A Novel Photometric Stereo Method with Nonisotropic Point Light Sources

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Abstract—This paper presents a photometric stereo method with nonisotropic point light sources. Subject to the non-uniform lighting conditions produced by the nonisotropic point sources, each incident light ray should be precisely determined so as to realize an accurate calculation of surface normal. In the proposed method, radiance model of the light source is firstly introduced to the classical photometric stereo framework. By considering the distance and angular attenuations of incident light rays, a precise description for the lighting field can be established. Based on the initial 3D reconstruction result, an iterative process is introduced to optimize the primary light model parameters with respect to the unknown distance factor. The experimental setup is quite simple, which only consists of some LEDs and one camera. And the experimental results show that, with the proposed method, accuracy of the reconstructed surface normal can be greatly improved in comparison with some conventional light models.

Keywords—photometric stereo; nonisotropic point light; radiance model; 3D reconstruction.

I. INTRODUCTION

Photometric stereo has been a classical computer vision technique for surface 3D reconstruction [1]. By observing the target under various illuminations, surface normal for each surface point can be estimated with respect to its lighting conditions. And then, 3D shape of the target can be retrieved via surface integration approaches [2-4]. Parallel or distant point light source are usually selected by existing photometric stereo systems, which can greatly simplify the calculation of incident light direction and intensity. In this work, some normal LED lamps are used to configure a near-field photometric stereo system, which make the system with advantages of low-cost and compact size. LED is a typical nonisotropic point light source, and a single LED lamp cannot produce a uniform lighting field in a short working distance subject to its radiance property [5-7]. And thus makes conventional photometric stereo methods incapable to get precise surface normal values.

To realize accurate surface normal calculation with the nonisotropic point light source-based photometric stereo system, direction and intensity of each incident light ray on the target surface should be precisely calculated. Major difficulties involved can be summarized as follows: 1) radiance property of the light source should be precisely modeled so as to represent the lighting field accurately; 2) irradiance at each surface point is not only determined by the incident light direction, but also related to the object distance which is usually unknown; 3) position and orientation of each point light source should be precisely estimated, slight deviations will cause distinct errors in the modeling of lighting field. To solve above problems, the critical issue is to construct a new light model which is capable to precisely describe the lighting condition for each surface point, including its incident light direction and intensity.

There are also some works have been proposed to address the problem of unideal lighting conditions in photometric stereo domain. In [8], a photometric stereo system was investigated for the underwater 3D reconstruction. Subject to the light absorption by water, a sparse depth map was first constructed for each object in the scene. And then the distance attenuation factor was considered to model the point light source more precisely. In [9-10], a straightforward means was introduced by using a reference plane. For each light source, the reference plane was illuminated and imaged. The average intensity of each image was calculated and assumed as the intensity under a virtual parallel light. For each image point, its intensity deviation with respect to the average intensity was calculated and stored in a compensation-map. However, such an approach can only work with the near planar surfaces which were placed at the similar distance as the reference plane. In [11-12], the Partial Differential Equations (PDE) was studied to represent the near-field lighting condition. A set of quasi-linear PDEs was used to approximate the general lighting scenarios, and several kinds of light attenuation models were tested with synthetic data which contained non-uniform albedo and extra noises. In [13], a mesh deformation-based method was introduced to improve the 3D reconstruction quality of a near-field photometric stereo system. Each facet on the target mesh was corresponded to a pixel in the captured image, and the mesh deformation was decoupled into an iteration of interlaced steps of local projection and global blending.

In this work, a nonisotropic point light model is investigated based on the proposed photometric stereo setup. To describe the lighting field more precisely, radiance model of the nonisotropic light source is firstly constructed. And then, both distance and angular attenuations are considered to model the
lighting condition for each surface point. In the 3D reconstruction stage, the target is firstly assumed as planar and reconstructed via conventional surface integration method. Based on the initially reconstructed 3D model, an iterative procedure is applied to optimize the lighting model with respect to the change of surface depth. To evaluate the feasibility and accuracy of the proposed method, extensive experiments are conducted with a comparison with conventional light models.

II. MODELING OF THE NONISOTROPIC LIGHTING FIELD

To realize a precise photometric stereo process based on the proposed system, the lighting condition for each surface point should be accurately determined. In the proposed light model, three major factors are considered, i.e., radiance property of the nonisotropic light source, the incident light angle and the incident light intensity.

