camera in "world" coordinates, the transform world to camera is expressed as

$$\mathbf{X}_{cam} = \begin{bmatrix} \mathbf{R} & -\mathbf{RC} \\ \mathbf{0}^T & 1 \end{bmatrix} \mathbf{X}$$

and then instead of projecting the camera coordinates towards the image frame :

$$\mathbf{x} = \mathbf{K}\left[\mathbf{I}|\mathbf{0}\right]\mathbf{X}_{cam}$$

we rely on the projection of the world coordinates directly towards the image frame :

$$\mathbf{x} = \mathbf{K}\left[\mathbf{R}| - \mathbf{RC}\right]\mathbf{X} = \mathbf{K}\left[\mathbf{R}|\mathbf{t}\right]\mathbf{X} = \mathbf{PX}$$

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- Result : the line through two points $\mathbf{x}$ and $\mathbf{x}'$ is $\mathbf{l} = \mathbf{x} \times \mathbf{x}'$.

Some quick vector operations

$$\mathbf{x} \times \mathbf{y} = \mathbf{x}_\times \cdot \mathbf{y} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} = \begin{pmatrix} x_2 y_3 - x_3 y_2 \\ x_3 y_1 - x_1 y_3 \\ x_1 y_2 - y_1 x_2 \end{pmatrix}$$

$$\mathbf{x}_\times = \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix}$$

Mixed product : $\mathbf{x}^T(\mathbf{y} \times \mathbf{z}) = |\mathbf{x} \;\; \mathbf{y} \;\; \mathbf{z}|$ (the volume of the parallelepiped defined by the three vectors)

Singular value decomposition

Theorem (SVD) :

Let $\mathbf{A}$ be an $m \times n$ matrix. $\mathbf{A}$ may be expressed as :

$$\mathbf{A} = \mathbf{U\Sigma V}^T = \sum_{i=1}^{\min(m,n)} \sigma_i U_i V_i^T$$

where $\mathbf{\Sigma}$ is a $m \times n$ diagonal matrix with $\sigma_i = \mathbf{\Sigma}_{ii} \geq 0$, and $\mathbf{U}$ $(m \times m)$ and $\mathbf{V}$ $(n \times n)$ are composed of orthornormal columns

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- By convention, the σ_i are aligned in descending order by the decomposition algorithms.

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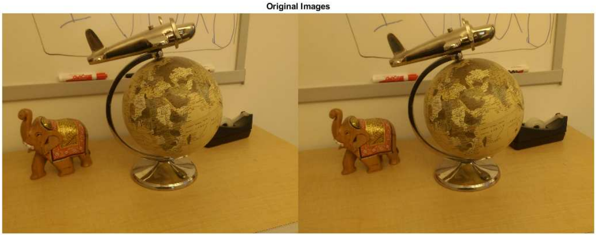
Why is this part “fundamental” ? (cheap joke)

What we can get from two views :



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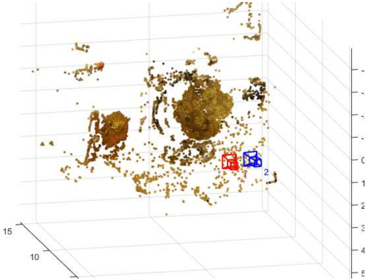
What we can get from two views :

- Sparse 3D reconstruction



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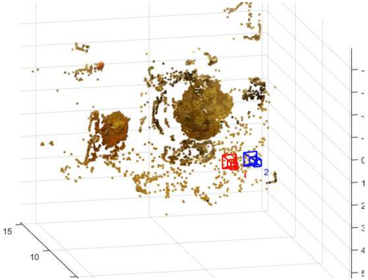
What we can get from two views :

- Sparse 3D reconstruction
- Relative camera pose estimation



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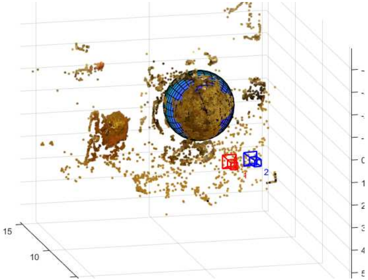

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What we can get from two views :

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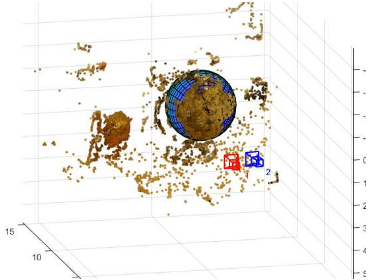
- Dense 3D reconstruction (more complex work required for this)



