Horn Clauses and SLD Resolution

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Horn Clauses (in L_P)

- Definition

  A ***Horn Clause*** is a wff in CF
  that contains at most <u>one</u> literal in positive form

- Three types of *Horn Clauses*:

  Rule: two or more literals, one positive

  Examples: $\{B, \neg D, \neg A, \neg C\}$, $\{A, \neg B\}$ (equivalent to: $(D \wedge A \wedge C) \to B$, $B \to A$)

  Facts: just one positive literal

  Examples: $\{B\}$, $\{A\}$

  Goal: one or more literals, all negative

  Examples: $\{\neg B\}$, $\{\neg A, \neg B\}$

  More terminology:

  Rules and facts are also called *definite clauses*

  Goals are allo called *negative clauses*

Lost in Translation…

Many wffs can be translated into Horn clauses:

$(A \land B) \to C$

$\neg(A \land B) \lor C$ (rewriting $\to$)

$\neg A \lor \neg B \lor C$ (De Morgan - CF – it is a rule)

$A \to (B \land C)$

$\neg A \lor (B \land C)$ (rewriting $\to$)

$(\neg A \lor B) \land (\neg A \lor C)$ (distributing $\lor$)

$(\neg A \lor B), (\neg A \lor C)$ (CF – <u>two</u> rules)

$(A \lor B) \to C$

$\neg(A \lor B) \lor C$ (rewriting $\to$)

$(\neg A \land \neg B) \lor C$ (De Morgan)

$(\neg A \lor C) \land (\neg B \lor C)$ (distributing $\lor$)

$(\neg A \lor C), (\neg B \lor C)$ (CF – <u>two</u> rules)

But not all of them:

$(A \land \neg B) \to C$

$\neg(A \land \neg B) \lor C$ (rewriting $\to$)

$\neg A \lor B \lor C$ (De Morgan)

$A \to (B \lor C)$

$\neg A \lor B \lor C$ (rewriting $\to$)

SLD Resolution (in L_P)

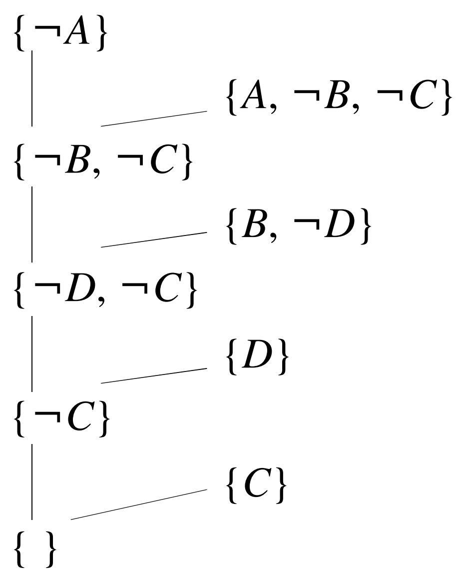
Linear resolution with Selection function for Definite clauses

- Algorithm

  Starts from a set of *definite clauses* (also the *program*) + a *goal*

  1) At each step, the *selection function* identifies a *literal* in the *goal* (i.e. *subgoal*)
  2) All *definite clause* applicable to the *subgoal* is selected
  3) The resolution rule is applied generating the *resolvent*

  Termination: either the empty clause $\{\ \}$ is obtained or step 2) fails.



Example:

Selection function: *leftmost subgoal first*

Definite clauses: $\{C\}, \{D\}, \{B, \neg D\}, \{A, \neg B, \neg C\}$

Goal: $\{\neg A\}$

SLD trees (in L_P)

SLD derivations

Example: $\{C\}, \{D\}, \{B, \neg D\}, \{A, \neg B, \neg C\}$ goal $\{\neg A\}$

In this example each subgoal can be resolved in one mode only

This is not true in general

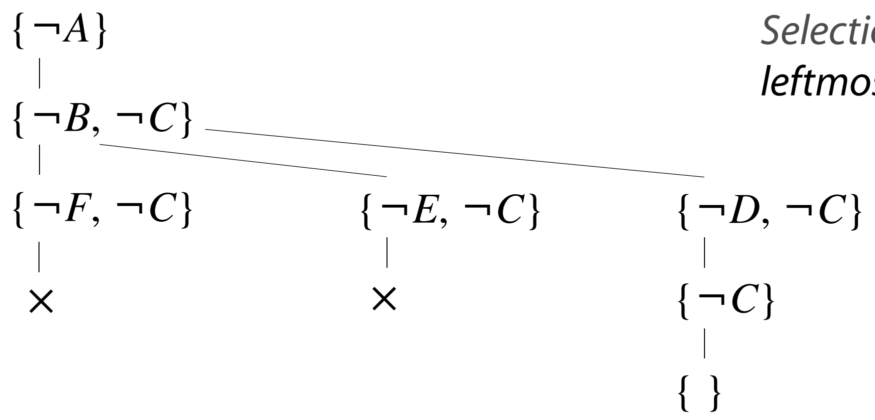
Sequence ↓

$$\{\neg A\}$$
$$\{\neg B, \neg C\}$$
$$\{\neg D, \neg C\}$$
$$\{\neg C\}$$
$$\{\ \}$$

