Plausible Reasoning

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Plausible (defeasible) reasoning

Why plausible reasoning?

Consider a generic entailment problem $\Gamma \models \varphi$?

Four possible answers:

1. $\Gamma \models \varphi$
   $\Gamma \not\models \neg\varphi$
2. $\Gamma \not\models \varphi$
   $\Gamma \models \neg\varphi$
3. $\Gamma \models \varphi$
   $\Gamma \models \neg\varphi$
   This case occurs only when Γ is contradictory, i.e. unsatisfiable
4. $\Gamma \not\models \varphi$
   $\Gamma \not\models \neg\varphi$

Case 4. is quite frequent: "our knowledge Γ does not allow deciding about φ "

Plausible (defeasible) reasoning

A reasoning process where the **relation** between formulae
is <u>rationally plausible</u> yet not necessarily <u>correct</u> (in the classical logical sense)

i.e. a specific reasoning method

Notation:
$\Gamma \mathrel{|\!\sim}_{<SysLog>} \varphi$ says that φ is a **plausible** derivation from Γ in $<SysLog>$

Properties of $\mathrel{|\!\sim}_{<SysLog>}$

$\Gamma \mathrel{|\!\sim}_{<SysLog>} \varphi \Rightarrow \Gamma \mathrel{\not|\!\sim}_{<SysLog>} \neg\varphi$ (coherence)

$\Gamma \vdash_{<SysLog>} \varphi \Rightarrow \Gamma \mathrel{|\!\sim}_{<SysLog>} \varphi$ (compatibility with derivation)

$\Gamma \mathrel{|\!\sim}_{<SysLog>} \varphi \nRightarrow \Gamma \vdash_{<SysLog>} \varphi \; (\Rightarrow \Gamma \models \varphi)$ (not necessarily correct)

It occurs very often in practice:

“The train schedule does not report a train to Milano at 06:55,
therefore we assume that such a train does <u>not</u> exist”

Most databases contain positive information only
Negative facts are typically derived ‘by default’

Closed-World Assumption (CWA)

$\{\Gamma \not\models \alpha\} \mathrel{|\!\!\sim}_{CWA} \neg\alpha$ (α is an *atom*)

Example (a program):

$\Pi \equiv \{\{Philosopher(socrates)\}, \{Philosopher(plato)\}, \{Cat(felix)\}\}$

The program Π can be rewritten in L_{FO} as:

$\forall x\ ((x = socrates) \rightarrow Philosopher\ (x))$
$\forall x\ ((x = plato) \rightarrow Philosopher\ (x))$
$\forall x\ ((x = felix) \rightarrow Cat(x))$

The *Closed-World Assumption* (*CWA)* means completing (i.e. extending) the program Π:

$\forall x\ ((x = felix) \leftrightarrow Cat(x))$
$\forall x\ ((x = socrates \vee x = plato) \leftrightarrow Philosopher\ (x))$ *Notice the double implication*

Then these plausible inferences become sound:

$\Pi \mathrel{|\!\!\sim}_{CWA} \neg Cat(socrates)$
$\Pi \mathrel{|\!\!\sim}_{CWA} \neg Cat(plato)$
$\Pi \mathrel{|\!\!\sim}_{CWA} \neg Philosopher\ (felix)$

Plausible (defeasible) reasoning

- Inference in *defeasible reasoning* is

Non-monotonic

$$\Gamma \mid\!\sim_{<SysLog>} \varphi \;\not\Rightarrow\; \Gamma \cup \Delta \mid\!\sim_{<SysLog>} \varphi$$

The arrival of new pieces of information may falsify inferences that used to be justified

e.g. an extra train to Milano at 06:55 is announced …

Systemic

In classical logic, the soundness of all inferences schema depend only on the few formulae involved.

e.g. $\varphi \to \psi, \varphi \vdash \psi$

In defeasible reasoning, inferences are justified by an entire theory Γ

One must check the entire database (see CWA): $\Gamma \nvdash \varphi \quad \mid\!\sim_{<SysLog>} \neg\varphi$

Inference and reasoning (according to C. S. Peirce, 1870 c.a.)

- Different types of reasoning

__Deductive__ inference (*sound*)

Derive only what is justified in terms of ***entailment***

"All beans in this bag are white"
"This handful of beans comes from this bag"
"This is a handful of white beans"

$$\frac{\forall x\, \varphi(x) \rightarrow \psi(x) \qquad \varphi(a)}{\psi(a)}$$

__Inductive__ inference (*plausible*)

From repeated occurrences, derive rules

"This handful of beans comes from this bag"
"This is a handful of white beans"
"All beans in this bag are white"

$$\frac{\psi(a) \qquad \varphi(a)}{\forall x\, \varphi(x) \rightarrow \psi(x)}$$

__Abductive__ inference (*plausible*)

From rules and outcomes, derive premises

"All beans in this bag are white"
"This is a handful of white beans"
"This handful of beans comes from this bag"

$$\frac{\forall x\, \varphi(x) \rightarrow \psi(x) \qquad \psi(a)}{\varphi(a)}$$