Symbolic representation and inference: the intuitive idea

Marco Piastra

An aside: *solving analytical equations*

- Solving quadratic equations

$$x^2 + ax + b = 0$$
$$x^2 + 2(a/2)x + a^2/4 - a^2/4 + b = 0$$
$$(x + a/2)^2 - a^2/4 + b = 0$$
$$(x + a/2)^2 = a^2/4 - b$$
$$x = - a/2 \pm (a^2/4 - b)^{1/2}$$

  A sequence of steps: at each step a *transformation rule* is applied

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  A sequence of steps: at each step a *transformation rule* is applied

  From start to end

  Start point: a premise (*i.e. we assume the truth of something*)

  End point: a conclusion (*i.e. we state that something else must <u>also</u> be true*)

  (*both points are decided by us*)

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From start to end

Start point: a premise (*i.e. we assume the truth of something*)

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Sequence of steps

Each transformation rule is based on a *semantic equivalence*

Each step must be *correct*, in the above sense

Symbolic Reasoning

Symbolic descriptions can *abstract*

- Symbolic descriptions have *abstraction* capabilities:
  for instance, many linguistic phenomena are *systemic*
  (i.e. their structural complexity goes beyond a *pattern-matching*)

Someone may understand English or not.

But no human being can understand the sentence:
"*Sally* ***likes*** *Harry*"
without being able to understand:
"*Harry* ***likes*** *Sally*"
or any other sentences of the kind:
"X ***likes*** Y"
where X and Y can be any nouns or definite descriptions:
"*Ronald's girlfriend* ***likes*** *the cat that Linda brought home yesterday*"

(freely adapted from [Fodor e Phylyshyn, 1988])

Reasoning and schemas: *syllogism*

Many (intuitively) valid arguments obey an abstract schema

All (*humans*) are (*mortals*)
All (*Greeks*) are (*humans*)   hence

All (*Greeks*) are (*mortals*)

Abstract schema:

All ***H*** are ***M***
All ***G*** are ***H***   hence

All ***G*** are ***M***

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All ***G*** are ***H***

All ***G*** are ***M***

CAUTION!
The position of the line IS relevant:
in this case the schema does not work

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All (*Greeks*) are (*mortals*) hence

All (*Greeks*) are (*humans*)

Abstract schema:

All ***H*** are ***M***
All ***G*** are ***M*** hence

All ***G*** are ***H***

CAUTION!
The ordering of sentences IS relevant:
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Reasoning and schemas: *syllogism*

Many (intuitively) valid arguments obey an abstract schema

All (*humans*) are (*mortals*)
All (*Greeks*) are (*humans*) — hence
All (*Greeks*) are (*mortals*)

Abstract schema:

All ***H*** are ***M***
All ***G*** are ***H*** — hence
All ***G*** are ***M***

The validity of schemas does not depend on *meaning*:

All (*enchanted frogs*) are (*princesses*)
All (*princesses*) are (*young and beautiful*) — hence
All (*enchanted frogs*) are (*young and beautiful*)

Same schema,
different impression

Fallacies (*paralogisms*)

Wrong sequence:

All (*humans*) are (*mortals*)
All (*Greeks*) are (*mortals*)
____
All (*Greeks*) are (*humans*)

The last two sentences have been switched

Referential ambiguities:

(*Nothing*) is better than (*eternal happiness*)
(*One ham sandwich*) is better than (*nothing*)
____
(*One ham sandwich*) is better than (*eternal happiness*)

'Obscure' subtleties (obscure for now):

All (*enchanted frogs*) are (*princesses*)
All (*enchanted frogs*) are (*young and beautiful*)
There is an (*enchanted frog*) which is (*a young and beautiful princess*)

What do we mean by "all"? Do we need at least one specimen to say "all"?
In such case, the schema would be valid
(*in formal logic it is not, as we will see*)

What is the purpose of *symbolic* logic?

To distinguish correct reasoning from incorrect reasoning

What is the purpose of *symbolic* logic?

