## Artificial Intelligence

## Formal Logic

Marco Piastra

## Symbolic descriptions can abstract

- Symbolic descriptions have abstraction capabilities: for instance, many linguistic phenomena are systemic
(i.e. their complexity goes beyond a pattern-matching)

Someone may understand English or not.
But no human being can understand the sentence:
"Sally likes Harry"
without being able to understand:
"Harry likes Sally"
or any other sentences of the kind:
"X likes Y"
where $X$ and $Y$ can be any nouns or definite descriptions:
"Ronald's girlfriend likes the cat that Linda brought home yesterday"
(freely adapted from [Fodor e Phylyshyn, 1988])

## Reasoning and schemas: syllogism

Many (intuitively) valid arguments obey an abstract schema

| All (humans) are (mortals) |
| :--- |
| All (Greeks) are (humans) |
| All (Greeks) are (mortals) |

Abstract schema:
All $H$ are $M$
All $G$ are $H$
All $G$ are $M$

## Reasoning and schemas: syllogism

Many (intuitively) valid arguments obey an abstract schema
All (humans) are (mortals)
hence
All (Greeks) are (humans)
All (Greeks) are (mortals)
Abstract schema:
All $\boldsymbol{H}$ are $\boldsymbol{M}$ hence
All $G$ are $H$
All $G$ are $M$

CAUTION!
The position of the line IS relevant: in this case the schema does not work

## Reasoning and schemas: syllogism

Many (intuitively) valid arguments obey an abstract schema

| All (humans) are (mortals) |  |
| :--- | ---: |
| All (Greeks) are (mortals) | hence |
| All (Greeks) are (humans) |  |

Abstract schema:
All $H$ are $M$
All $G$ are $M$
hence
All $G$ are $H$
CAUTION!
The ordering of sentences IS relevant: in this case the schema does not work

## Reasoning and schemas: syllogism

Many (intuitively) valid arguments obey an abstract schema

> All (humans) are (mortals)

All (Greeks) are (humans) hence
All (Greeks) are (mortals)
Abstract schema:

## All H are M

All $G$ are $H$
hence

## All $G$ are $M$

The validity of schemas does not depend on meaning:
All (enchanted frogs) are (princesses)
All (princesses) are (young and beautifu) hence
All (enchanted frogs) are (young and beautiful)
Same schema, different impression

## Fallacies (paralogisms)

```
Wrong sequence:
    All (humans) are (mortals)
    All (Greeks) are (mortals)
    All (Greeks) are (humans)
                                    The last two sentences
                                    have been switched
Referential ambiguities:
    (Nothing) is better than (eternal happiness)
    (A ham sandwich) is better than (nothing)
    (A ham sandwich) is better than (eternal happiness)
'Obscure' subtleties (obscure for now):
    All (enchanted frogs) are (princesses)
    All (enchanted frogs) are (young and beautiful)
    There is an (enchanted frog) which is (a young and beautiful princess)
```

    What do we mean by "all"? Do we need at least one specimen to say "all"?
    In such case, the schema would be valid
    (in formal logic it is not, as we will see)
    
## What is the purpose of forma/logic?

## To distinguish correct reasoning from incorrect reasoning

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To distinguish correct reasoning from incorrect reasoning

by the formal structure

## An aside: equations

- Solving quadratic equations

$$
\begin{aligned}
& x^{2}+a x+b=0 \\
& x^{2}+2(a / 2) x+a^{2} / 4-a^{2} / 4+b=0 \\
& (x+a / 2)^{2}-a^{2} / 4+b=0 \\
& (x+a / 2)^{2}=a^{2} / 4-b \\
& x=-a / 2 \pm\left(a^{2} / 4-b\right)^{1 / 2} \\
& \text { A sequence of steps: at each step a transformation rule is applied }
\end{aligned}
$$

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## From start to end

Start point: a premise (i.e. we assume the truth of something)
End point: a conclusion (i.e. we state that something else must also be true)
(both points are decided by us)

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## Sequence of steps

Each transformation rule is based on equivalence
Each step must be correct, in this particular sense

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A sequence of steps: at each step a transformation rule is applied

## From start to end

Start point: a premise (i.e. we assume the truth of something)
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## Sequence of steps

Each transformation rule is based on equivalence
Each step must be correct, in this particular sense

## Abstraction and Correctness

Symbols like $x, a$ and $b$ represent any number, equivalences are valid for all of them
What does it mean for the whole sequence to be correct?

