

# TEST - 16<sup>th</sup> SEPTEMBER 2016 - TRIGONOMETRY AND ODE's

**SURNAME:**

**NAME:**

**Instructions:** Please read carefully the following instructions:

- remember to write your name;
- you must complete the test within 2 hours;
- neither calculators nor notes or books are allowed;
- if some request is unclear, please, ask;
- fill in the blanks with your answer and return both this solution sheet and the detailed computations you have worked out.

## PROBLEMS

**Problem 1.** Write a unit-speed parameterization of the line in  $\mathbb{R}^3$  which passes through the points

$$P_1 = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} \quad \text{and} \quad P_2 = \begin{pmatrix} -1 \\ 5 \\ 4 \end{pmatrix}.$$

$$\gamma(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} =$$

**Problem 2.** Consider the circle  $\Gamma$  in  $\mathbb{R}^2$  centered at  $P = (1, 3)$  with radius  $R = 7$ .

- Write a parameterization of  $\Gamma$
  
  
  
  
  
  
  
  
  
  
- Write the tangent vector of your parameterization.

- Compute the speed of your parameterization.

**Problem 3.** Compute the following indefinite integrals:

$$\int \frac{e^x}{\sqrt{1 - e^{2x}}} dx =$$

$$\int \tanh(x) dx =$$

**Problem 4.** Write the Taylor polynomial at  $x_0 = 0$  of the function

$$f(x) = \frac{1}{\cosh(2x)}$$

up to the 2nd order.

$$P_f(x) =$$

**Problem 5.** Find the solution of the following ODE with initial Cauchy data:

$$\begin{cases} y'(x) = \sqrt{1 + y(x)^2} \\ y(0) = \frac{1}{2}(e^4 - e^{-4}) \end{cases} .$$

- $y(x) =$
- Draw the graph of the solution.

**Problem 6.** Consider the following ODE

$$y''(x) - 9y(x) = 0.$$

- Write the general solution.

$$y(x) =$$

- Write the solution which satisfies the initial conditions

$$\begin{cases} y(1) = y(-1) \\ y(0) = 7. \end{cases}.$$

$$y(x) =$$

- Draw the graph of the solution.