Formal Logic

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Symbolic descriptions can *abstract*

- Symbolic descriptions have *abstraction* capabilities:
  for instance, many linguistic phenomena are *systemic*
  (i.e. their complexity goes beyond a *pattern-matching*)

Someone may understand English or not.

But no human being can understand the sentence:

"*Sally **likes** Harry*"

without being able to understand:

"*Harry **likes** Sally*"

or any other sentences of the kind:

"X ***likes*** Y"

where X and Y can be any nouns or definite descriptions:

"*Ronald's girlfriend **likes** the cat that Linda brought home yesterday*"

(freely adapted from [Fodor e Phylyshyn, 1988])

Reasoning and schemas: *syllogism*

Many (intuitively) valid arguments obey an abstract schema

All (*humans*) are (*mortals*)
All (*Greeks*) are (*humans*) hence
All (*Greeks*) are (*mortals*)

Abstract schema:

All ***H*** are ***M***
All ***G*** are ***H*** hence
All ***G*** are ***M***

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All ***H*** are ***M*** hence

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CAUTION!
The position of the line IS relevant:
in this case the schema does not work

Reasoning and schemas: *syllogism*

Many (intuitively) valid arguments obey an abstract schema

All (*humans*) are (*mortals*)
All (*Greeks*) are (*mortals*) hence
———
All (*Greeks*) are (*humans*)

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All ***H*** are ***M***
All ***G*** are ***M*** hence
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CAUTION!
The ordering of sentences IS relevant:
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Reasoning and schemas: *syllogism*

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All (*humans*) are (*mortals*)
All (*Greeks*) are (*humans*) hence

All (*Greeks*) are (*mortals*)

Abstract schema:

All ***H*** are ***M***
All ***G*** are ***H*** hence

All ***G*** are ***M***

The validity of schemas does not depend on *meaning*:

All (*enchanted frogs*) are (*princesses*)
All (*princesses*) are (*young and beautiful*) hence

All (*enchanted frogs*) are (*young and beautiful*)

Same schema,
different impression

Fallacies (*paralogisms*)

Wrong sequence:

All (*humans*) are (*mortals*)
All (*Greeks*) are (*mortals*)

All (*Greeks*) are (*humans*)

The last two sentences have been switched

Referential ambiguities:

(*Nothing*) is better than (*eternal happiness*)
(*A ham sandwich*) is better than (*nothing*)

(*A ham sandwich*) is better than (*eternal happiness*)

'Obscure' subtleties (obscure for now):

All (*enchanted frogs*) are (*princesses*)
All (*enchanted frogs*) are (*young and beautiful*)
There is an (*enchanted frog*) which is (*a young and beautiful princess*)

What do we mean by "all"? Do we need at least one specimen to say "all"?

In such case, the schema would be valid
(*in formal logic it is not, as we will see*)

What is the purpose of *formal* logic?

To distinguish correct reasoning from incorrect reasoning

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To distinguish correct reasoning from incorrect reasoning

by the formal structure

An aside: equations

- Solving quadratic equations

$$x^2 + ax + b = 0$$
$$x^2 + 2(a/2)x + a^2/4 - a^2/4 + b = 0$$
$$(x + a/2)^2 - a^2/4 + b = 0$$
$$(x + a/2)^2 = a^2/4 - b$$
$$x = -a/2 \pm (a^2/4 - b)^{1/2}$$

A sequence of steps: at each step a *transformation rule* is applied

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  From start to end

  Start point: a premise (*i.e. we assume the truth of something*)

  End point: a conclusion (*i.e. we state that something else must* <u>*also*</u> *be true*)

  (*both points are decided by us*)

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Sequence of steps

Each transformation rule is based on *equivalence*

Each step must be *correct*, in this particular sense

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Each transformation rule is based on *equivalence*

Each step must be *correct*, in this particular sense

Abstraction and *Correctness*

Symbols like x, a and b represent any number, *equivalences* are valid for *all* of them

What does it mean for the whole sequence to be *correct*?

What is *true*?