## What is true?

- A world of cats

| likes | $\begin{aligned} & -1 \\ & 3 \end{aligned}$ | $\begin{aligned} & \text { n } \\ & 0 \\ & 0 \end{aligned}$ | 交 |  |
| :---: | :---: | :---: | :---: | :---: |
| Tom | X |  |  |  |
| Spot | X |  | X |  |
| Kitty |  | X | X |  |
| Felix |  |  | X |  |

- Sentences about this world
"Spot likes Tom" and "Tom does not like Spot"
"Tom likes himself"
"Kitty likes Spot" and "Spot likes Kitty"
"Kitty likes herself"
"Felix likes Kitty"

All these sentences are true, in the world above

## What is true?

- A world of cats

| likes | $\begin{aligned} & -1 \\ & 3 \end{aligned}$ | $\begin{aligned} & \text { n } \\ & 0 \\ & 0 \end{aligned}$ | 交 |  |
| :---: | :---: | :---: | :---: | :---: |
| Tom | X |  |  |  |
| Spot | X |  | X |  |
| Kitty |  | X | X |  |
| Felix |  |  | X |  |

- Sentences about this world
"Spot likes Felix"
"Spot likes himself"
"Kitty does not like herself"
"Felix likes Spot"

All these sentences are false, in the world above

## What is true?

- A world of cats

| likes | $\frac{1}{3}$ | $\begin{aligned} & \text { n } \\ & 0 \\ & 0 \end{aligned}$ | 京 | $\frac{\Pi}{\text { D }}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom | x |  |  |  |
| Spot | X |  | X |  |
| Kitty |  | X | X |  |
| Felix |  |  | X |  |

- Sentences about this world
"Every cat likes a cat" is true
"Every cat likes another cat" is false
"Tom does not like any other cat" is true
"Kitty is liked by every cat" is false


## How to make a sentence true?

- Consider the sentence
"Kitty likes Spot" and "Spot likes Tom"

It can be made true in many different ways
"It may be true in many different possible worlds"
Examples:

| likes | $\frac{1}{3}$ | $\begin{aligned} & n \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | 잢 | $\frac{T}{\bar{D}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom |  |  |  |  |
| Spot | X |  |  |  |
| Kitty |  | X |  |  |
| Felix |  |  |  |  |


| likes | $\frac{-1}{3}$ | $\begin{aligned} & 000 \\ & 0 \\ & \hline \end{aligned}$ | 交 | $\frac{\Pi}{\text { D }}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom | X |  |  |  |
| Spot | X |  | X |  |
| Kitty |  | X | X |  |
| Felix |  |  | X |  |


| likes | $\frac{-1}{3}$ | $\begin{aligned} & \text { n } \\ & 0 \\ & \hline \end{aligned}$ | 交 | $\frac{T}{\bar{x}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom | X | X | X | x |
| Spot | x | X | X | X |
| Kitty | X | X | X | X |
| Felix | X | X | X | X |

## How to make a sentence true?

- Consider the sentence
"Kitty likes Spot" and "Kitty does not like Spot"

There is no way to make it true
"There is no possible world where this can be true"

## Sentences may be related

- Three sentences

1) "Every cat that likes Kitty likes Spot as well"
2) "Tom likes Kitty"
3) "Tom likes Spot"

There is no way to make true sentences 1) and 2) without making true sentence 3) as well... (just give it a try...)

| likes | $\begin{aligned} & -1 \\ & \frac{-1}{3} \end{aligned}$ | \% | $\begin{aligned} & \text { 즟 } \\ & \hline \end{aligned}$ | $\frac{\Gamma}{\bar{\top}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom |  |  |  |  |
| Spot | x |  |  |  |
| Kitty |  | x |  |  |
| Felix |  | x | x |  |


| likes | $\begin{aligned} & -1 \\ & 3 \end{aligned}$ | $\begin{aligned} & \text { n } \\ & \hline \end{aligned}$ |  | $\stackrel{\text { D }}{\text { D }}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom |  |  |  |  |
| Spot | x | x | x |  |
| Kitty |  | x | x |  |
| Felix |  | x | x |  |