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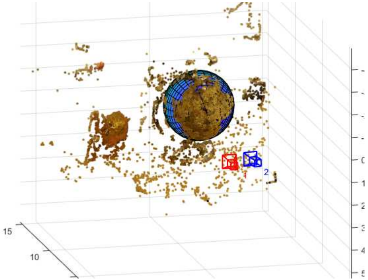
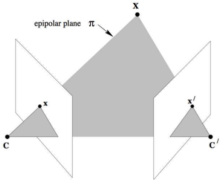
- ... but also many multi-view algorithms extend nicely from two-view analysis



The anatomy of two views

Some important observations :

- the pixel projection is along the ray defined by the 3D point and the camera center (i.e. as for **x**, **X** and **C**)



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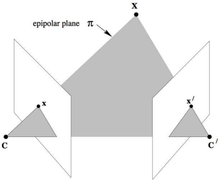
Some important observations :

- the pixel projection is along the ray defined by the 3D point and the camera center (i.e. as for **x**, **X** and **C**)
- conversely, if **x** and **x**′ do correspond to the same 3D point, the two rays intersect



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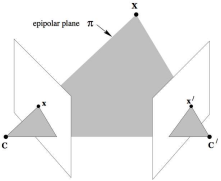

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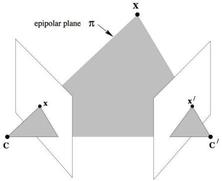

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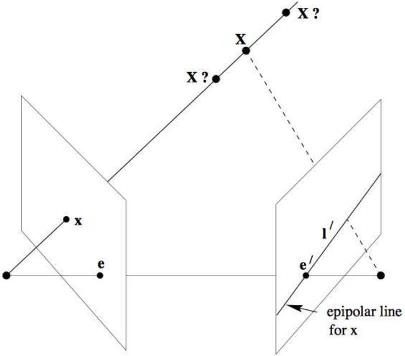

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Finally, by transposing $\mathbf{K}'^{-1}\mathbf{x}'$ and ignoring the scalar λ we get :

$$\mathbf{x}'^T \underbrace{\mathbf{K}'^{-T}\mathbf{t}_\times \mathbf{R}\mathbf{K}^{-1}}_{\mathbf{F}} \mathbf{x} = 0$$

The fundamental matrix F

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- The fundamental matrix constraint translates to a search along the epipolar line ...
- ... but also $\mathbf{F} = \mathbf{K}'^{-T} \mathbf{t}_\times \mathbf{R}\mathbf{K}^{-1}$ encodes, along with the calibration matrices, *the rotation and translation* between views

The fundamental matrix F

Theorem

The condition which is necessary and sufficient for a matrix **F** to be a fundamental matrix is that

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Multiple ways to notice that **F** is rank deficient :

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- Proof :
  $\left\| \mathbf{UDV}^T \mathbf{f} \right\| = \left\| \mathbf{DV}^T \mathbf{f} \right\|$, and $\left\| \mathbf{f} \right\| = \left\| \mathbf{V}^T \mathbf{f} \right\|$. We have to minimize $\left\| \mathbf{DV}^T \mathbf{f} \right\|$ subject to $\left\| \mathbf{V}^T \mathbf{f} \right\| = 1$. If $\mathbf{y} = \mathbf{V}^T \mathbf{f}$, then we minimize $\left\| \mathbf{Dy} \right\|$ subject to $\left\| \mathbf{y} \right\| = 1$. Since $\mathbf{D}$ is diagonal with values in descending order, it means that $\mathbf{y} = (0, 0 \ldots, 1)$, and $\mathbf{f} = \mathbf{Vy}$ is the last column of $\mathbf{V}$. (*A5.3, Hartley and Zisserman*)

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- $\det(\mathbf{F_1} + \alpha \mathbf{F_2})$ is a third degree polynomial, so up to three potential solutions may be recovered
- This algorithm is also preferred as fewer observations are needed

Outline

- The 3D representation of points
- The pinhole camera model
- Applying a coordinate transformation
- Homogeneous representations and algebraic operations
- The fundamental matrix
- The essential matrix
- Rectification