- **A world of *cats***

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot	x		x	
Kitty		x	x	
Felix			x	

- **Sentences about this world**

  "Spot likes Tom" and "Tom does not like Spot"
  "Tom likes himself"
  "Kitty likes Spot" and "Spot likes Kitty"
  "Kitty likes herself"
  "Felix likes Kitty"

  *All these sentences are **true**, in the world above*

What is *true*?

- **A world of *cats***

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot	x		x	
Kitty		x	x	
Felix			x	

- **Sentences about this world**

  "Spot likes Felix"
  "Spot likes himself"
  "Kitty does not like herself"
  "Felix likes Spot"

  All these sentences are ***false****, in the world above*

What is *true*?

- **A world of *cats***

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot	x		x	
Kitty		x	x	
Felix			x	

- **Sentences about this world**

  “Every cat likes a cat” *is **true***
  “Every cat likes another cat” *is **false***
  “Tom does not like any other cat” *is **true***
  “Kitty is liked by every cat” *is **false***

How to make a sentence *true*?

- **Consider the sentence**

  "Kitty likes Spot" and "Spot likes Tom"

  It can be made true in many different ways

  "It may be true in many different *possible worlds*"

  Examples:

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x			
Kitty		x		
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot	x		x	
Kitty		x	x	
Felix			x	

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	x
Spot	x	x	x	x
Kitty	x	x	x	x
Felix	x	x	x	x

How to make a sentence *true*?

- **Consider the sentence**

  "Kitty likes Spot" and "Kitty does not like Spot"

  There is no way to *make it true*

  "There is no *possible world* where this can be true"

Sentences may be related

- **Three sentences**

    1) “Every cat that likes Kitty likes Spot as well”
    2) “Tom likes Kitty”
    3) “Tom likes Spot”

    There is no way to make true sentences 1) *and* 2) *without making true sentence* 3) *as well…*

    (just give it a try…)

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	
Kitty		x	x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot			x	
Kitty		x		
Felix				x

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	x
Kitty			x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot				
Kitty			x	
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom		x	x	
Spot	x	x	x	
Kitty	x	x		
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x		x	
Spot	x			
Kitty		x		
Felix		x		x

likes	Tom	Spot	Kitty	Felix
Tom			x	
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Kitty		x		
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Tom	x	x	x	x
Spot		x		
Kitty			x	
Felix				x

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Sentence 1) is true in these worlds

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	
Kitty		x	x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom		x	x	
Spot	x	x	x	
Kitty	x	x		
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot			x	
Kitty		x		
Felix				x

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	x
Kitty			x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
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Kitty			x	
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likes	Tom	Spot	Kitty	Felix
Tom	x		x	
Spot	x			
Kitty		x		
Felix		x		x

likes	Tom	Spot	Kitty	Felix
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Tom	x	x	x	x
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Spot	x			
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Spot	x	x	x	
Kitty		x	x	
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Tom	x			
Spot			x	
Kitty		x		
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Tom				
Spot	x	x	x	x
Kitty			x	
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likes	Tom	Spot	Kitty	Felix
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Spot		x		
Kitty			x	
Felix				x

Sentence 2) is true in these worlds

Sentences may be related

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1) "Every cat that likes Kitty likes Spot as well"
2) "Tom likes Kitty"
3) "Tom likes Spot"

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(just give it a try)

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Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	
Kitty		x	x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom	x			
Spot			x	
Kitty		x		
Felix				x

likes	Tom	Spot	Kitty	Felix
Tom				
Spot	x	x	x	x
Kitty			x	
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom				
Spot				
Kitty			x	
Felix				

Sentence 3) is true in these worlds

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	
Spot	x			
Kitty		x		
Felix		x	x	

likes	Tom	Spot	Kitty	Felix
Tom		x	x	
Spot	x	x	x	
Kitty	x	x		
Felix				

likes	Tom	Spot	Kitty	Felix
Tom	x	x	x	
Spot	x			
Kitty		x		
Felix		x		x

likes	Tom	Spot	Kitty	Felix
Tom			x	
Spot	x			
Kitty		x		
Felix		x	x	

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Tom	x	x	x	x
Spot		x		
Kitty			x	
Felix				x

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  (just give it a try)

Questions:

Was this just coincidence?

How many possible worlds

must we consider in order to be sure?