| likes | $\stackrel{-1}{3}$ | O | $\begin{aligned} & \text { 즟 } \\ & \text { 俍 } \end{aligned}$ | $\frac{\Gamma}{\bar{\top}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom |  |  |  |  |
| Spot |  |  |  |  |
| Kitty |  |  | x |  |
| Felix |  |  |  |  |


| likes | $\begin{aligned} & -7 \\ & \frac{-1}{3} \end{aligned}$ | $\begin{aligned} & \text { n } \\ & \hline 0 \end{aligned}$ | 즈₹ | $\frac{\Gamma}{\bar{D}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom | x | x | x |  |
| Spot | x |  |  |  |
| Kitty |  | x |  |  |
| Felix |  | x | x |  |


| likes | $\begin{aligned} & -1 \\ & 3 \end{aligned}$ | $\stackrel{\sim}{0}$ | 잧 | $\frac{\text { D }}{\text { N }}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom |  | x | x |  |
| Spot | x | x | x |  |
| Kitty | x | x |  |  |
| Felix |  |  |  |  |


| likes | $\begin{aligned} & -1 \\ & \frac{-1}{3} \end{aligned}$ | O | 잧 | $\frac{\pi}{\bar{x}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom | x |  | x |  |
| Spot | x |  |  |  |
| Kitty |  | x |  |  |
| Felix |  | x |  | x |


| likes | $\stackrel{-1}{3}$ | $\begin{aligned} & 0 \\ & 0 \\ & \hline \end{aligned}$ | 잧 | $\frac{\Gamma}{\frac{D}{X}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom |  |  | x |  |
| Spot | x |  |  |  |
| Kitty |  | x |  |  |
| Felix |  | x | x |  |


| likes | $\begin{aligned} & -7 \\ & 3 \end{aligned}$ | $$ | 즟 | $\frac{\text { D }}{\text { D }}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom | X | x | x | x |
| Spot |  | x |  |  |
| Kitty |  |  | x |  |
| Felix |  |  |  | x |

## Sentences may be related

- Three sentences

1) "Every cat that likes Kitty likes Spot as well"
2) "Tom likes Kitty"
3) "Tom likes Spot"

There is no way to make true sentences 1) and 2) without making true sentence 3) as well... (just give it a try...)

Sentence 1) is true in these worlds

| likes | $\begin{aligned} & -1 \\ & 3 \end{aligned}$ | $\begin{aligned} & n \\ & 0 \\ & \hline \end{aligned}$ | 즑 | $\frac{\bar{D}}{\bar{X}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom |  |  |  |  |
| Spot | x |  |  |  |
| Kitty |  | x |  |  |
| Felix |  | x | x |  |


| likes | $\begin{aligned} & -1 \\ & 3 \end{aligned}$ | $\begin{aligned} & \text { n } \\ & \hline \end{aligned}$ |  | $\stackrel{\text { D }}{\text { D }}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom |  |  |  |  |
| Spot | x | x | x |  |
| Kitty |  | x | x |  |
| Felix |  | x | x |  |



| likes | $\stackrel{-1}{3}$ | O | $\begin{aligned} & \text { 즟 } \\ & \text { 俍 } \end{aligned}$ | $\frac{\Gamma}{\bar{\top}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom |  |  |  |  |
| Spot |  |  |  |  |
| Kitty |  |  | x |  |
| Felix |  |  |  |  |


| likes | $\begin{aligned} & -1 \\ & \frac{-1}{3} \end{aligned}$ | $\begin{aligned} & \underline{n} \\ & \stackrel{\sim}{\circ} \end{aligned}$ | 즟 | $\begin{aligned} & \frac{D}{\bar{x}} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom | x | x | x |  |
| Spot | x |  |  |  |
| Kitty |  | x |  |  |
| Felix |  | x | x |  |


| likes | $\stackrel{-1}{3}$ | $\begin{aligned} & \underset{\sim}{0} \\ & \hline \end{aligned}$ | 즟 | $\frac{\bar{D}}{\bar{x}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom |  | x | x |  |
| Spot | x | x | x |  |
| Kitty | x | x |  |  |
| Felix |  |  |  |  |


| likes | $\frac{-1}{3}$ | $\begin{aligned} & \text { n } \\ & \hline \end{aligned}$ | 즠 | $\frac{\text { D }}{\text { N }}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom | x |  | x |  |
| Spot | x |  |  |  |
| Kitty |  | x |  |  |
| Felix |  | x |  | x |


