

Artificial Intelligence

*Symbolic representation and inference:
the intuitive idea*

Marco Piastra

An aside: equations

- Solving quadratic equations

$$x^2 + ax + b = 0$$

$$x^2 + 2(a/2)x + a^2/4 - a^2/4 + b = 0$$

$$(x + a/2)^2 - a^2/4 + b = 0$$

$$(x + a/2)^2 = a^2/4 - b$$

$$x = -a/2 \pm (a^2/4 - b)^{1/2}$$

A sequence of steps: at each step a *transformation rule* is applied

An aside: equations

- Solving quadratic equations

$$x^2 + ax + b = 0$$

$$x^2 + 2(a/2)x + a^2/4 - a^2/4 + b = 0$$

$$(x + a/2)^2 - a^2/4 + b = 0$$

$$(x + a/2)^2 = a^2/4 - b$$

$$x = -a/2 \pm (a^2/4 - b)^{1/2}$$

A sequence of steps: at each step a *transformation rule* is applied

From start to end

Start point: a premise (*i.e. we assume the truth of something*)

End point: a conclusion (*i.e. we state that something else must also be true*)

(*both points are decided by us*)

An aside: equations

- Solving quadratic equations

$$x^2 + ax + b = 0$$

$$x^2 + 2(a/2)x + a^2/4 - a^2/4 + b = 0$$

$$(x + a/2)^2 - a^2/4 + b = 0$$

$$(x + a/2)^2 = a^2/4 - b$$

$$x = -a/2 \pm (a^2/4 - b)^{1/2}$$

A sequence of steps: at each step a *transformation rule* is applied

From start to end

Start point: a premise (*i.e. we assume the truth of something*)

End point: a conclusion (*i.e. we state that something else must also be true*)

(*both points are decided by us*)

An aside: equations

- Solving quadratic equations

$$x^2 + ax + b = 0$$

$$x^2 + 2(a/2)x + a^2/4 - a^2/4 + b = 0$$

$$(x + a/2)^2 - a^2/4 + b = 0$$

$$(x + a/2)^2 = a^2/4 - b$$

$$x = -a/2 \pm (a^2/4 - b)^{1/2}$$

A sequence of steps: at each step a *transformation rule* is applied

From start to end

Start point: a premise (*i.e. we assume the truth of something*)

End point: a conclusion (*i.e. we state that something else must also be true*)

(*both points are decided by us*)

Sequence of steps

Each transformation rule is based on a *semantic equivalence*

Each step must be *correct*, in the above sense

Symbolic Reasoning

Symbolic descriptions can *abstract*

- Symbolic descriptions have *abstraction* capabilities:
for instance, many linguistic phenomena are *systemic*
(i.e. their structural complexity goes beyond a *pattern-matching*)

Someone may understand English or not.

But no human being can understand the sentence:

*“Sally **likes** Harry”*

without being able to understand:

*“Harry **likes** Sally”*

or any other sentences of the kind:

*“X **likes** Y”*

where X and Y can be any nouns or definite descriptions:

*“Ronald’s girlfriend **likes** the cat that Linda brought home yesterday”*

(freely adapted from [Fodor e Phylyshyn, 1988])

Reasoning and schemas: *syllogism*

Many (intuitively) valid arguments obey an abstract schema

All (*humans*) are (*mortals*)

All (*Greeks*) are (*humans*)

hence

All (*Greeks*) are (*mortals*)

Abstract schema:

All **H** are **M**

All **G** are **H**

hence

All **G** are **M**

Reasoning and schemas: *syllogism*

Many (intuitively) valid arguments obey an abstract schema

All (*humans*) are (*mortals*) hence

All (*Greeks*) are (*humans*)

All (*Greeks*) are (*mortals*)

Abstract schema:

All **H** are **M** hence

All **G** are **H**

All **G** are **M**

CAUTION!

*The position of the line IS relevant:
in this case the schema does not work*

Reasoning and schemas: *syllogism*

Many (intuitively) valid arguments obey an abstract schema

All *(humans)* are *(mortals)*

All *(Greeks)* are *(mortals)*

hence

All *(Greeks)* are *(humans)*

Abstract schema:

All **H** are **M**

All **G** are **M**

hence

All **G** are **H**

CAUTION!

*The ordering of sentences IS relevant:
in this case the schema does not work*

Reasoning and schemas: *syllogism*

Many (intuitively) valid arguments obey an abstract schema

All (*humans*) are (*mortals*)
All (*Greeks*) are (*humans*) hence

All (*Greeks*) are (*mortals*)

Abstract schema:

All **H** are **M**
All **G** are **H** hence

All **G** are **M**

The validity of schemas does not depend on *meaning*:

All (*enchanted frogs*) are (*princesses*)
All (*princesses*) are (*young and beautiful*) hence

All (*enchanted frogs*) are (*young and beautiful*)

*Same schema,
different impression*

Fallacies (*paralogisms*)

Wrong sequence:

All (*humans*) are (*mortals*)

All (*Greeks*) are (*mortals*)

All (*Greeks*) are (*humans*)

*The last two sentences
have been switched*

Referential ambiguities:

(*Nothing*) is better than (*eternal happiness*)

(*One ham sandwich*) is better than (*nothing*)

(*One ham sandwich*) is better than (*eternal happiness*)

'Obscure' subtleties (obscure for now):

All (*enchanted frogs*) are (*princesses*)

All (*enchanted frogs*) are (*young and beautiful*)

There is an (*enchanted frog*) which is (*a young and beautiful princess*)

What do we mean by "all"? Do we need at least one specimen to say "all"?