- SLD trees (= trace of all SLD derivations from a goal)

Example: $\{C\}, \{D\}, \{B, \neg F\}, \{B, \neg E\}, \{B, \neg D\}, \{A, \neg B, \neg C\}$ goal $\{\neg A\}$

A few new rules have been added: there are now different possibilities



Selection function:
leftmost subgoal first

Each branch correspond to a possible resolution for a *subgoal*

SLD Resolution (in L_P)

- A resolution method <u>for Horn clauses</u> in L_P

    It always terminates

    It is *correct*: $\Gamma \vdash \varphi \;\Rightarrow\; \Gamma \models \varphi$

    It is *complete*: $\Gamma \models \varphi \;\Rightarrow\; \Gamma \vdash \varphi$

- Computationally efficient

  It has polynomial time complexity (w.r.t the # of propositional symbols occurring in Γ and φ)

- Limitations

  Not all problems can be translated into Horn clauses

    The “Harry is happy” problem does not translate

    Γ : only a set of *rules* and *facts*

    φ : only a conjunction of *facts*

Horn Clauses in L_{FO}

The definition is very similar to the propositional case

- Horn Clauses (of the skolemization of a set *sentences*)

  Each clause contains at most one literal in positive form

Facts, rules and goals

Fact: a clause with just an individual *atom*

$\{Human(socrates)\}$, $\{Pyramid(x)\}$, $\{Sister(sally, motherOf(paul))\}$

Rule: a clause with at least two literals, exactly one in positive form

$\{Human(x), \neg Philosopher(x)\}$,
$\forall x\ (Philospher(x) \rightarrow Human(x))$

$\{\neg Female(x), \neg Parent(k(x),x), \neg Parent(k(y),y), Sister(x,y)\}$
$\forall x \forall y\ ((Female(x) \land \exists z\ (Parent(z,x) \land Parent(z,y))) \rightarrow Sister(x,y))$

$\{\neg Above(x,y), On(x,k(x))\}$, $\{\neg Above(x,y), On(j(y),y)\}$
$\forall x \forall y\ (Above(x,y) \rightarrow (\exists z\ On(x,z) \land \exists v\ On(v,y)))$

Goal: a clause containing negative literals only

$\{\neg Human(socrates)\}$

$\{\neg Sister(sally,x), \neg Sister(x,paul)\}$
Negation of $\exists x\ (Sorella(sally,x) \land Sorella(x,paul))$

SLD Resolution in L_{FO}

Linear resolution with Selection function for Definite clauses

- Description

Program (a set of *definite clauses:* rules + facts):

Rule: $\beta \vee \neg\gamma_1 \vee \neg\gamma_2 \vee \ldots \vee \neg\gamma_n$

Fact: δ

Goal (a conjunction of facts in negated form:

Goal: $\neg\alpha_1 \vee \neg\alpha_2 \vee \ldots \vee \neg\alpha_k$

Procedure:

- Starting point: a program Π and a goal ϕ
- The subgoals are considered according to the *selection function* of choice
- For each subgoal $\neg\alpha_i$ the resolution (with unification) is attempted with all rules and facts in Π whose positive literal is compatible

SLD Trees

- Example:

$$\Pi \equiv \{\{Human(x), \neg Philosopher(x)\}, \{Mortal(y), \neg Human(y)\},$$
$$\{Philosopher(socrates)\}, \{Philosopher(plato)\}, \{Philosopher(aristotle)\}\}$$

$$goal \equiv \{\neg Mortal(x), \neg Human(x)\}$$

"Is there anyone who is both human and mortal?"

