

Artificial Intelligence

Formal Logic

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Symbolic descriptions can *abstract*

- Symbolic descriptions have *abstraction* capabilities:
for instance, many linguistic phenomena are *systemic*
(i.e. their complexity goes beyond a *pattern-matching*)

Someone may understand English or not.

But no human being can understand the sentence:

*“Sally **likes** Harry”*

without being able to understand:

*“Harry **likes** Sally”*

or any other sentences of the kind:

*“X **likes** Y”*

where X and Y can be any nouns or definite descriptions:

*“Ronald’s girlfriend **likes** the cat that Linda brought home yesterday”*

(freely adapted from [Fodor e Phylyshyn, 1988])

Reasoning and schemas: *syllogism*

Many (intuitively) valid arguments obey an abstract schema

All (*humans*) are (*mortals*)

All (*Greeks*) are (*humans*)

hence

All (*Greeks*) are (*mortals*)

Abstract schema:

All **H** are **M**

All **G** are **H**

hence

All **G** are **M**

Reasoning and schemas: *syllogism*

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All (*humans*) are (*mortals*) hence

All (*Greeks*) are (*humans*)

All (*Greeks*) are (*mortals*)

Abstract schema:

All **H** are **M** hence

All **G** are **H**

All **G** are **M**

CAUTION!

*The position of the line IS relevant:
in this case the schema does not work*

Reasoning and schemas: *syllogism*

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Abstract schema:

All **H** are **M**

All **G** are **M**

hence

All **G** are **H**

CAUTION!

*The ordering of sentences IS relevant:
in this case the schema does not work*

Reasoning and schemas: *syllogism*

Many (intuitively) valid arguments obey an abstract schema

All (*humans*) are (*mortals*)
All (*Greeks*) are (*humans*) hence

All (*Greeks*) are (*mortals*)

Abstract schema:

All **H** are **M**
All **G** are **H** hence

All **G** are **M**

The validity of schemas does not depend on *meaning*:

All (*enchanted frogs*) are (*princesses*)
All (*princesses*) are (*young and beautiful*) hence

All (*enchanted frogs*) are (*young and beautiful*)

*Same schema,
different impression*

Fallacies (*paralogisms*)

Wrong sequence:

All (*humans*) are (*mortals*)

All (*Greeks*) are (*mortals*)

All (*Greeks*) are (*humans*)

*The last two sentences
have been switched*

Referential ambiguities:

(*Nothing*) is better than (*eternal happiness*)

(*A ham sandwich*) is better than (*nothing*)

(*A ham sandwich*) is better than (*eternal happiness*)

'Obscure' subtleties (obscure for now):

All (*enchanted frogs*) are (*princesses*)

All (*enchanted frogs*) are (*young and beautiful*)

There is an (*enchanted frog*) which is (*a young and beautiful princess*)

What do we mean by "all"? Do we need at least one specimen to say "all"?

In such case, the schema would be valid

(*in formal logic it is not, as we will see*)

What is the purpose of *formal* logic?

*To distinguish correct reasoning
from incorrect reasoning*

What is the purpose of *formal* logic?

*To distinguish correct reasoning
from incorrect reasoning
by the formal structure*

An aside: equations

- Solving quadratic equations

$$x^2 + ax + b = 0$$

$$x^2 + 2(a/2)x + a^2/4 - a^2/4 + b = 0$$

$$(x + a/2)^2 - a^2/4 + b = 0$$

$$(x + a/2)^2 = a^2/4 - b$$

$$x = -a/2 \pm (a^2/4 - b)^{1/2}$$

A sequence of steps: at each step a *transformation rule* is applied

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From start to end

Start point: a premise (*i.e. we assume the truth of something*)

End point: a conclusion (*i.e. we state that something else must also be true*)

(*both points are decided by us*)

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Sequence of steps

Each transformation rule is based on *equivalence*

Each step must be *correct*, in this particular sense

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Sequence of steps

Each transformation rule is based on *equivalence*

Each step must be *correct*, in this particular sense

Abstraction and Correctness

Symbols like x , a and b represent any number, *equivalences* are valid for *all* of them

What does it mean for the whole sequence to be *correct*?

What is *true*?

- **A world of cats**

| | Tom | Spot | Kitty | Felix |
|--------------|-----|------|-------|-------|
| likes | | | | |
| Tom | x | | | |
| Spot | x | | x | |
| Kitty | | x | x | |
| Felix | | | x | |

- **Sentences about this world**

“Spot likes Tom” and “Tom does not like Spot”

“Tom likes himself”

“Kitty likes Spot” and “Spot likes Kitty”

“Kitty likes herself”

“Felix likes Kitty”

*All these sentences are **true**, in the world above*

What is *true*?

