*Symbolic representation and inference:
the intuitive idea*

Marco Piastra

- Solving quadratic equations

$$x^2 + ax + b = 0$$
$$x^2 + 2(a/2)x + a^2/4 - a^2/4 + b = 0$$
$$(x + a/2)^2 - a^2/4 + b = 0$$
$$(x + a/2)^2 = a^2/4 - b$$
$$x = -a/2 \pm (a^2/4 - b)^{1/2}$$

A sequence of steps: at each step a *transformation rule* is applied

An aside: *solving equations*

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  A sequence of steps: at each step a *transformation rule* is applied

  From start to end

  Start point: a premise (*i.e. we assume the truth of something*)

  End point: a conclusion (*i.e. we state that something else must <u>also</u> be true*)

  (*both points are decided by us*)

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  (*both points are decided by us*)

  Sequence of steps

  Each transformation rule is based on a *semantic equivalence*

  Each step must be *correct*, in the above sense

Symbolic Reasoning

Symbolic descriptions can *abstract*

- Symbolic descriptions have *abstraction* capabilities:
  for instance, many linguistic phenomena are *systemic*
  (i.e. their structural complexity goes beyond a *pattern-matching*)

  Someone may understand English or not.

  But no human being can understand the sentence:
    *"Sally **likes** Harry"*

  without being able to understand:
    *"Harry **likes** Sally"*

  or any other sentences of the kind:
    "X **likes** Y"

  where X and Y can be any nouns or definite descriptions:
    *"Ronald's girlfriend **likes** the cat that Linda brought home yesterday"*

    (freely adapted from [Fodor e Phylyshyn, 1988])

Reasoning and schemas: *syllogism*

Many (intuitively) valid arguments obey an abstract schema

All (*humans*) are (*mortals*)
All (*Greeks*) are (*humans*)   hence
_____
All (*Greeks*) are (*mortals*)

Abstract schema:

All *H* are *M*
All *G* are *H*   hence
_____
All *G* are *M*

Reasoning and schemas: *syllogism*

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All (*Greeks*) are (*mortals*)

Abstract schema:

All **H** are **M**   hence
All **G** are **H**
All **G** are **M**

CAUTION!
The position of the line IS relevant:
in this case the schema does <u>not</u> work

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All (*Greeks*) are (*mortals*)   hence
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Abstract schema:

All **H** are **M**
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All **G** are **H**

CAUTION!
The ordering of sentences IS relevant:
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Reasoning and schemas: *syllogism*

Many (intuitively) valid arguments obey an abstract schema

> All (*humans*) are (*mortals*)
> All (*Greeks*) are (*humans*)   hence
> _____
> All (*Greeks*) are (*mortals*)

Abstract schema:

> All **H** are **M**
> All **G** are **H**   hence
> _____
> All **G** are **M**

The validity of schemas does not depend on *meaning*:

> All (*enchanted frogs*) are (*princes*)
> All (*princes*) are (*young and beautiful*)   hence
> _____
> All (*enchanted frogs*) are (*young and beautiful*)

*Same schema,
different impression*

Fallacies (*paralogisms*)

Wrong sequence:

All (*humans*) are (*mortals*)
All (*Greeks*) are (*mortals*)
—————————————————
All (*Greeks*) are (*humans*)

The last two sentences have been switched

Referential ambiguities:

(*Nothing*) is better than (*eternal happiness*)
(*One ham sandwich*) is better than (*nothing*)
—————————————————————————
(*One ham sandwich*) is better than (*eternal happiness*)

'Obscure' subtleties (obscure for now):

All (*enchanted frogs*) are (*princes*)
All (*enchanted frogs*) are (*young and beautiful*)
There is an (*enchanted frog*) which is (*a young and beautiful princes*)

What do we mean by "all"? Do we need at least one specimen to say "all"?

In such case, the schema would be valid
(*in formal logic it is not, as we will see*)

What is the purpose of *symbolic* logic?

*To distinguish correct reasoning
from incorrect reasoning*

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*To distinguish correct reasoning
from incorrect reasoning*

*by the **formal, symbolic** structure alone*

Truth and Possible Worlds:
the quest for formal semantics

What is *true*?

- **A world of *cats***

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot	x		x	
Kitty		x	x	
Felix			x	

- **Sentences about this world**

  "Spot likes Tom" and "Tom does not like Spot"
  "Tom likes himself"
  "Kitty likes Spot" and "Spot likes Kitty"
  "Kitty likes herself"
  "Felix likes Kitty"

  All these sentences are **true**, *in the world above*

What is *true*?