A. Radiance Model of Light Source with Angular Attenuation

For a single LED lamp, it can be treated as a nonisotropic point light source, which usually has three intrinsic attributes, the principal optical axis \( \mathbf{l}_0 \), the principle energy \( E_a \) along the principal axis, and the angular distribution function \( F \). \( E_a \) is the maximum radiance emitted by the light source, and its direction is defined as the principal optical axis \( \mathbf{l}_0 \). \( F \) is an angular distribution function which describes the radiance with respect to the included angle \( \theta \) and any emergent light ray direction \( \mathbf{l} \). Therefore the light source can be represented as \( L \leq E_a, \mathbf{l}_0, F > \).

According to the rotational symmetry property of light source radiance, the angular distribution function \( F \) can be illustrated by Fig. 1 and formulated as:

\[
F(\theta) = \cos^g \theta,
\]

where \( g \) is an angular attenuation factor related to the luminous property of light sources. If \( g=0 \), the light source is an ideal point light source, which indicates the radiance in all directions are the same. If \( g=1 \), the light source is a Lambertian one, which indicates the distribution of radiance obey cosine law. The parameter of \( g \) is usually not provided with the product specification of light source. To calculate \( g \), we usually use another attribute named half angle \( \theta_{half} \), which is the included angle between \( \mathbf{l}_{half} \) and the principal optical axis \( \mathbf{l}_0 \). Only considering the angular attenuation, the radiant energy along direction \( \mathbf{l}_{half} \) is always half of that on \( \mathbf{l}_0 \), i.e. \( \cos^g \theta_{half} = 0.5 \).

Therefore, once the half angle \( \theta_{half} \) is known, the parameter of \( g \) can be calculated as:

\[
g = \frac{\ln(0.5)}{\ln(\cos(\theta_{half}))}.
\]

The half angle is an inherent attribute for any LED lamps, and it can be obtained from the product specification.

Under the near-field lighting condition, the incident light at any surface point is mainly attenuated by two factors, i.e. the angle \( \theta \) of emergent light ray reaches at this point and the distance from the point to light source. According to (1), energy of the emergent light with angle \( \theta \) can be expressed as:

\[
E_\theta = E_a \cdot \cos^g \theta.
\]

B. Distance Attenuation of Emergent Light Rays

Distance attenuation is more easily to understand, which means the energy of the emergent light rays will decrease with the increasing of transmission distance. The inverse-square law is a typical distance attenuation way for ideal point light source. For general situations, we suppose to use an exponential factor of distance to express the distance attenuation effect as:

\[
E = E_\theta / d^\sigma,
\]

where \( E \) is emergent light energy along direction \( \mathbf{l} \), and \( d \) is the distance from the light source to the surface point, \( \sigma \) is the distance attenuation factor and \( \sigma=2 \) indicates the distance attenuation obeys inverse-square law. For any surface point, its incident light energy can be expressed as:

\[
E = E_a / d^\sigma.
\]

To simplify subsequent formula deduction, the surface reflectance model is assumed as Lambertian. Thus, for any surface point, its image intensity \( I \) can be expressed as:

\[
F(\theta) = \cos^g \theta,\]

\[
g = \frac{\ln(0.5)}{\ln(\cos(\theta_{half}))}.
\]
\[ I = E_p \sum_{i=1}^{n} I_i, \] 

where \( I \) is the incident light direction and \( n \) is the surface normal at any surface point \( P \) as shown by Fig. 2. By substituting (1) to (6), for any surface point under the illumination of nonisotropic point light source, its lighting condition can be expressed as:

\[ I = E_n \cos \theta \frac{d}{\pi} - n \cdot I. \] 

### III. LIGHT MODEL OPTIMIZATION AND SURFACE NORMAL CALCULATION

In the proposed light model, for each surface point \( P \), its incident light ray can be expressed as \( L_r = E_r \cdot l \), where \( E_r \) refers to the incident light intensity, and \( l \) indicates the incident light direction. To calculate \( E_r \) and \( l \), we have to know the relative position and distance between \( P \) and the light source \( L \). Obviously, this information is not available. Otherwise, it is unnecessary to do the 3D reconstruction. To realize accurate normal estimation of any surface point, an iterative approach is introduced as follows.

#### A. Initial Estimation of Surface Normal

Assume the target surface as a plane \( \Pi \subset \Delta, I_{\text{inc}} > \), which is perpendicular with the camera optical axis \( I_{\text{inc}} \), and \( D \) is its distance to the camera optical center \( O \). The camera is defined as the world coordinate frame. Consider surface point \( P(x, y, z) \) is illuminated by the \( i \)-th light source \( L_i(x'_i, y'_i, z'_i) \) with principal axis \( l'_i \) and the principle energy of \( E'_i \), its incident light direction \( l_i \) can be computed as:

\[ l_i = [x - x'_i, y - y'_i, z - z'_i]^T, \]

where the parameters of light source position \( L_i(x'_i, y'_i, z'_i) \) and the principle axis \( l'_i \) can be obtained via the calibration method as described in [13].