- **A world of cats**

| | Tom | Spot | Kitty | Felix |
|--------------|-----|------|-------|-------|
| <i>likes</i> | | | | |
| Tom | x | | | |
| Spot | x | | x | |
| Kitty | | x | x | |
| Felix | | | x | |

- **Sentences about this world**

“Spot likes Felix”

“Spot likes himself”

“Kitty does not like herself”

“Felix likes Spot”

*All these sentences are **false**, in the world above*

What is *true*?

- **A world of cats**

| | Tom | Spot | Kitty | Felix |
|--------------|-----|------|-------|-------|
| likes | | | | |
| Tom | x | | | |
| Spot | x | | x | |
| Kitty | | x | x | |
| Felix | | | x | |

- **Sentences about this world**

“Every cat likes a cat” *is true*

“Every cat likes another cat” *is false*

“Tom does not like any other cat” *is true*

“Kitty is liked by every cat” *is false*

How to make a sentence *true*?

- Consider the sentence

“Kitty likes Spot” and “Spot likes Tom”

It can be made true in many different ways

“It may be true in many different *possible worlds*”

Examples:

| | Tom | Spot | Kitty | Felix |
|--------------|-----|------|-------|-------|
| <i>likes</i> | | | | |
| Tom | | | | |
| Spot | x | | | |
| Kitty | | x | | |
| Felix | | | | |

| | Tom | Spot | Kitty | Felix |
|--------------|-----|------|-------|-------|
| <i>likes</i> | | | | |
| Tom | x | | | |
| Spot | x | | x | |
| Kitty | | x | x | |
| Felix | | | x | |

| | Tom | Spot | Kitty | Felix |
|--------------|-----|------|-------|-------|
| <i>likes</i> | | | | |
| Tom | x | x | x | x |
| Spot | x | x | x | x |
| Kitty | x | x | x | x |
| Felix | x | x | x | x |

How to make a sentence *true*?

- **Consider the sentence**

“Kitty likes Spot” and “Kitty does not like Spot”

There is no way to *make it true*

“There is no *possible world* where this can be true”

Sentences may be related

■ Three sentences

- 1) "Every cat that likes Kitty likes Spot as well"
- 2) "Tom likes Kitty"
- 3) "Tom likes Spot"

*There is no way to make true sentences 1) and 2) without making true sentence 3) as well...
(just give it a try...)*

| likes | Tom | Spot | Kitty | Felix |
|--------------|-----|------|-------|-------|
| Tom | | | | |
| Spot | x | | | |
| Kitty | | x | | |
| Felix | | x | x | |

| likes | Tom | Spot | Kitty | Felix |
|--------------|-----|------|-------|-------|
| Tom | | | | |
| Spot | x | x | x | |
| Kitty | | x | x | |
| Felix | | x | x | |

| likes | Tom | Spot | Kitty | Felix |
|--------------|-----|------|-------|-------|
| Tom | x | | | |
| Spot | | | x | |
| Kitty | | x | | |
| Felix | | | | x |

| likes | Tom | Spot | Kitty | Felix |
|--------------|-----|------|-------|-------|
| Tom | | | | |
| Spot | x | x | x | x |
| Kitty | | | x | |
| Felix | | x | x | |

| likes | Tom | Spot | Kitty | Felix |
|--------------|-----|------|-------|-------|
| Tom | | | | |
| Spot | | | | |
| Kitty | | | x | |
| Felix | | | | |

| likes | Tom | Spot | Kitty | Felix |
|--------------|-----|------|-------|-------|
| Tom | x | x | x | |
| Spot | x | | | |
| Kitty | | x | | |
| Felix | | x | x | |

| likes | Tom | Spot | Kitty | Felix |
|--------------|-----|------|-------|-------|
| Tom | | x | x | |
| Spot | x | x | x | |
| Kitty | x | x | | |
| Felix | | | | |

| likes | Tom | Spot | Kitty | Felix |
|--------------|-----|------|-------|-------|
| Tom | x | | x | |
| Spot | x | | | |
| Kitty | | x | | |
| Felix | | x | | x |

| likes | Tom | Spot | Kitty | Felix |
|--------------|-----|------|-------|-------|
| Tom | | | x | |
| Spot | x | | | |
| Kitty | | x | | |
| Felix | | x | x | |

| likes | Tom | Spot | Kitty | Felix |
|--------------|-----|------|-------|-------|
| Tom | x | x | x | x |
| Spot | | x | | |
| Kitty | | | x | |
| Felix | | | | x |