| likes | $\frac{-1}{3}$ | $\begin{aligned} & \text { n } \\ & \hline 0 \end{aligned}$ | 즠 | $\frac{\text { D }}{\text { D }}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom |  |  | x |  |
| Spot | x |  |  |  |
| Kitty |  | x |  |  |
| Felix |  | x | x |  |


| likes | $\begin{aligned} & -7 \\ & \frac{-1}{3} \end{aligned}$ | $\begin{aligned} & \mathrm{n} \\ & \stackrel{\sim}{\circ} \end{aligned}$ | 잢 | $\frac{\text { D }}{\text { D }}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom | x | x | x | x |
| Spot |  | x |  |  |
| Kitty |  |  | x |  |
| Felix |  |  |  | x |

## Sentences may be related

- Three sentences

1) "Every cat that likes Kitty likes Spot as well"
2) "Tom likes Kitty"
3) "Tom likes Spot"

There is no way to make true sentences 1) and 2) without making true sentence 3) as well... (just give it a try...)

Sentence 1) is true in these worlds

| likes | $\begin{aligned} & -1 \\ & 3 \end{aligned}$ | $\underset{\sim}{n}$ | $\overline{\text { 즟 }}$ | $\frac{\Gamma}{\bar{D}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom |  |  |  |  |
| Spot | x |  |  |  |
| Kitty |  | x |  |  |
| Felix |  | x | x |  |


| likes | $\begin{aligned} & -1 \\ & 3 \end{aligned}$ | $\begin{aligned} & \mathrm{n} \\ & \underset{\sim}{\circ} \end{aligned}$ | 즟 |  |
| :---: | :---: | :---: | :---: | :---: |
| Tom |  |  |  |  |
| Spot | x | x | x |  |
| Kitty |  | x | x |  |
| Felix |  | x | x |  |


| likes | $\stackrel{-1}{3}$ | $\begin{aligned} & \text { n } \\ & \hline 0 \end{aligned}$ | 잟 |  |
| :---: | :---: | :---: | :---: | :---: |
| Tom | x |  |  |  |
| Spot |  |  | x |  |
| Kitty |  | x |  |  |
| Felix |  |  |  | x |


| likes | $\begin{aligned} & -1 \\ & 3 \end{aligned}$ | $\begin{aligned} & n \\ & \underset{\sim}{n} \end{aligned}$ | 즟 | $\stackrel{\text { T }}{\text { N }}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom |  |  |  |  |
| Spot | x | X | X | x |
| Kitty |  |  | x |  |
| Felix |  | x | x |  |


| likes | $\stackrel{-1}{3}$ | $\stackrel{\sim}{0}$ | 京 | $\frac{\pi}{\frac{D}{X}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom |  |  |  |  |
| Spot |  |  |  |  |
| Kitty |  |  | x |  |
| Felix |  |  |  |  |


| likes | $\begin{aligned} & -7 \\ & \frac{-1}{3} \end{aligned}$ | $\begin{aligned} & \mathrm{n} \\ & \stackrel{\sim}{\circ} \end{aligned}$ |  | $\begin{aligned} & \frac{D}{\bar{x}} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom | x | x | x |  |
| Spot | x |  |  |  |
| Kitty |  | x |  |  |
| Felix |  | x | x |  |


| likes | $\frac{-1}{3}$ | $\begin{aligned} & \text { n } \\ & \hline \end{aligned}$ | $\begin{array}{\|c} \text { 즟 } \\ \hline \end{array}$ | $\frac{\text { D }}{\text { N }}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom |  | x | x |  |
| Spot | x | x | x |  |
| Kitty | x | x |  |  |
| Felix |  |  |  |  |


| likes | $\frac{-1}{3}$ | $\underset{O}{n}$ | 즠 | $\frac{\pi}{\bar{x}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom | x |  | x |  |
| Spot | x |  |  |  |
| Kitty |  | x |  |  |
| Felix |  | x |  | x |


| likes | $\frac{-1}{3}$ | $\begin{aligned} & \underline{\sim} \\ & \hline \end{aligned}$ | 즑 | $\frac{\text { D }}{\text { D }}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom |  |  | x |  |
| Spot | x |  |  |  |
| Kitty |  | x |  |  |
| Felix |  | x | x |  |


| likes | $\begin{aligned} & -7 \\ & \frac{-1}{3} \end{aligned}$ | $\stackrel{n}{0}$ | 즟 | $\frac{\text { D }}{\text { D }}$ |
| :---: | :---: | :---: | :---: | :---: |
| Tom | X | x | x | x |
| Spot |  | x |  |  |
| Kitty |  |  | x |  |
| Felix |  |  |  | x |