- **A world of *cats***

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot	x		x	
Kitty		x	x	
Felix			x	

- **Sentences about this world**

  "Spot likes Felix"
  "Spot likes himself"
  "Kitty does not like herself"
  "Felix likes Spot"

  *All these sentences are **false**, in the world above*

What is *true?*

- **A world of *cats***

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot	x		x	
Kitty		x	x	
Felix			x	

- **Sentences about this world** (i.e. with *generalization*)

  "Every cat likes a cat" *is **true***
  "Every cat likes another cat" *is **false***
  "Tom does not like any other cat" *is **true***
  "Kitty is liked by every cat" *is **false***

  "Every cat that likes another cat also likes her/himself" *is **?***

How to make a sentence *true?*

- **Consider the sentence**

    "Kitty likes Spot" and "Spot likes Tom"

    It can be made true in many different ways
    - "It may be true in many different *possible worlds*"

    Examples (i.e. of other *possible worlds* of cats):

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x			
Kitty		x		
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot	x		x	
Kitty		x	x	
Felix			x	

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	x
Spot	x	x	x	x
Kitty	x	x	x	x
Felix	x	x	x	x

How to make a sentence *true*?

- **Consider the sentence**

    "Kitty likes Spot" and "Kitty does <u>not</u> like Spot"

    There is no way to *make it true*
    "There is no *possible world* where this can be true"

- **Three sentences**

  1) "Every cat that likes Kitty likes Spot as well"
  2) "Tom likes Kitty"

  3) "Tom likes Spot"

  There is no way to make true sentences 1) *and* 2) *without making sentence* 3) *true as well…*

    (just give it a try…)

  *This is just a small subset of **all** possible worlds …*

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	
Kitty		x	x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot			x	
Kitty		x		
Felix				x

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	x
Kitty			x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot				
Kitty			x	
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom		x	x	
Spot	x	x	x	
Kitty	x	x		
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x		x	
Spot	x			
Kitty		x		
Felix		x		x

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	x
Spot		x		
Kitty			x	
Felix				x

likes	Tom	Spot	Kitty	Felix
Tom			x	
Spot	x			
Kitty		x		
Felix		x	x	

- **Three sentences**

    1) "Every cat that likes Kitty likes Spot as well"
    2) "Tom likes Kitty"

    3) "Tom likes Spot"

    There is no way to make true sentences 1) *and* 2) *without making sentence* 3) *true as well…*
    (just give it a try…)

Sentence 1) is **true** in these worlds

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	
Kitty		x	x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot			x	
Kitty		x		
Felix				x

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	x
Kitty			x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot				
Kitty			x	
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom		x	x	
Spot	x	x	x	
Kitty	x	x		
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x		x	
Spot	x			
Kitty		x		
Felix		x		x

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	x
Spot		x		
Kitty			x	
Felix				x

likes	Tom	Spot	Kitty	Felix
Tom			x	
Spot	x			
Kitty		x		
Felix		x	x	

- **Three sentences**

  1) "Every cat that likes Kitty likes Spot as well"
  2) "Tom likes Kitty"

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  There is no way to make true sentences 1) and 2) without making sentence 3) true as well…

  (just give it a try…)

Sentence 1) is **true** in these worlds

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	
Kitty		x	x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot			x	
Kitty		x		
Felix				x

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Tom				
Spot	x	x	x	x
Kitty			x	
Felix		x	x	

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Tom	x	x	x	
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Tom		x	x	
Spot	x	x	x	
Kitty	x	x		
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Tom	x		x	
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Felix		x		x

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	x
Spot		x		
Kitty			x	
Felix				x

likes	Tom	Spot	Kitty	Felix
Tom			x	
Spot	x			
Kitty		x		
Felix		x	x	

*Sentence 2) is **true** in these worlds*

Sentences may be related

- **Three sentences**

  1) "Every cat that likes Kitty likes Spot as well"
  2) "Tom likes Kitty"

  3) "Tom likes Spot"

  There is no way to make true sentences 1) *and* 2) *without making sentence* 3) *true as well…*
  (just give it a try)

Sentence 1) is **true** in these worlds

Sentence 3) is **true** in these worlds

Sentence 2) is **true** in these worlds

Top row

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	
Kitty		x	x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot			x	
Kitty		x		
Felix				x

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	x
Kitty			x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot				
Kitty			x	
Felix				

Bottom row

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom		x	x	
Spot	x	x	x	
Kitty	x	x		
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	
Spot	x			
Kitty		x		
Felix		x		x

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	x
Spot			x	
Kitty			x	
Felix				x

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Tom			x	
Spot	x			
Kitty		x		
Felix		x	x	

- **Three sentences**

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  There is no way to make true sentences 1) *and* 2) *without making sentence* 3) *true as well…*
  (just give it a try)

Questions:
Was this just coincidence?
How many possible worlds must we consider in order to be sure?

Sentences may be related

- **Three sentences**

    1) "Every cat that likes Kitty likes Spot as well"
    2) "Tom likes Kitty"

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    There is no way to make true sentences 1) *and* 2) *without making sentence* 3) *true as well…*
    (just give it a try)

Questions:
Was this just coincidence?
How many possible worlds must we consider in order to be sure?

Hint:
All of them.