In such case, the schema would be valid

(*in formal logic it is not, as we will see*)

What is the purpose of *symbolic* logic?

*To distinguish correct reasoning
from incorrect reasoning*

What is the purpose of *symbolic* logic?

*To distinguish correct reasoning
from incorrect reasoning
by the formal structure*

Truth and Possible Worlds: the quest for formal semantics

What is *true*?

- **A world of cats**

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot	x		x	
Kitty		x	x	
Felix			x	

- **Sentences about this world**

“Spot likes Tom” and “Tom does not like Spot”

“Tom likes himself”

“Kitty likes Spot” and “Spot likes Kitty”

“Kitty likes herself”

“Felix likes Kitty”

*All these sentences are **true**, in the world above*

What is *true*?

- **A world of cats**

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot	x		x	
Kitty		x	x	
Felix			x	

- **Sentences about this world**

“Spot likes Felix”

“Spot likes himself”

“Kitty does not like herself”

“Felix likes Spot”

*All these sentences are **false**, in the world above*

What is *true*?

- **A world of cats**

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot	x		x	
Kitty		x	x	
Felix			x	

- **Sentences about this world** (i.e. with *generalization*)

“Every cat likes a cat” *is true*

“Every cat likes another cat” *is false*

“Tom does not like any other cat” *is true*

“Kitty is liked by every cat” *is false*

“Every cat that likes another cat also likes her/himself” *is ?*

How to make a sentence *true*?

- **Consider the sentence**

“Kitty likes Spot” and “Spot likes Tom”

It can be made true in many different ways

“It may be true in many different *possible worlds*”

Examples (i.e. of other *possible worlds* of cats):

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x			
Kitty		x		
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot	x		x	
Kitty		x	x	
Felix			x	

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	x
Spot	x	x	x	x
Kitty	x	x	x	x
Felix	x	x	x	x

How to make a sentence *true*?

- **Consider the sentence**

“Kitty likes Spot” and “Kitty does not like Spot”

There is no way to *make it true*

“There is no *possible world* where this can be true”

Sentences may be related

■ Three sentences

- 1) "Every cat that likes Kitty likes Spot as well"
- 2) "Tom likes Kitty"
- 3) "Tom likes Spot"

*There is no way to make true sentences 1) and 2) without making true sentence 3) as well...
(just give it a try...)*

This is just a small subset of all possible worlds ...

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	
Kitty		x	x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot			x	
Kitty		x		
Felix				x

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	x
Kitty			x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot				
Kitty			x	
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom		x	x	
Spot	x	x	x	
Kitty	x	x		
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x		x	
Spot	x			
Kitty		x		
Felix		x		x

likes	Tom	Spot	Kitty	Felix
Tom			x	
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	x
Spot		x		
Kitty			x	
Felix				x

Sentences may be related

■ Three sentences

- 1) "Every cat that likes Kitty likes Spot as well"
- 2) "Tom likes Kitty"
- 3) "Tom likes Spot"

*There is no way to make true sentences 1) and 2) without making true sentence 3) as well...
(just give it a try...)*

Sentence 1) is true in these worlds

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	
Kitty		x	x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot			x	
Kitty		x		
Felix				x

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	x
Kitty			x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot				
Kitty			x	
Felix				

likes	Tom	Spot	Kitty	Felix
Tom		x	x	
Spot	x	x	x	
Kitty	x	x		
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x		x	
Spot	x			
Kitty		x		
Felix		x		x

likes	Tom	Spot	Kitty	Felix
Tom			x	
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	x
Spot		x		
Kitty			x	
Felix				x

Sentences may be related

■ Three sentences

- 1) "Every cat that likes Kitty likes Spot as well"
- 2) "Tom likes Kitty"
- 3) "Tom likes Spot"

*There is no way to make true sentences 1) and 2) without making true sentence 3) as well...
(just give it a try...)*

Sentence 1) is true in these worlds

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	
Kitty		x	x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot			x	
Kitty		x		
Felix				x

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	x
Kitty			x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot				
Kitty			x	
Felix				

likes	Tom	Spot	Kitty	Felix
Tom		x	x	
Spot	x	x	x	
Kitty	x	x		
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x		x	
Spot	x			
Kitty		x		
Felix		x		x

likes	Tom	Spot	Kitty	Felix
Tom			x	
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	x
Spot		x		
Kitty			x	
Felix				x

Sentence 2) is true in these worlds

Sentences may be related

■ Three sentences

- 1) "Every cat that likes Kitty likes Spot as well"
- 2) "Tom likes Kitty"
- 3) "Tom likes Spot"

*There is no way to make true sentences 1) and 2) without making true sentence 3) as well...
(just give it a try)*

Sentence 1) is true in these worlds

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	
Kitty		x	x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot			x	
Kitty		x		
Felix				x

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	x
Kitty			x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot				
Kitty			x	
Felix				

Sentence 3) is true in these worlds

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom		x	x	
Spot	x	x	x	
Kitty	x	x		
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	
Spot	x			
Kitty		x		
Felix		x	x	x

likes	Tom	Spot	Kitty	Felix
Tom			x	
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	x
Spot		x		
Kitty			x	
Felix				x

Sentence 2) is true in these worlds

Sentences may be related

- **Three sentences**

- 1) "Every cat that likes Kitty likes Spot as well"
- 2) "Tom likes Kitty"
- 3) "Tom likes Spot"

*There is no way to make true sentences 1) and 2) without making true sentence 3) as well...
(just give it a try)*

Questions:

Was this just coincidence?

How many possible worlds must we consider in order to be sure?