Artificial Intelligence

A course about foundations

Plausible Reasoning

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Plausible (defeasible) reasoning

Why plausible reasoning?

Consider a generic entailment problem $\Gamma \models \varphi$?

Four possible answers:

1. $\Gamma \models \varphi$
   $\Gamma \not\models \neg\varphi$
2. $\Gamma \not\models \varphi$
   $\Gamma \models \neg\varphi$
3. $\Gamma \models \varphi$
   $\Gamma \models \neg\varphi$

   This case occurs only when Γ is contradictory, i.e. unsatisfiable
4. $\Gamma \not\models \varphi$
   $\Gamma \not\models \neg\varphi$

Case 4. is quite frequent: "our knowledge Γ does not allow deciding about φ "

Plausible (defeasible) reasoning

A reasoning process where the **relation** between formulae
is rationally plausible yet not necessarily correct (in the classical logical sense)

i.e. a specific reasoning method

Notation:
$\Gamma \vdash_{<SysLog>} \varphi$ says that φ is a **plausible** derivation from Γ in <*SysLog*>

Properties of $\vdash_{<SysLog>}$

$\Gamma \vdash_{<SysLog>} \varphi \Rightarrow \Gamma \nvdash_{<SysLog>} \neg\varphi$ (coherence)

$\Gamma \vdash_{<SysLog>} \varphi \Rightarrow \Gamma \vdash_{<SysLog>} \varphi$ (compatibility with derivation)

$\Gamma \vdash_{<SysLog>} \varphi \not\Rightarrow \Gamma \vdash_{<SysLog>} \varphi \ (\Rightarrow \Gamma \models \varphi)$ (not necessarily correct)

It occurs very often in practice:

"The train schedule does not report a train to Milano at 06:55,
therefore we assume that such a train does not exist"

Most databases contain positive information only
Negative facts are typically derived 'by default'

Closed-World Assumption (CWA)

$\{\Gamma \not\models \alpha\} \vdash_{CWA} \neg\alpha$ (α is an *atom*)

Example (a program):

$\Pi \equiv \{\{Philosopher(socrates)\}, \{Philosopher(plato)\}, \{Cat(felix)\}\}$

The program Π can be rewritten in L_{FO} as:

$\forall x\,((x = socrates) \rightarrow Philosopher\,(x))$
$\forall x\,((x = plato) \rightarrow Philosopher\,(x))$
$\forall x\,((x = felix) \rightarrow Cat(x))$

The *Closed-World Assumption* (*CWA*) means <u>completing</u> (i.e. extending) the program Π:

$\forall x\,((x = felix) \leftrightarrow Cat(x))$
$\forall x\,((x = socrates \vee x = plato) \leftrightarrow Philosopher\,(x))$ *Notice the double implication*

Then these plausible inferences become sound:

$\Pi \vdash_{CWA} \neg Cat(socrates)$
$\Pi \vdash_{CWA} \neg Cat(plato)$
$\Pi \vdash_{CWA} \neg Philosopher\,(felix)$

Plausible (defeasible) reasoning

- Inference in *defeasible reasoning* is

Non-monotonic

$\Gamma \mid\sim_{<SysLog>} \varphi \;\not\Rightarrow\; \Gamma \cup \Delta \mid\sim_{<SysLog>} \varphi$

The arrival of new pieces of information may falsify inferences that used to be justified

e.g. an extra train to Milano at 06:55 is announced …

Systemic

In classical logic, the soundness of all inferences schema depend only on the few formulae involved.

e.g. $\varphi \rightarrow \psi, \varphi \vdash \psi$

In defeasible reasoning, inferences are justified by an entire theory Γ

One must check the entire database (see CWA): $\Gamma \nvdash \varphi \;\; \mid\sim_{<SysLog>} \neg\varphi$

Inference and reasoning (according to C. S. Peirce, 1870 c.a.)

- Different types of reasoning

  Deductive inference (*sound*)

  Derive only what is justified in terms of ***entailment***

  "All beans in this bag are white"
  "This handful of beans comes from this bag"
  "This is a handful of white beans"

$$\frac{\forall x\, \varphi(x) \rightarrow \psi(x) \quad \varphi(a)}{\psi(a)}$$

  Inductive inference (*plausible*)

  From repeated occurrences, derive rules

  "This handful of beans comes from this bag"
  "This is a handful of white beans"
  "All beans in this bag are white"

$$\frac{\psi(a) \quad \varphi(a)}{\forall x\, \varphi(x) \rightarrow \psi(x)}$$

  Abductive inference (*plausible*)

  From rules and outcomes, derive premises

  "All beans in this bag are white"
  "This is a handful of white beans"
  "This handful of beans comes from this bag"

$$\frac{\forall x\, \varphi(x) \rightarrow \psi(x) \quad \psi(a)}{\varphi(a)}$$