To distinguish correct reasoning from incorrect reasoning

by the formal structure

Truth and Possible Worlds: the quest for formal semantics

What is *true*?

- **A world of *cats***

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot	x		x	
Kitty		x	x	
Felix			x	

- **Sentences about this world**

“Spot likes Tom” and “Tom does not like Spot”
“Tom likes himself”
“Kitty likes Spot” and “Spot likes Kitty”
“Kitty likes herself”
“Felix likes Kitty”

*All these sentences are **true**, in the world above*

What is *true*?

- **A world of *cats***

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot	x		x	
Kitty		x	x	
Felix			x	

- **Sentences about this world**

  "Spot likes Felix"
  "Spot likes himself"
  "Kitty does not like herself"
  "Felix likes Spot"

  *All these sentences are **false**, in the world above*

What is *true*?

- **A world of *cats***

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot	x		x	
Kitty		x	x	
Felix			x	

- **Sentences about this world** (i.e. with *generalization*)

  "Every cat likes a cat" ***is true***
  "Every cat likes another cat" ***is false***
  "Tom does not like any other cat" ***is true***
  "Kitty is liked by every cat" ***is false***

  "Every cat that likes another cat also likes her/himself" ***is ?***

How to make a sentence *true*?

- **Consider the sentence**

  "Kitty likes Spot" and "Spot likes Tom"

  It can be made true in many different ways

  "It may be true in many different *possible worlds*"

  Examples (i.e. of other *possible worlds* of cats):

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x			
Kitty		x		
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot	x		x	
Kitty		x	x	
Felix			x	

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	x
Spot	x	x	x	x
Kitty	x	x	x	x
Felix	x	x	x	x

How to make a sentence *true*?

- **Consider the sentence**

  “Kitty likes Spot” and “Kitty does <u>not</u> like Spot”

  There is no way to *make it true*

  “There is no *possible world* where this can be true”

Sentences may be related

- **Three sentences**

  1) "Every cat that likes Kitty likes Spot as well"
  2) "Tom likes Kitty"
  3) "Tom likes Spot"

  There is no way to make true sentences 1) *and* 2) *without making true sentence* 3) *as well…*

  (just give it a try…)

This is just a small subset of all possible worlds …

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	
Kitty		x	x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot			x	
Kitty		x		
Felix				x

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	x
Kitty			x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot				
Kitty			x	
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom		x	x	
Spot	x	x	x	
Kitty	x	x		
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likes	Tom	Spot	Kitty	Felix
Tom	x		x	
Spot	x			
Kitty		x		
Felix		x		x

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Tom	x	x	x	x
Spot		x		
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Sentence 1) is true in these worlds

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	
Kitty		x	x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot			x	
Kitty		x		
Felix				x

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	x
Kitty			x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot				
Kitty			x	
Felix				

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Tom	x	x	x	
Spot	x			
Kitty		x		
Felix		x	x	

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Tom		x	x	
Spot	x	x	x	
Kitty	x	x		
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x		x	
Spot	x			
Kitty		x		
Felix		x		x

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	x
Spot		x		
Kitty			x	
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Tom			x	
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Spot	x			
Kitty		x		
Felix		x	x	

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Spot	x	x	x	
Kitty		x	x	
Felix		x	x	

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Tom	x			
Spot			x	
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Spot	x	x	x	x
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Spot		x		
Kitty			x	
Felix				x

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Spot	x			
Kitty		x		
Felix		x	x	

Sentence 2) is true in these worlds

Sentences may be related

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  3) "Tom likes Spot"

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  (just give it a try)

Sentence 1) is true in these worlds

Sentence 3) is true in these worlds

Sentence 2) is true in these worlds

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Tom				
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	
Kitty		x	x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot			x	
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Tom				
Spot	x	x	x	x
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Tom	x	x	x	
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Kitty	x	x		
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Felix		x		x

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Tom	x	x	x	x
Spot		x		
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Questions:

Was this just coincidence?

How many possible worlds must we consider in order to be sure?