FINAL TEST - ANALYTICAL MECHANICS - OCTOBER, 19^{th} 2016

LAST NAME:

FIRST NAME:

ID NUMBER:

Instructions: *Please read the following instructions carefully*

- remember to write (in block letters) your last name, first name and the ID number given to you by UniPV;
- note that neither calculators nor notes or books are allowed during the test;
- if anything is unclear in the description of the test problems, please ask the instructors;

- in the test problems, questions marked with the symbol ♣ are intended for *extra bonus*: these are not mandatory and you can get the maximum grade even by answering correctly and in full to all other questions, i.e. without the symbol ♣;
- do not try answering the questions marked with ♣ before having completed the answers to all other questions in the same problem;
- given what above, correct answers to questions marked with ♣ will get you an extra evaluation (or a *cum laude* – if everything else is correct).

Problem 1. Consider the tensor $\mathbf{F} = \mathbf{I} - 2\mathbf{e}_1 \otimes \mathbf{e}_2 + 3\mathbf{e}_2 \otimes \mathbf{e}_3$.

(1) Compute $\det \mathbf{F}$ and $\operatorname{tr} \mathbf{F}$.

(2) What is the image under $\mathbf{F}$ of the vector $\mathbf{u} := -2\mathbf{e}_1 + 3\mathbf{e}_2 + \mathbf{e}_3$?

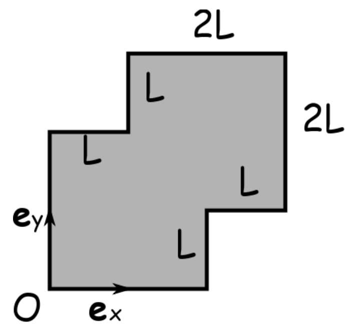
(3) Decompose $\mathbf{F}$ into its symmetric part $\operatorname{sym} \mathbf{F}$ and its skew-symmetric part $\operatorname{skw} \mathbf{F}$.

(4) What is the axial vector of $\operatorname{skw} \mathbf{F}$?

(5) Write the tensors $\mathbf{F}^{-1}$ and $\mathbf{F}^*$.

♣ (6) What is the area dilation factor of $\mathbf{F}$ for surfaces orthogonal to the vector $\mathbf{e}_1$?

Problem 2. Consider the following body (in grey) in $\mathbb{R}^3$ as having uniform mass density ρ:



(1) Write the vector position $C - O$ of the center of mass.

(2) Write a principal basis for the central tensor of inertia $\mathbf{I}_C$ (that is: a basis of $\mathbb{R}^3$, made of principal versors of the tensor of inertia at C).

(3) Write the central tensor of inertia $\mathbf{I}_C$ and verify that $\mathrm{tr}\, \mathbf{I}_C = \dfrac{55}{3}\rho L^4$.

♣ (4) Write the tensor of inertia $\mathbf{I}_O$.

Problem 3. Consider the curve defined as follows:

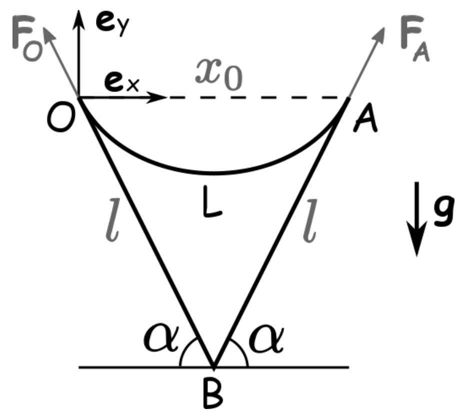
$$P(t) := (\cos(\cosh(t)), \sin(\cosh(t)), t) \qquad t \in \mathbb{R}$$

(1) Write the arc-length parameter s of the curve.

(2) Write the arc-length parametrization of the curve.

(3) Compute the curvature $c(s)$ of the curve.

Problem 4. A cable with length L and linear mass density λ is hung between two points O and A by means of two poles of equal length hinged at B (Note: *consider the two poles as being totally rigid and with negligible mass*).



Assuming that the only distributed force acting on the cable is due to the gravitational acceleration $\mathbf{g} = -g\mathbf{e}_y$ is, we want to find an equilibrium solution such that:

- the two poles make a given angle α with the ground;
- the cable is tangent to each pole in O and A, respectively, so that the force $\mathbf{F}_O$ exerted on the cable at O is parallel to OB and the force $\mathbf{F}_A$ exerted on the cable at A is parallel to AB.

(1) Let $y(x)$ be the function that describes the shape of the cable. What is the condition for $\dot{y}(0)$, at equilibrium?

(2) Write the distance x_0 between point O and point A as a function of the given data α, λ and L and of the constant K of the model.

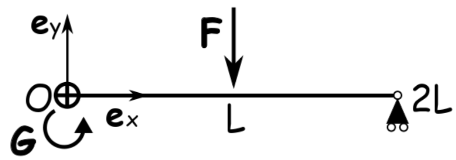
(3) Write the constant K as a function of the given data α, λ and L.

(4) Write the function $y(x)$.

♣ (4) What should it be the length l of the two poles OB and AB?.

♣ (5) What is the norm $|\mathbf{F}_O|$ of the force $\mathbf{F}_O$?

Problem 5. A uniformly elastic beam with negligible mass of length $2L$ is hinged at one end and supported by a roller at the other end. Assume $EI_2 = B$ is known. A force $\mathbf{F} = -4F\mathbf{e}_y$ (with $F > 0$) is exerted at the midpoint of the beam and a couple $\mathbf{G} = 2FL\mathbf{e}_z$ is exerted on the hinged end O:



(Recall: *the hinge exerts a reactive force that can have any direction in the plane* span$(\mathbf{e}_x, \mathbf{e}_y)$ *and the roller exerts a vertical reactive force only*).

(1) Let $P(s) = (x(s), y(s))$ be the curve that describes the shape of the beam at equilibrium, $\boldsymbol{\varphi}(s)$ and $\boldsymbol{\gamma}(s)$ be the stress force and couple at equilibrium and $\vartheta(s)$ be the angle between $\mathbf{e}_x$ and the tangent to $P(s)$ at equilibrium. What are the boundary conditions for $y(s)$, $\boldsymbol{\varphi}(s)$, $\boldsymbol{\gamma}(s)$ and $\vartheta'(s)$ at the ends $s = 0$ and $s = 2L$?

(2) Write the balance equations for forces and torques at equilibrium at the middle point $s = L$.

(3) What are the reactive forces $\mathbf{F}_O$ and $\mathbf{F}_{2L}$ exerted on the beam at equilibrium, respectively, by the hinge (at $s = 0$) and by the roller support (at $s = 2L$) (Hint: *use the total balance of forces acting on the beam and the total balance of torques at* $s = 0$).

(4) Write the function $y(x)$ which represents the shape of the beam at equilibrium, in case of small deflections.

♣ (5) What is the maximum slope $\vartheta_{\max} := \max\{|\vartheta(x)|\}$ of the beam, and for which values of x is this attained?

♣ (6) What is the condition on the value of F that makes the small deflections approximation viable?