

## ADDITIONAL EXERCISES - TRIGONOMETRY AND ODE's

SURNAME:

NAME:

### PROBLEMS

**Problem 1.** Write a unit-speed parameterization of the line in  $\mathbb{R}^3$  which passes through the points

$$P_1 = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} \quad \text{and} \quad P_2 = \begin{pmatrix} 0 \\ 6 \\ -3 \end{pmatrix} .$$

$$\gamma(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} =$$

**Problem 2.** Consider the circle  $\Gamma$  in  $\mathbb{R}^2$  centered at  $P = (0, 5)$  with radius  $R = 5$ .

- Write a parameterization of  $\Gamma$
  
  
  
  
  
  
  
  
  
  
- Write the tangent vector of your parameterization.
  
  
  
  
  
  
  
  
  
  
- Compute the speed of your parameterization.

**Problem 3.** Compute the following indefinite integrals:

$$\int \frac{e^{3x}}{1 + e^{9x}} dx =$$

$$\int \tan(x) dx =$$

**Problem 4.** Write the Taylor polynomial at  $x_0 = 0$  of the function

$$f(x) = e^x + \sin(3x)$$

up to the 2nd order.

$$P_f(x) =$$

**Problem 5.** Find the solution of the following ODE with initial Cauchy data:

$$\begin{cases} y'(x) = (1 - y(x)^2)x \\ y(0) = \tanh(1) \end{cases} .$$

- $y(x) =$

**Problem 6.** Consider the following ODE

$$y''(x) - 3y'(x) + 2y(x) = 0 .$$

- Write the general solution.

$$y(x) =$$

- Write the solution which satisfies the initial conditions

$$\begin{cases} y(0) = 3 \\ y'(0) = 4 . \end{cases} .$$

$$y(x) =$$