Using the camera calibration and the essential matrix

If the calibration matrices $\mathbf{K}$ and $\mathbf{K}'$ are known :

- we may recover the pose information from $\mathbf{F} = \mathbf{K}'^{-T}\mathbf{t}_\times\mathbf{R}\mathbf{K}^{-1}$:

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- Knowing $\mathbf{E}$: interesting for relative pose estimation
- Main disadvantage : $\mathbf{K}$ and $\mathbf{K}'$ are required to get to $\mathbf{E}$

Recovering R and t from E

It has been shown that the decomposition of **E** is possible and there are actually four valid solutions (*9.6.2, Hartley and Zisserman*) :

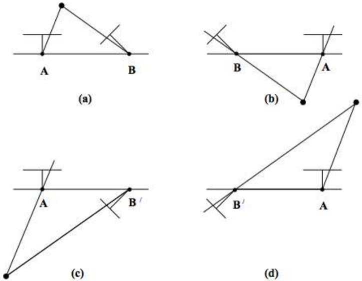

Fig. 9.12. **The four possible solutions for calibrated reconstruction from E.** *Between the left and right sides there is a baseline reversal. Between the top and bottom rows camera B rotates 180° about the baseline. Note, only in (a) is the reconstructed point in front of both cameras.*

- Identify the correct solution : cheirality check (the 3D points have to be in front of the camera) with an additional match from the two views

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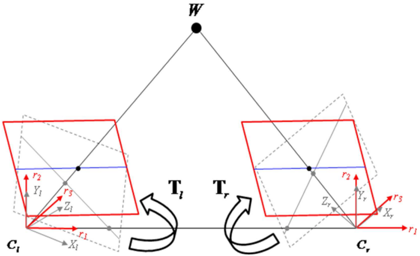
Rectification

Using **F**, we restrict the search for the corresponding projection $\mathbf{x}'$ of a point $\mathbf{x}$ to a line (the epipolar line $\mathbf{l}' = \mathbf{F}\mathbf{x}$).

Stereo rectification

- Apply an adjustment to the images in order to get horizontal epipolar lines in both views



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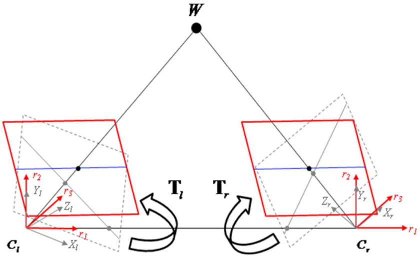

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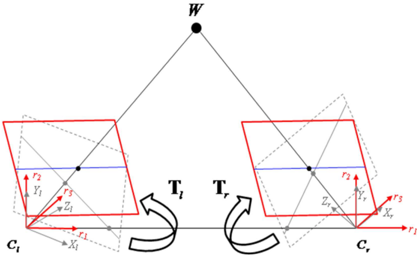

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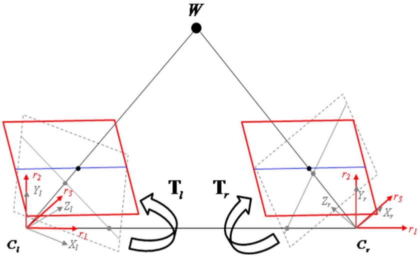
- This implies that epipoles are at horizontal infinity : $\mathbf{e} = \mathbf{e}' = [1\ 0\ 0]^T$
- Apply a virtual rotation of cameras (Fusiello, A. ; Trucco, E. ; Verri, A. A compact algorithm for rectification of stereo pairs. Mach. Vision Appl 2000)



Rectification

Using **F**, we restrict the search for the corresponding projection $\mathbf{x}'$ of a point $\mathbf{x}$ to a line (the epipolar line $\mathbf{l}' = \mathbf{F}\mathbf{x}$).

Stereo rectification

- Apply an adjustment to the images in order to get horizontal epipolar lines in both views
- The search for $\mathbf{x}'$ takes place simply along the same corresponding row in the second image : interesting for dense correspondence
- This implies that epipoles are at horizontal infinity : $\mathbf{e} = \mathbf{e}' = [1\ 0\ 0]^T$
- Apply a virtual rotation of cameras (Fusiello, A. ; Trucco, E. ; Verri, A. A compact algorithm for rectification of stereo pairs. Mach. Vision Appl 2000)
- An interpolation is required for creating the new images, but high computation gain overall