COMPUTER VISION
Multi-view Geometry

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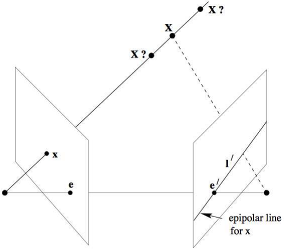
Triangulation - the building block of 3D reprojections

We have the pose $\mathbf{R}, \mathbf{t}'$ between cameras and the projection locations $\mathbf{x}, \mathbf{x}'$. What now ?



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- There are other algorithms which are more accurate, but costlier

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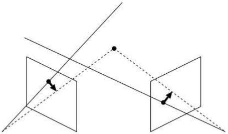
- The linear approach is reasonably good, and it is effective especially if used as an initialization for a nonlinear refinement (as we will see in the following slides)

Triangulation - how to use multiple views

If we have multiple views, the unknown $\mathbf{X}_j$ may be constrained by multiple observations $\mathbf{z}_{j,\tau}$ from cameras C_τ characterized by some pose parametrization $\mathbf{s}_\tau$. How to use them effectively together ?

Nonlinear optimization



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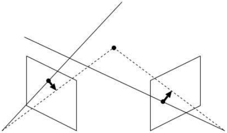

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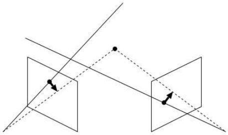

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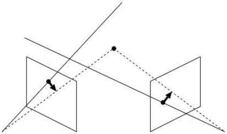
- More than one 3D point may be refined, but in this way the optimizations are decoupled



Pose estimation - how to use multiple views

Opposite problem : we have a set of 3D points $\mathbf{X_j}$ (computed previously) which are visible from camera C_τ. Based on current observations $\mathbf{z}_{j,\tau}$ from C_τ we would like to estimate its pose $\mathbf{s}_\tau$.

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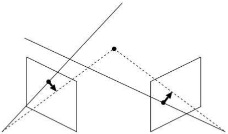

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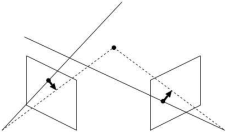

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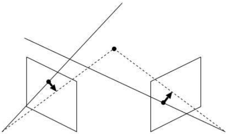

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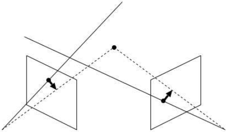

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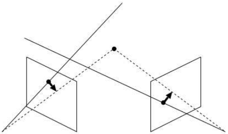
  - if the camera is moving, predict the current location based on its previous trajectory
  - from the projection of three 3D points in space and their projections, one may compute the camera pose in a closed form (the P3P problem)



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- for triangulation : we assume that the pose is correctly estimated
- for pose estimation : we assume that the 3D locations are accurate
- in reality all estimations we perform are noisy
- if we also apply the process iteratively (triangulation, pose estimation and repeat) the errors will be amplified (drift)

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- observation : this step does not modify $\mathbf{X}$
- the interest of the initial step is just to provide a quality initialization for $\mathbf{s}_\tau$ as $\hat{\mathbf{s}}_t$

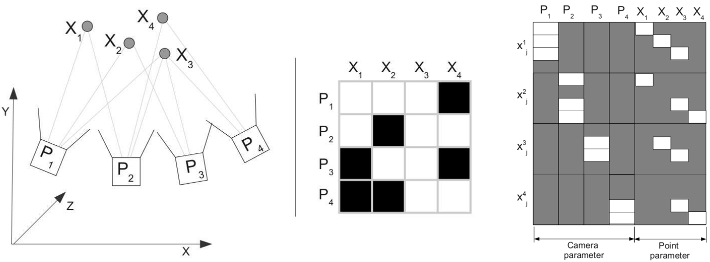
Global optimization - final step

We compute the MAP (Maximum A Posteriori) for the maximum amount of preliminary estimations and observations that we have at that moment (brutal, massive optimization). The solution we search this time is provided by :

$$\tilde{\mathbf{S}}_{0:t}, \tilde{\mathbf{X}} = \underset{\mathbf{S}_{0:t}, \mathbf{X}}{\arg\min} \sum_{\tau=0}^{T} \sum_{j=1}^{M} e(\mathbf{s}_\tau, \mathbf{X}_{j,\tau}, z_{j,\tau})^T \; e(\mathbf{s}_\tau, \mathbf{X}_{j,\tau}, z_{j,\tau})$$

The complexity of this algorithm, once we exploit the sparseness of its Jacobian : $O(T^3 + MT^2)$, which is very interesting since $M \gg T$.

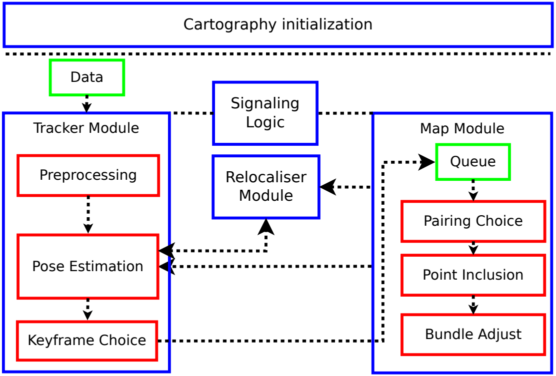
Towards real time reconstruction



An example of configuration : 5207 3D points, 54 poses, 24609 projections, 15945 variables, 21 it., 7.99 sec.
Not fast enough !

- Selection of key-frames
- Parallel execution of tracking et BA (initial and final steps)
- Limit the number of iterations (when needed)
- Local Bundle Adjustment

Typical architecture for RT optimization



Appendix - nonlinear optimization