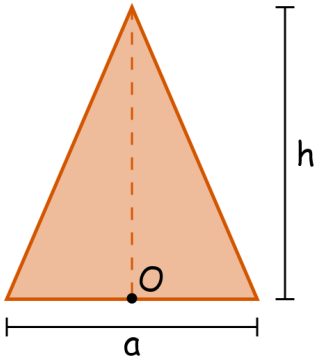
EXERCISES ON INERTIA - 1

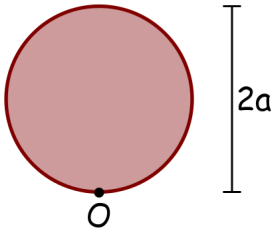
2D-BODIES

Problem 1. Consider the planar isosceles triangle in the picture and suppose it has total mass m. Write the tensor of inertia $\mathbf{I}_O$ at the point O of the disc. Find the vector $C - O$, where C is the center of mass of the disc, and write the central tensor of inertia $\mathbf{I}_C$.



HINT1: Find $\mathbf{I}_O$ first, and then use the Huygens-Steiner Theorem to find $\mathbf{I}_C$.
HINT2: Be careful in writing the intervals of your variables in the integrals computation. For example, if you take $\mathbf{e}_x$ parallel to the basis (a) of the triangle and $\mathbf{e}_y$ parallel to the height (h), for a generic point P of the triangle you can write $P - O = x\mathbf{e}_x + y\mathbf{e}_y$ with x in what interval? And when you fix x, what is the interval for y?

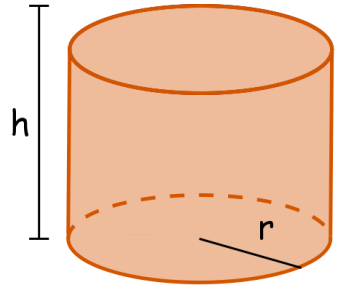
Problem 2. Consider a planar disc with radius a (as in the following figure) and suppose it has total mass m. Write the vector $C - O$, where C is the center of mass of the disc. Write the central tensor of inertia $\mathbf{I}_C$ and the tensor of inertia $\mathbf{I}_O$ at O of the disc.



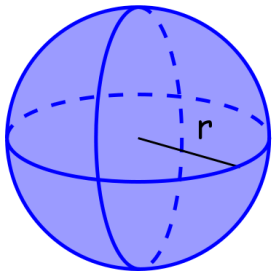
HINT1: Find $\mathbf{I}_C$ first, and then use the Huygens-Steiner Theorem to find $\mathbf{I}_O$.
HINT2: What are, in this case, the intervals of integration?
HINT3: If your integrals are too difficult to compute, try to find some cleaver change of variables.

3D-BODIES

Problem 3. Find the center of mass and the central tensor of inertia of a cylinder with radius r, height h and total mass m:



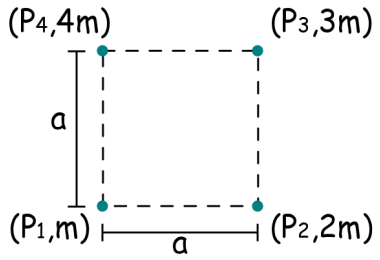
Problem 4. Find the center of mass and the central tensor of inertia of a 3D-ball with radius r and total mass m:



DISCRETE BODIES

Problem 5. Find the center of mass C and the central tensor of inertia $\mathbf{I}_C$ of the following configurations of 4 points:

(a)



(b)