Since the target surface is assumed planar and perpendicular with the camera optical axis, with the pinhole model assumption, we have \( P(x, y, z) = (u, v, f) \cdot D / f \), where \( f \) refers to the camera focal length. By writing the distance from point \( P \) to the light source as \( d'_i = |l_i|^2 \), the incident light energy at \( P \) can be expressed as:

\[ E'_i = E_0 \frac{l'_i \cdot l'_i}{d'_i}. \]

As mentioned above, the lighting condition at point \( P \) can be expressed as:

\[ I'_r = E'_0 \frac{l'_i \cdot l_i}{d'_i} \cdot l_i, \]

where \( i \) indicates the index of the light source and camera image, \( E'_0 \) refers to the principal energy of the \( i \)-th light source.

Assume there are \( m \) light sources, for each image point \( P \), we have the expressions: \( L_s = [1, I_1, \cdots, I_m] \) and \( I_s = [I_1, I_2, \cdots, I_m] \). According to (3), by assuming the surface reflectance as Lambertian, surface normal of point \( P \) can be calculated as:

\[ n_p = L_p^{-1} I_p. \]

With the calculated surface normal for all image points, the classical Frankot-Chellappa algorithm [11] is employed to reconstruct the initial surface shape.

#### B. Iterative Process for Light Model Optimization

While the target surface is relative flat, accurate surface normal can be obtained with above calculations. However, the depth change on the surface makes the calculation inaccurate. In this section, an iterative process is introduced to refine the calculation of surface normal with respect to free-form surface.

Based on the initially reconstructed 3D surface model, relative depth \( w \) for each image point can be expressed by a set of integral Fourier basis functions \( F \) as:

\[ w = \mathbf{P}^{-1} \left( - f \cdot \frac{\xi^2 \cdot f (\xi) + \eta^2 \cdot f (\eta)}{\xi^2 + \eta^2} \right). \]

On the image sensor plane, the corresponding image point of surface point \( P \) can be represented as \( p(u, v, w) \). Since the depth \( w \) recovered by the FC algorithm has no metric meaning, to use it in the consequent iterative procedure, it should be transferred to the world coordinate with real metric firstly.

As shown by Fig. 3, a mean plane \( \Pi_{\text{mean}}(\overline{w}, \overline{1}_{\text{mean}}) \) and a reference plane \( \Pi_{\text{ref}}(\overline{w}, \overline{1}_{\text{ref}}) \) are defined in the camera coordinate system. For any image point \( p \), its integrated surface depth value along \( 1_{\text{mean}} \) can be expressed as \( \Delta w_p = w_p - \overline{w} \). According to the pinhole model, a scale factor can be used to relate \( \Delta w_p \) and \( \Delta z \) as:

\[ k_a = \frac{f}{z} \frac{\Delta w}{\Delta z}. \]

Fig. 3. A reference plane and a mean plane are defined in the camera coordinate system, which are used to transform the integrated surface depth value to real metric depth approximately.
where \( z = \Delta z + D \). So the value of \( z \) can be approximately calculated as:

\[
z = \frac{f \cdot D}{f - \Delta w}.
\]

\[(13)\]

With above calculation, surface depth obtained by FC algorithm can be approximately transferred to the metric space. By the usage of updated surface coordinate \((x, y, z)\), the lighting condition \(L_p\) at \(P\) can be updated accordingly. And then, a new set of surface normal can be obtained. In addition, to eliminate the residuals and increase the rate of convergence, the updating manner of \(w\) can be defined as:

\[
w_t = 2w_{t-1} - w_{t-2}, t > 2,
\]

\[(14)\]

where \(t\) indicates the iteration number. The iteration continues until the Euclidean distance between two adjacent reconstruction results satisfy a given threshold.

IV. EXPERIMENTS AND DISCUSSIONS

The experimental setup consists of one camera and 6 LED lamps as shown by Fig. 4. To reduce affect from ambient lights, infrared LED is adopted in our system. According to the product specification, half angle \(\theta_{\text{max}}\) of the LED is 60\(^\circ\). The LEDs are mounted circularly around the camera with a radius of about 150 mm. The adopted camera has a resolution of 1280\times1024 pixel and a maximum speed of 120 fps with USB3.0 interface. Focal length of the camera lens is 10 mm, and a narrow band-pass filter with 850 nm wavelength is attached on the lens. An I/O controller is developed to synchronize the camera and each LED. Exposure time of the camera is set to 3 ms, and 6 images can be captured within 0.1s. In addition, the LED lamp is attached on a metal plant, which can be freely rotated. All the LED lamps are adjusted to point to the target center and then fixed. The target is placed with a distance of about 400 mm to the camera.