$goal\ 1$: $\neg Mortal(x)$ $[]$

$\{\neg Mortal(x)\}, \{Mortal(y_1), \neg Human(y_1),\}$ $[]$

$goal\ 2$: $\{\neg Human(y_1)\}$ $[x/y_1]$

$\{\neg Human(y_1)\}, \{Human(x_1), \neg Philosopher(x_1)\}$ $[x/y_1]$

$\{\neg Philosopher(x_1)\}$ $[x/y_1][y_1/x_1]$

$\{\neg Philosopher(x_1)\}$ $\{Philosopher(socrates)\}$ $[x/y_1][y_1/x_1]$

$\{\neg Philosopher(x_1)\}$ $\{Philosopher(plato)\}$ $[x/y_1][y_1/x_1]$

$\{\neg Philosopher(x_1)\}$ $\{Philosopher(aristotle)\}$ $[x/y_1]$ $[y_1/x_1]$

$\{\}$ $[x/y_1][y_1/x_1][x_1/socrates]$

$\{\}$ $[x/y_1][y_1/x_1][x_1/plato]$

$\{\}$ $[x/y_1][y_1/x_1][x_1/aristotle]$

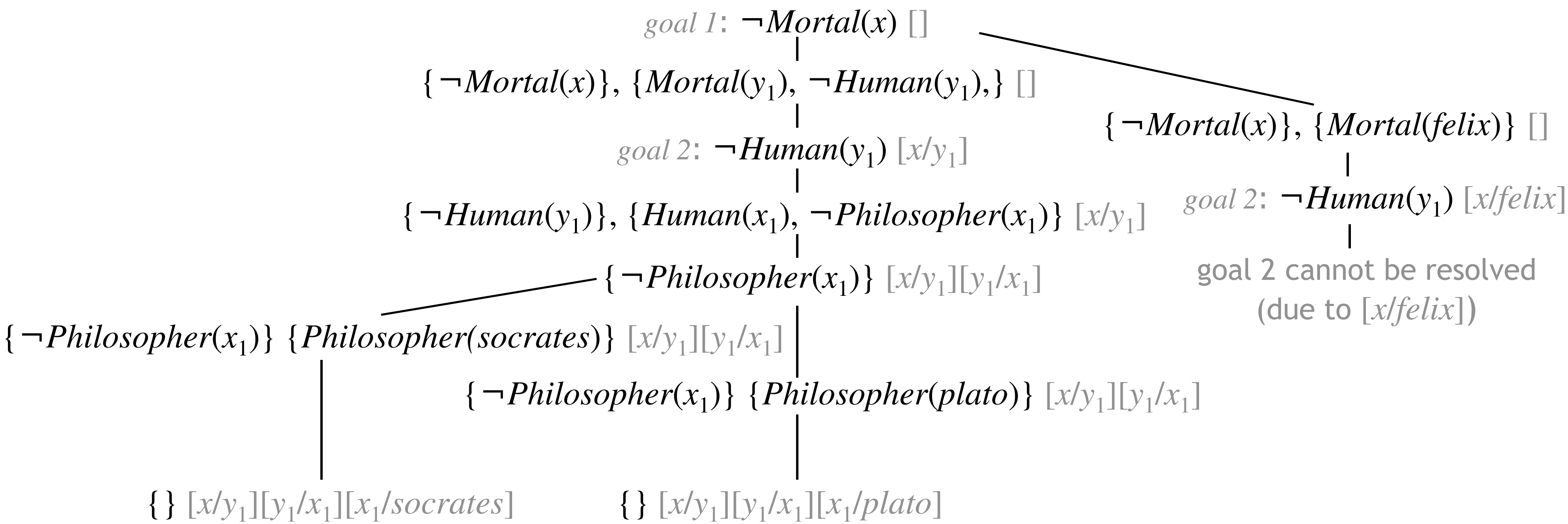
SLD Trees

- Another example

$$\Pi \equiv \{\{Human(x), \neg Philosopher(x)\}, \{Mortal(y), \neg Human(y)\},$$
$$\{Philosopher(socrates)\}, \{Philosopher(plato)\}, \{Mortal(felix)\}\}$$

$$goal \equiv \{\neg Mortal(x), \neg Human(x)\}$$

"Is there anyone who is both human and mortal?"



Infinite SLD Trees

- A first example:

$$\Pi \equiv \{\{P(x), \neg P(x)\}\}$$
$$\neg\phi \equiv \{\neg P(x)\}$$

$$\begin{array}{c}
\text{goal: } \neg P(x)\ [\,] \\
| \\
\{\neg P(x)\}, \{P(x_1), \neg P(x_1),\}\ [\,] \\
| \\
\{\neg P(x_1)\}\ [x/x_1] \\
| \\
\{\neg P(x_1)\}, \{P(x_2), \neg P(x_2),\}\ [x/x_1] \\
| \\
\{\neg P(x_2)\}\ [x/x_1]\ [x_1/x_2] \\
| \\
\ldots
\end{array}$$

Since $\Pi \not\models \phi$, the method can *diverge* (and it does…)

Infinite SLD Trees

- A second example:

$$\Pi \equiv \{\{P(x), \neg P(x)\}, \{P(a)\}\}$$
$$\neg\phi \equiv \{\neg P(x)\}$$

$$\text{goal: } \neg P(x) \; []$$

$$\{\neg P(x)\}, \{P(x_1), \neg P(x_1),\} \; []$$
$$\{\neg P(x_1)\} \; [x/x_1]$$
$$\{\neg P(x_1)\}, \{P(x_2), \neg P(x_2),\} \; [x/x_1]$$
$$\{\neg P(x_2)\} \; [x/x_1] \; [x_1/x_2]$$
$$\ldots$$

$$\{\neg P(x)\}, \{P(a)\} \; [x/a]$$
$$\{\} \; [x/a]$$

In this case $\Pi \models \phi$, so the method should *not* diverge.

However, when a *depth-first* selection function is used, the infinite branch in the SLD-tree makes the method diverge anyway.

A ***fair*** selection function is such that no possible resolution will be postponed indefinitely: that is, any possible resolution will be performed, eventually.