Sentences may be related

■ Three sentences

- 1) "Every cat that likes Kitty likes Spot as well"
- 2) "Tom likes Kitty"
- 3) "Tom likes Spot"

*There is no way to make true sentences 1) and 2) without making true sentence 3) as well...
(just give it a try...)*

Sentence 1) is true in these worlds

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | | | | |
| Spot | x | | | |
| Kitty | | x | | |
| Felix | | x | x | |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | | | | |
| Spot | x | x | x | |
| Kitty | | x | x | |
| Felix | | x | x | |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | x | | | |
| Spot | | | x | |
| Kitty | | x | | |
| Felix | | | | x |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | | | | |
| Spot | x | x | x | x |
| Kitty | | | x | |
| Felix | | x | x | |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | | | | |
| Spot | | | | |
| Kitty | | | x | |
| Felix | | | | |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | | x | x | |
| Spot | x | x | x | |
| Kitty | x | x | | |
| Felix | | | | |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | x | | x | |
| Spot | x | | | |
| Kitty | | x | | |
| Felix | | x | | x |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | | | x | |
| Spot | x | | | |
| Kitty | | x | | |
| Felix | | x | x | |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | x | x | x | x |
| Spot | | x | | |
| Kitty | | | x | |
| Felix | | | | x |

Sentences may be related

■ Three sentences

- 1) "Every cat that likes Kitty likes Spot as well"
- 2) "Tom likes Kitty"
- 3) "Tom likes Spot"

*There is no way to make true sentences 1) and 2) without making true sentence 3) as well...
(just give it a try...)*

Sentence 1) is true in these worlds

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | | | | |
| Spot | x | | | |
| Kitty | | x | | |
| Felix | | x | x | |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | | | | |
| Spot | x | x | x | |
| Kitty | | x | x | |
| Felix | | x | x | |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | x | | | |
| Spot | | | x | |
| Kitty | | x | | |
| Felix | | | | x |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | | | | |
| Spot | x | x | x | x |
| Kitty | | | x | |
| Felix | | x | x | |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | | | | |
| Spot | | | | |
| Kitty | | | x | |
| Felix | | | | |

Sentence 2) is true in these worlds

Sentences may be related

■ Three sentences

- 1) "Every cat that likes Kitty likes Spot as well"
- 2) "Tom likes Kitty"
- 3) "Tom likes Spot"

*There is no way to make true sentences 1) and 2) without making true sentence 3) as well...
(just give it a try)*

Sentence 1) is true in these worlds

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | | | | |
| Spot | x | | | |
| Kitty | | x | | |
| Felix | | x | x | |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | | | | |
| Spot | x | x | x | |
| Kitty | | x | x | |
| Felix | | x | x | |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | x | | | |
| Spot | | | x | |
| Kitty | | x | | |
| Felix | | | | x |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | | | | |
| Spot | x | x | x | x |
| Kitty | | | x | |
| Felix | | x | x | |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | | | | |
| Spot | | | | |
| Kitty | | | x | |
| Felix | | | | |

3) is true in these worlds

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | x | x | x | |
| Spot | x | | | |
| Kitty | | x | | |
| Felix | | x | x | |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | | x | x | |
| Spot | x | x | x | |
| Kitty | x | x | | |
| Felix | | | | |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | x | x | x | |
| Spot | x | | | |
| Kitty | | x | | |
| Felix | | x | | x |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | | | x | |
| Spot | x | | | |
| Kitty | | x | | |
| Felix | | x | x | |

| likes | Tom | Spot | Kitty | Felix |
|-------|-----|------|-------|-------|
| Tom | x | x | x | x |
| Spot | | x | | |
| Kitty | | | x | |
| Felix | | | | x |

Sentence

Sentence 2) is true in these worlds

Sentences may be related

▪ Three sentences

- 1) "Every cat that likes Kitty likes Spot as well"
- 2) "Tom likes Kitty"
- 3) "Tom likes Spot"

*There is no way to make true sentences 1) and 2) without making true sentence 3) as well...
(just give it a try)*

Questions:

Was this just coincidence?
How many possible worlds
must we consider in order to be sure?