![Fig. 4. The proposed photometric stereo setup is composed with six infrared LEDs, one camera and one controller that used to synchronize the camera and each LED lamp. It takes less than 0.1s to capture six images.](image)

To evaluate the proposed light model quantitatively, the reference plane is also used in this experiment. By capturing six images of the reference plane with illumination from six light sources sequentially, different light models are applied for the surface normal calculation and comparison. The parallel light model (PLM), where neither angular and distance attenuation factors are considered, i.e. \(g=0, \sigma=0\). In this model, all the surface points are assumed with the same incident light direction and intensity. The ideal point light model (IPLM), where the emergent light energy is only related to the distance factor, i.e. \(g=0, \sigma=2\). In this model, for each surface point, its incident light direction is calculated separately, and its incident light intensity is calculated with distance attenuation. The proposed quasi-point light model (QPLM), where both angular and distance attenuation factors are counted, i.e. \(g=1, \sigma=2\).

The reconstructed images of surface normal map as well as the 3D shapes by various light models are displayed in Fig. 5. Subject to the inaccurate light models, the planes reconstructed

![Fig. 5. Reconstruction results of the reference plane by various light models. From left to right is: image of surface normal map, front view of the 3D model, and side view of the 3D model.](image)

<table>
<thead>
<tr>
<th>Light Model</th>
<th>Parameters</th>
<th>Residual (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLM</td>
<td>(g=0, \sigma=0)</td>
<td>17.2516</td>
</tr>
<tr>
<td>IPSM</td>
<td>(g=0, \sigma=2)</td>
<td>13.7292</td>
</tr>
<tr>
<td>QPLM</td>
<td>(g=1, \sigma=2)</td>
<td>1.1810</td>
</tr>
</tbody>
</table>

TABLE 1. ACCURACY EVALUATION OF VARIOUS LIGHT MODELS.
by PLM and IPLM are with huge distortions. In comparison, more accurate surface can be obtained by the proposed model. Table 1 shows the statistic results of the surface normal by various methods. An absolute mean error of 1.18° with a std. of only 0.018° can be obtained by the QPLM model. With this experiment, high accuracy of the proposed nonisotropic light model can be demonstrated.

B. Reconstruction of Free-form Surfaces

In this experiment, more objects with free-form surfaces and various reflectance properties are adopted. The experimental targets include an oil painting, an adornment and real human face. For the experiments with the oil painting and the adornment, different light models are also applied for the comparison. Fig. 6 shows the experiment with oil painting, which has a flat surface but with much more surface details. The reconstructed surface normal images by the model of PLM, IPLM and QPLM are as shown from left to right. In comparison, the image of surface normal map produced by the proposed model is more homogeneous. Fig. 7 shows the experiment with an adornment. All the mentioned four light models are applied for the comparison. The calculated images of surface normal map and the reconstructed 3D models are provided for visual comparison. By observation, reconstructed shapes by the PLM and IPLM are globally distorted. In comparison, both global and local shapes can be well reconstructed by the proposed light model. A real human face with closed eyes is also experimented as shown by Fig. 8. And the results show that, accurate 3D models with satisfied local details can be reconstructed by the proposed method.

V. Conclusion and Future Work

This paper presents a novel photometric stereo method which adopts the nonisotropic LED lamps as light sources. Subject to the non-uniform lighting condition produced by the nonisotropic light source, conventional photometric stereo light models are incapable to obtain precise surface normal values. To realize an accurate photometric stereo procedure, the radiance property of the light source is firstly introduced to the photometric stereo framework. With the light source radiance model and considering both distance and angular attenuation factors, lighting condition at target surface point can be precisely described. Based on the initial 3D reconstruction result, an iterative process is proposed for the optimization of system parameters with respect to the unknown distance values. And finally, precise 3D reconstruction result can be obtained. In the experiments, accuracy of the reconstructed surface normal values are first evaluated and compared with some conventional light models. Some free-form objects including real human face are used for the visual comparison. The experimental results show that, more accurate surface normal and higher quality 3D reconstructed models can be obtained by the proposed method. Future work can address how to further improve the model precision, especially for the marginal image areas where the incident light rays with huge attenuations.
ACKNOWLEDGMENT
This work was supported in part by the National Natural Science Foundation of China (No. 61375041), Shenzhen Science Plan (Nos. JCYJ20130402113127502, JCYJ20140509174140685, JCYJ20150401150223645 and JSGG20150925164740726), DongGuan IUR Project (no. 201550913400193) and CAS Key Laboratory of Human-Machine Intelligence-Synergy Systems.

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