Sentence 2) is true in these worlds

## Sentences may be related

－Three sentences
1）＂Every cat that likes Kitty likes Spot as well＂
2）＂Tom likes Kitty＂
3）＂Tom likes Spot＂
There is no way to make true sentences 1）and 2）without making true sentence 3）as well．．． （just give it a try）

Sentence 1）is true in these worlds

|  | likes | $\frac{-1}{3}$ | $\frac{0}{0}$ | 즟 | $\begin{aligned} & \frac{T}{N} \\ & \frac{D}{\bar{x}} \end{aligned}$ | likes | $\frac{-1}{3}$ | $\frac{\pi}{0}$ | 晾 | $\begin{array}{\|l\|} \hline \frac{D}{x} \\ \frac{D}{x} \end{array}$ | likes | $\stackrel{-1}{3}$ | $\frac{\pi}{0}$ | 䂞 | $\stackrel{\text { ¢ }}{\text { ¢ }}$ | likes | $\begin{aligned} & -1 \\ & 3 \\ & \hline \end{aligned}$ | $\frac{n}{0}$ | 즟 | $\begin{aligned} & \text { T } \\ & \frac{D}{x} \end{aligned}$ | likes | $\stackrel{-1}{3}$ | $\stackrel{\sim}{0}$ | 京 | $\frac{\square}{\text { D }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\sim$ | Tom |  |  |  |  | Tom |  |  |  |  | Tom | x |  |  |  | Tom |  |  |  |  | Tom |  |  |  |  |
| $\bigcirc$ | Spot | x |  |  |  | Spot | x | x | x |  | Spot |  |  | x |  | Spot | x | x | x | x | Spot |  |  |  |  |
|  | Kitty |  | x |  |  | Kitty |  | x | x |  | Kitty |  | x |  |  | Kitty |  |  | x |  | Kitty |  |  | x |  |
| j | Felix |  | x | x |  | Felix |  | x | x |  | Felix |  |  |  | x | Felix |  | x | x |  | Felix |  |  |  |  |
| 2 | likes | $\frac{-1}{3}$ | $\frac{0}{0}$ | 즠 | $\frac{\Gamma}{\frac{D}{x}}$ | likes | $\frac{-1}{3}$ | $\begin{aligned} & \text { n } \\ & \hline 0 \end{aligned}$ | 茯 | $\frac{\pi}{\frac{D}{x}}$ | likes | $\begin{aligned} & -1 \\ & 3 \end{aligned}$ | $\begin{aligned} & \text { n } \\ & \hline \end{aligned}$ | 즟 | $\frac{\pi}{\frac{D}{x}}$ | likes | $\frac{-1}{3}$ | $\frac{n}{0}$ | 잙 | $\frac{\pi}{\frac{D}{x}}$ | likes | $\begin{aligned} & -1 \\ & 3 \end{aligned}$ | $\stackrel{\sim}{0}$ | 䂞 | $\frac{\grave{0}}{\overline{\bar{x}}}$ |
| ． | Tom | x | x | x |  | Tom |  | x | x |  | Tom | x | x | x |  | Tom |  |  | x |  | Tom | x | x | x | x |
| m | Spot | x |  |  |  | Spot | x | x | x |  | Spot | X |  |  |  | Spot | x |  |  |  | Spot |  | x |  |  |
|  | Kitty |  | x |  |  | Kitty | x | x |  |  | Kitty |  | x |  |  | Kitty |  | x |  |  | Kitty |  |  | x |  |
| $\underset{\sim}{\sim}$ | Felix |  | x | x |  | Felix |  |  |  |  | Felix |  | x |  | x | Felix |  | x | x |  | Felix |  |  |  | x |

## Sentences may be related

- Three sentences

1) "Every cat that likes Kitty likes Spot as well"
2) "Tom likes Kitty"
3) "Tom likes Spot"

There is no way to make true sentences 1) and 2) without making true sentence 3) as well...
(just give it a try)

## Questions:

Was this just coincidence?
How many possible worlds must we consider in order to be sure?

